Academy of Pedagogical Sciences
of the RSFSR

Elementary School

Teacher's Handbook

EDITED BY
PROF. M. A. MELNIKOV

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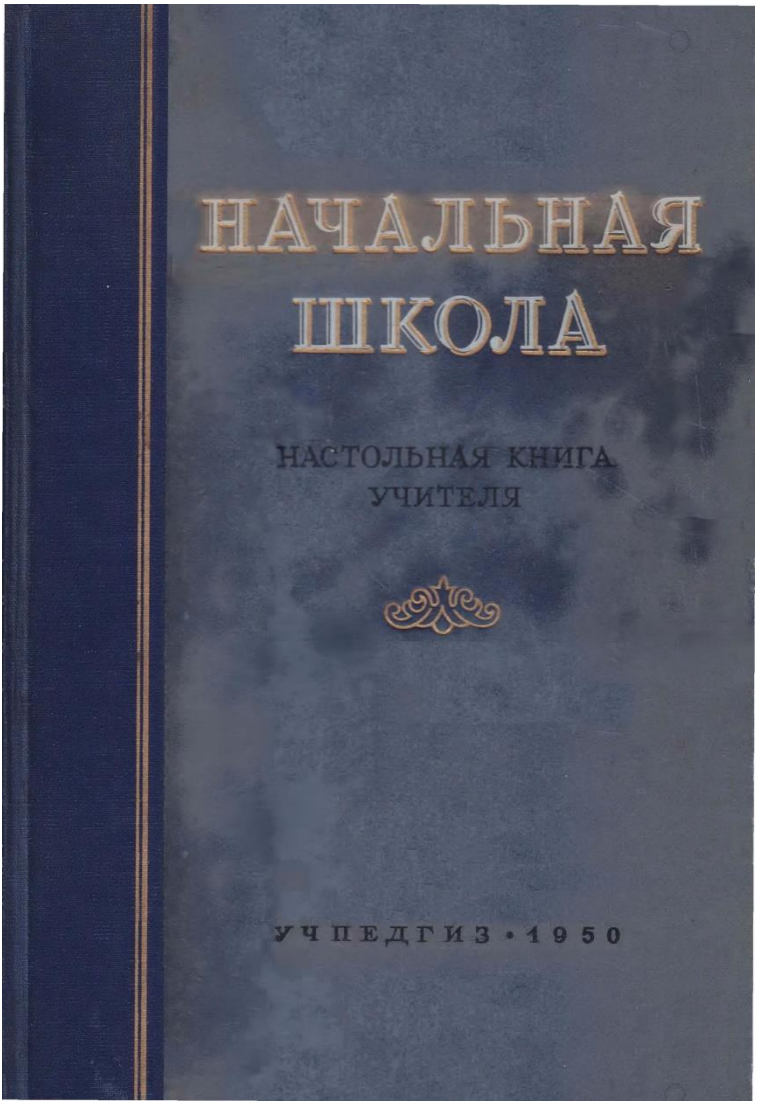
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НАЧАЛЬНАЯ ШКОЛА

НАСТОЛЬНАЯ КНИГА
УЧИТЕЛЯ

ПОД РЕДАКЦИЕЙ
ПРОФ. М.А. МЕЛЬНИКОВА

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MAIN QUESTIONS OF ARITHMETIC METHODOLOGY

TEACHING COUNTING AND PROBLEM SOLVING IN JUNIOR GRADES OF PRIMARY SCHOOL

Teaching counting and problem solving in grades **I** and **II** aims to equip children with a certain range of knowledge, skills and abilities, develop their logical thinking, instil in them the skills of independent work, accustom them to cleanliness and neatness, develop their activity and creativity, and promote the communist education of children.

Teaching arithmetic in the junior grades should be conducted with particular care and **methodical consistency**, since each subsequent stage in teaching counting and problem solving is based on the previous ones: actions in the limit of 20 are reduced to actions in the limit of 10, actions in the limit of 100 are reduced to actions in the limit of 20, teaching numbering precedes teaching calculations, etc.

Exceptionally great attention is paid to the formation of clear, distinct mathematical concepts in the minds of children. The formation of precise concepts can only be achieved on the basis of clear

visual representations. For better assimilation of arithmetic concepts by children, it is therefore necessary to widely use visual aids, choosing and using visual aids so that with their help, gradually moving from complete visual aids to partial ones, children are brought as quickly as possible to an understanding of the essence of a particular arithmetic operation.

The counting material used as visual aids should be varied, since under this condition the process of generalisation proceeds more successfully.

The development of independent work skills in children is of great importance in teaching arithmetic. In the junior grades, the development of these skills occurs in a strict methodological sequence with a gradual complication of the content and forms of work. First, the assigned exercise is performed on the blackboard under the guidance of the teacher. Then the assigned exercise is analysed in class, after which the children perform it in their notebooks under the supervision of the teacher. And only after that do the children perform similar exercises completely independently.

The development of children's mathematical thinking at this stage of education is facilitated by solving examples using various techniques.

Each arithmetic operation can be performed using several techniques. Let's take an example of subtraction in the limit of 20 with a transition through a dozen: 16-9. This example can be solved using a number of techniques:

1) $16 - 9 = 16 - (6 + 3) = 16 - 6 - 3 = 10 - 3 = 7$
2) $16 - 9 = (10 + 6) - 9 = 10 - 9 + 6 = 1 + 6 = 7$
3) $16 - 9 = (16 - 10) + 1 = 6 + 1 = 7$, etc.

The use of several techniques when solving one example gives children the opportunity to better understand the relationship between the data and the results of the action. But at the same time, it can also lead to the fact that children will not firmly learn any technique. Therefore, when studying each case of a particular action, one should choose the main technique, the easiest for children and suitable for a number of cases of the studied action. Children should learn this technique thoroughly, and only after that it is advisable to use other techniques, in every way encouraging those children who offer their own techniques for performing a given action, which is important for the development of their initiative, intelligence, and resourcefulness.

For the successful teaching of computational techniques, a rational selection of examples is of great importance. The latter should be selected so that the calculations required for their solution do not complicate the children, so that the students can fully concentrate their attention on the computational technique being studied.

Better assimilation of a new technique can be facilitated by so-called similar examples, which are solved using one technique and which also have a lot in common in the given numbers.

Let's take the case of subtracting a single-digit number from a two-digit number in the limit of 100 with a transition through ten. If we begin studying this operation with dissimilar examples (let's say: 32—5, 53—8, 61— etc.), then it will not be easy for children to learn the new technique. It will be learned much easier if we begin with similar examples, preceding each group

of such examples with a similar example of subtraction in the limit of 20. For example:

11 — 2	12 — 4	14 — 6
41 — 2	52 — 4	34 — 6
81 — 2	92 — 4	74 — 6 etc.

After solving similar examples, you can move on to solving dissimilar ones:

23 — 6; 45 — 8; 32 — 6; 81 — 5, etc.

Only after mastering a new technique can you move on to solving mixed simple examples that require the use of both the technique just learned and those previously learned, for example:

28 + 40; 32 — 9; 23 + 25; 64 —8 —4,

and, finally, to solving compound examples that include a new action in combination with previously studied ones, for example:

41 — 4 + 12; 63 — 7 — 24, etc.

If, when solving an example or problem, children have the opportunity to see the numbers, it is easier for them to complete the task than in the case when they perceive numerical data only by ear and are forced, as a result, to keep the numbers in memory.

Considering the difficulties that auditory perception of numerical data presents for children, at the beginning of learning a new calculation technique, they should be written on the board or examples from the

problem book should be used. The execution of an operation on numbers perceived by ear can be introduced only after the children have mastered the technique being studied. The execution of any calculation consists of replacing the operation on the given numbers with an operation on other numbers formed from the data. Children will more quickly understand and learn what parts the given numbers are divided into and in what order the operations on the resulting parts are performed if the oral explanation of the new technique is accompanied by a record of auxiliary calculations. Thus, when explaining the case of subtraction in the limit of 100 with a transition through a decade (for example: 62 — 34), it is advisable to supplement the oral explanation with the following entry:

62 — 34 = 28 [1]
62 — 30 = 32
32 — 4 = 28.

For a better understanding of the features of each action and the connections between them and for the development of children's thinking, it is useful to practice solving examples and problems that help to clarify the relationship between data and the results of actions, for example: 3 X 6; 6 X 3; 18 X 3; 18: 6. This goal can also be served by parallel study of direct and inverse actions, in which each case of inverse action (for example, subtraction) is studied following the

[1] The result of the action is recorded after performing auxiliary calculations.

corresponding case of direct action (for example, addition).

In order for teaching elementary arithmetic to promote the development of children's creative abilities, it is necessary, along with solving ready-made examples and problems, to exercise children in composing their own problems and examples, provided, however, that this does not lead to a violation of the systematic nature of teaching.

Taking into account the age characteristics of seven-year-old children, when studying new material, especially when reinforcing what was previously covered, great attention should be paid to entertaining exercises and games.

The first ten

The teaching system of elementary arithmetic should be based on the study of operations, not numbers. This applies to all sections of arithmetic, including the first ten.

But successful study of actions in the limit of 10 is possible only if children have a clear idea of each number and know how to designate it using digits. Therefore, the study of actions in the limit of 10 should be preceded by the study of numbers in this limit.

In the methodological literature, there are different points of view on the development of a child's numerical representations, which in turn determine the differences in the methods of studying the numbers of the first ten. Some authors believe that the basis for the formation of a child's numerical representations is the simultaneous perception of the number of objects under

consideration. Based on this, they recommend that each of the numbers of the first (and even the second) ten be presented to children in the form of a visual image of a numerical figure, which is a set of circles arranged in a certain way.

Other authors find that the child's numerical concepts develop on the basis of sequential counting of objects. According to these authors, when studying the numbers of the first ten, the main attention should be paid to counting objects.

Other authors propose to consider a number as a result of measurement (as a ratio).

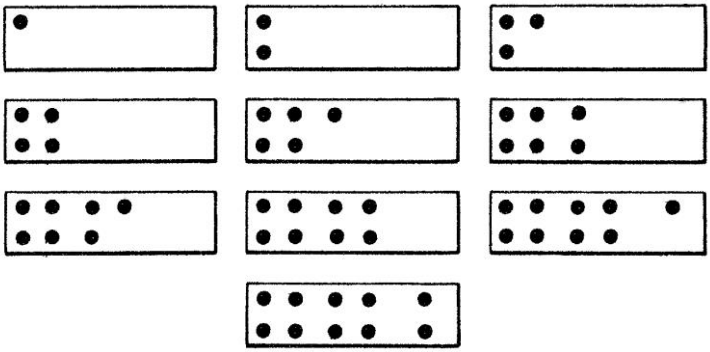
The fallacy of the first point of view is obvious. As studies have shown, children's ideas even about the numbers of the first five, not to mention larger numbers, become quite distinct only as a result of counting. The importance of counting for the development of children's numerical ideas is confirmed by numerous observations and experience of the Soviet school.

When studying the numbers of the first ten, one should therefore pay primary attention to **counting objects** , forming **a clear concept of a given number as a set of the corresponding concept of a given number** as a set of the corresponding number of units, **clarifying the place of the number in the numerical series, and clarifying the composition of each number.**

The perception of a number as a result of measurement can also be of undoubted benefit. This enriches the student's numerical representations, making them more complete and more general.

But at the same time, the use of numerical figures is also useful. With repeated perception of circles arranged in a certain way, their shape is imprinted in

memory, which helps children remember the composition of numbers and, thus, facilitates the assimilation of addition and subtraction tables. Thus, by repeatedly perceiving the numerical figure 8, the student gradually remembers that the number 8 consists of 4 and 4, 6 and 2, etc.



The study of the composition of each number is carried out on specific counting materials (circles, cubes, sticks) and on drawings.

As a result of this study, children should learn the composition of each number from two terms. Thus, as a result of studying the number 7, students should learn that 7 is 6 and 1, 5 and 2, 4 and 3, etc. Special attention should be paid to learning the composition of the number 10, without which it is impossible to successfully study addition with the transition through ten in the limit of 20.

The difficulty of addition depends mainly on the size of the second term, the difficulty of subtraction on the size of the subtrahend. Therefore, we should first consider the case of addition when the second term is 1, then the case when it is 2, etc. In the same way, we

should first consider the case of subtraction when the subtrahend is 1, then when it is 2, etc.

Observations show that when addition and subtraction are studied in parallel, when each case of subtraction is considered after a similar case of addition, children master the comparatively difficult action of subtraction much more easily than when these actions are studied separately, when subtraction is studied after all cases of addition have been considered. It is therefore advisable to study addition and subtraction in the limit of 10 in parallel (the case of subtraction 1 after the case of addition 1, the case of subtraction 2 after the case of addition 2, etc.).

The basic method **of addition** in the limit of 10 consists of successively adding to the first term as many units as there are in the second term. Thus, when adding 1, you need to add 1 unit to the first term, when adding 2, you need to successively add 2 units to it, when adding 3, you need to successively add 3 units (one unit each or 1 and 9 units), when adding 4, you need to successively add 4 units (one unit each or 2 and 2 units), etc.

When teaching addition to the limit of 10, it is useful, along with other visual aids (classroom abacus, cubes, sticks, circles, etc.), to use a table of numbers of the first ten, arranged horizontally and vertically, teaching children that adding one is a transition to the next number in the number series, adding two is a transition to the number behind the next number, etc.

When adding, it is easy to find the sum only in those cases where the second term contains a small number of units, since when adding a large number of them, it is easy to make a mistake about how many units have already been added and how many more are left to add.

Therefore, in those cases where the second term is greater than the first, for example: 2 + 5, 3 + 7, it is advisable, based on the commutative property, to rearrange the terms and add the smaller number to the larger number (add 2 to 5, add 3 to 7). In some cases, when solving a more difficult example (4+5), it is useful to turn to the closest easier example (4+4), having solved which, by appropriately changing the resulting sum (by adding 1 to 8), you can find the desired result of addition (9).

The basic technique of subtraction in the limit of 10 consists of successively counting from the minuend as many units as there are in the subtrahend. When subtracting one, one should count 1 unit from the minuend (or take the previous number in the numerical series), when subtracting 2, one should successively count 2 units, etc. Thus, when subtracting, one can successfully use the above table of numbers of the first ten. However, even when counting a relatively small number of units, it is easy to make a mistake in how many units have already been counted and how many remain to be counted. Therefore, when solving many subtraction problems, it is advisable to consider them as the inverse of the corresponding addition problems and find the remainder by selecting a number that, when added to the subtrahend, would give the minuend in sum (7—4)=3, because 3+4=7; 9—7=2, because 7+2=9, etc.).

It is obvious that children can successfully use this method of subtraction only when they have mastered the corresponding case of addition well. Therefore, one can move on to studying each case of subtraction only after thoroughly mastering the corresponding case of addition.

When considering a new case of addition or subtraction, it is advisable to explain the technique of this action first using visual aids (sticks, cubes, abacus, etc.), then using problems or examples with named numbers, and only after that move on to solving examples with abstract numbers.

Visual aids should be used in such a way as to facilitate the acquisition of the technique being studied to the maximum extent. Thus, when explaining the case of adding 2, it is necessary to strictly ensure that when solving examples using counting material (sticks, cubes, etc.), children add units of the second added to the first, and do not combine both groups of added objects and then count their total number, counting from 1, since in the latter case the use of visual aids does not facilitate the acquisition of the technique of this action.

When explaining actions, and especially when reinforcing them, **problem solving** should occupy a prominent place, since problems help children better understand the meaning of each action, facilitate the assimilation of computational techniques, and teach how to apply actions.

In the limit of 10, the easiest types of simple addition and subtraction problems are solved and children are prepared to solve problems in two steps.

Among the simple problems solved here are addition problems, in which you need to find a number equal to two given numbers taken together, and subtraction problems, in which you need to find the remainder.

Each of these types of problems is first considered separately, and then addition and subtraction problems are presented in a mixed order.

In order to help students better understand which problems are solved by addition and which by

subtraction, it is advisable to initially offer children problems with a common theme, one solved by addition and the other by subtraction, or vice versa, for example:

"There were 7 linden trees in front of the school. One linden tree was broken by a storm. How many linden trees are left?"

"There were 7 apple trees in the garden. In the spring, another one was planted. How many apple trees are there now in the garden?"

Achieving this goal can also be facilitated by having children practice creating addition and subtraction problems with the same numerical data, for example: creating a problem in which you need to add 2 apples to 7 apples (or add 2 to 7); creating a problem in which you need to subtract 2 apples from 7 apples (or subtract 2 from 7).

At first, the problems are offered in a ready-made form, then children gradually begin to be involved in composing the problems, while in some cases students are required to compose the problem completely, and in others—partially (to select the missing question or the missing numerical data).

Among simple problems, special attention should be paid to problems involving subtraction, when the remainder is zero, for example: "There were 3 books on the shelf. 3 books were taken off the shelf. How many books are left on the shelf?"

To prepare students to solve problems in two steps, it is useful to gradually introduce so-called "chains of simple problems," in which each subsequent problem is a continuation of the previous one, for example:

“There were 5 buckets of water in the barrel. Three more buckets were poured into it. How many buckets of water were there in the barrel?”

“To water the flowers, 4 buckets of water were taken from the barrel. How many buckets of water are left in the barrel?”

The solution of some problems in the limit of 10 is done orally. The students write down the solution of other problems in their notebooks. Writing down the solution helps the student better understand how the given problem was solved,

Second ten

When going through the numbering of numbers in the second ten, just as when studying numbers in the limit of 10, you should start by counting real objects (pencils, cubes, sticks, etc.) and only then move on to abstract counting.

Particular attention should be paid to clarifying the meaning of the names of the numbers of the second ten (eleven = one-by-ten, twelve = two-by-ten, etc.). For these purposes, at first, the units of the counting material should be placed not next to the ten, but on the ten. Later, when illustrating the numbers of the second ten, the units are placed next to the ten, but here too they should not be placed haphazardly, but so that they are located to the right of the ten, in accordance with the local value of the units of the first two digits.

When studying oral numeration in the limit of 20, exercises are carried out in forward and backward counting, in determining the place of a particular

number in the natural series (Between which numbers is the number 17? What number follows the number 13? What numbers are greater than 15, but less than 19? Which is greater: 14 or 13? How much greater is 14 than 13? etc.), exercises in group counting (counting by twos, fives), finding out the decimal composition of the numbers under consideration (how many tens and units does a given number consist of).

When teaching written numeration, it is useful to use a numeration table. Having put aside 11 sticks and having found out the decimal composition of the number 11, it is first written in the numeration table, and then outside the table. The written numeration of other numbers of the second ten is explained in the same way.

For a clearer representation of the natural series of numbers from 1 to 20, it is useful for children to look at these numbers on scale rulers, and it is even better if each student makes a ruler 20 cm long for himself. Using rulers as a visual aid when studying numeration, children, on the teacher's instructions, hold them not only horizontally, but also vertically, and read the numbers written on them not only forward, but also in reverse order.

When learning written numeration to 20, it is useful for children to write the numbers in two rows in addition to writing them in a continuous row, as follows:

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20.

Practicing writing and reading numbers arranged in this way will help children better understand the

relationship between the numbers in the first and second tens.

with addition and subtraction in the limit of 20 in parallel, considering individual cases of subtraction following the corresponding cases of addition, while it is necessary to single out in special stages only those cases that differ in the methods of calculation. Guided by these provisions, addition and subtraction in the limit of 20 can be divided into the following stages:

1. Addition and subtraction without going through tens:

a) addition of a single-digit number to 10, for example: 10 + 6; 6 + 10;

b) subtraction of a full [1] two-digit number from its ten or units, for example: 14 — 10; 14 — 4;

c) adding a single-digit number to a full two-digit number and vice versa, for example: 12 + 3; 12 + 8; 3 + 12; 8 + 12;

d) subtraction of a single-digit number from a full two-digit number, for example: 15 — 3;

d) subtracting a single-digit number from 20, for example: 20 — 4;

e) subtraction of a full two-digit number from a two-digit number, for example: 19 — 13; 20 — 14;

2. Addition and subtraction with transition through tens:

a) addition with transition through ten, for example: 9 + 2; 8 + 4;

b) subtraction with transition through ten, for example: 11 — 3; 13 — 5.

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[1] A complete two-digit number will be defined as a number represented by two significant digits, for example: 16, 19, 24.

The technique for adding a single-digit number to 10 is based on knowledge of numbering and consists of combining the given decimal groups into one number, for example: 10 + 8 = 18.

The method of subtracting a ten or units from a full two-digit number consists of decomposing the minuend into two decimal groups, from which one of the groups is then subtracted, for example:

$$18 - 8 = (10 + 8) - 8 = 10;$$
$$18 - 10 = (10 + 8) - 10 = 8.$$

When adding without going through ten, first add the units, then add their sum to 10, for example: 12+3; 10+2+3; 2+3=5; 10+5=15. So, 12+3=15. Adding a single-digit number to a two-digit number is reduced, based on the commutative property, to adding a two-digit number to a single-digit number.

When subtracting a single-digit number from a full two-digit number without going through tens, the units of the subtrahend are subtracted from the units of the minuend and the resulting remainder is added to 10, for example: 18—6; 8—6=2; 10+2=12. Thus, 18—6=12.

When subtracting a single-digit number from 20, one ten is taken from the minuend, the units of the subtrahend are subtracted from it, and the resulting remainder is added to 10, for example: 20—4; 10—4=6; 10+6=16. So, 20—4=16.

When subtracting a full two-digit number from a two-digit number, first subtract ten from the minuend, then the units of the subtrahend, for example: 18—13; 18—10=8; 8—3=5. So, 18—13=5.

When adding through a ten, the first term is added to 10, then the remaining units of the second term are

added to the resulting ten, for example: 9 + 4; 9 +1 + 3; 9 + 1 = 10; 10 + 3 = 13. So, 9 + 4 = 13.

When subtracting by a ten, the units of the subtrahend are subtracted from the minuend, then the remaining units of the subtrahend are subtracted from the resulting ten, for example: 14—5; 14—4—1; 14—4=10; 10—1=9. So, 14—5=9.

Along with teaching the above calculation techniques, students should be encouraged to find the results of the operation by moving to the right or left along the number line in easy cases, such as adding or subtracting 1 and 2. When solving subtraction problems where the remainder is small, it is useful for children to find the results by comparing the places that the given numbers occupy in the number line, for example: 20 — 18 = 2, because 20 is 2 units away from 18 (in other words, because 20 is 2 more than 18).

In addition to the basic techniques listed above, when adding and subtracting within the limit of 20, it is useful to use the following additional techniques:

1. Rearrangement of terms. For example, instead of the action 3 + 8, the action 8 + 3 is performed.

2. Reducing difficult addition cases to the closest easier ones. Thus, when solving the example 7 + 8, add 7 + 7 and add one to the resulting sum.

3. Rounding off the addends. For example, when adding 7 and 9, add 7 and 10 and subtract one from the resulting sum.

When repeating addition and subtraction in the limit of 20, it is useful, along with solving subtraction problems when the remainder is zero (let’s say 12 - 12), to practice children solving addition problems when one of the terms is zero (let’s say 5 + 0; 14 — 0).

The methods of addition and subtraction should be explained using visual aids (sticks, cubes, abacus, etc.), and the aids should be used in such a way that their use facilitates the acquisition of the method being studied. Thus, when solving the example 13 + 4, children using sticks should find the required sum not by counting all the sticks being added, but in the way that follows from the method of this action, i.e. 3+4 = 7; 10+7=17.

In parallel with teaching addition and subtraction, much attention should be paid to children memorising the tables of these operations. However, it is sufficient to memorize only the addition table, since with thorough mastery of the latter, children simultaneously master the subtraction table, as an operation inverse to addition.

Multiplication in the limit of 20 can be performed using various techniques, of which the following are most frequently used: a) Successive addition of equal terms, for example:

$$5 \times 4 = 5 + 5 + 5 + 5 = 20.$$

b) Decomposition of the factor into terms, multiplication of the multiplicand by these terms and addition of the resulting incomplete products, for example:

$$3 \times 6 = (3 \times 3) = (3 \times 3) = 9 + 9 = 18.$$

c) Permutation of factors, for example:

$$2 \times 9 = 9 \times 2 = 18.$$

The main technique of table multiplication should be considered the successive addition of equal terms.

When using this technique, however, it is easy to make a mistake in the case where the number of equal terms is relatively large. Therefore, as the first technique is mastered, children should be introduced to the second technique, which makes it possible to reduce difficult cases of multiplication to easier ones.

For these reasons, the use of the third technique is also useful.

Each of the indicated methods of multiplication in the limit of 20 should be clarified using visual aids.

While striving to consciously master the techniques of multiplication, one should at the same time pay serious attention to memorising the table of this operation.

Division in the limit of 20 can be performed using the following two techniques:

a) Successive subtraction. So, in order to divide 12 feathers equally between 4 boys, each of them is first given 1 feather, then another 1, etc., until all the feathers are divided.

b) Finding a number that, when multiplied by a divisor, would give the dividend in the product. For example: 12 : 4 = 3, because 3 x 4 = 12.

When going through division, you should start with the first technique, then move on to the second, with each technique being clarified using visual aids by dividing a given number of objects (sticks, pencils, feathers, etc.) into equal parts.

To better understand the second division technique, it is useful to solve inverse examples, such as the following: 2 x 6; 6 x 2; 12: 2; 12: 6.

Of the two types of division—division by content and division into parts—in the limit of 20, only the second type is considered, as easier.

Just as when passing the first ten, together with teaching actions, the teacher systematically exercises children in **solving problems.**

In the second ten, new types of simple tasks and compound problems in two steps are introduced.

New types of simple problems are introduced partly in the process of learning addition and subtraction, partly in the process of learning multiplication and division.

When going through addition and subtraction, simple problems are considered that require:

a) increase the given number by several units,

b) reduce the given number by several units.

When introducing problems that require increasing a given number by several units, it is advisable to start with practical tasks, such as the following: “Give Misha 5 cubes, and give Kolya 2 cubes more.” In this case, you need to require the children to actually complete the task, bringing them to the understanding that Kolya needs to be given as many cubes as Misha, plus 2 cubes more. Drawing-related tasks are also useful here, for example, draw 4 tomatoes (4 circles, etc.) on one line, and 3 tomatoes (circle) more on the other.

In the same way, children should be introduced to problems that require decreasing a given number by several units.

For multiplication in the limit of 20, one should first solve problems in which the choice of action is clearly suggested by their conditions, for example:

“Misha brought 2 logs of firewood from the barn 3 times. How many logs did he bring in total?”

Due to the peculiarities of the presentation of the conditions of this problem, it is easy to understand that to solve it you need to take 2 3 times.

Similar requirements should be made for the formulation of the conditions of the first division problems. The following problem can serve as an example:

“Two boys caught 12 perches and divided them equally between themselves. How many perches did each boy get?”

The conditions of subsequent problems on multiplication and division, of course, should be presented in such a way that the choice of action is not so clearly suggested by the text of the problem, for example:

“3 rows of apple trees were planted in the garden, 6 apple trees in each row. How many apple trees were planted in the garden in total?”

Or: “18 apple trees were planted in the garden in 3 rows, equally in each row. How many apple trees were planted in each row?”

Particular attention should be paid to solving problems on finding half of a given number. The concepts of this share should be carefully clarified using visual aids (circles, strips of paper, sticks, cubes, etc.).

Two-step problems are introduced in the process of studying addition and subtraction and initially cover only those types of simple problems that are considered in the “First Ten” concentration. Later, the types of simple problems considered in the “Second Ten” concentration are gradually introduced.

It is advisable to presuppose the solution of the first two-step problems by solving the corresponding chains of simple problems. Thus, the compound problem: “Vanya had 8 kopecks, his mother gave him another 10 kopecks. He spent 12 kopecks on buying a pencil. How

many kopecks does Vanya have left?" is useful to presuppose the following chain of two simple problems:

"Vanya had 8 kopecks. His mother gave him another 10 kopecks. How much money did Vanya have?"

"Vanya spent 12 kopecks on buying a pencil. How much money does he have left?"

In the future, compound problems are solved without the help of preparatory simple problems.

The first hundred

When studying numeration in the limit of 100, it is necessary to bring to children's consciousness the expediency of singling out ten as a counting unit. Exercises in counting the same number of sticks (for example, 50) first by units, then by tens can help achieve this goal. Exercises in parallel counting of a certain number of simple units (for example, 8 sticks) and the same number of tens (8 tens of sticks) are also useful.

When teaching **oral** numeration, one should find out the decimal composition of the numbers being considered as often as possible.

For better assimilation of **written** numeration, it is useful to use a meter, an abacus and a numeration table. On a meter divided into centimetres, children count from 1 to 100, practice finding certain numbers (certain numbers of centimetres).

When studying written numeration, children set aside a certain number of cubes or sticks (for example, 43) indicated by the teacher, then depict this number on the abacus, write it down in the numbering table and outside of it.

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When going through written numeration, first take numbers consisting of tens and units, then numbers consisting of only tens. In order to clarify children's understanding of the natural series of numbers in the limit of 100, it is advisable to practice students writing down the numbers of the first hundred in their notebooks in the following way:

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 etc.

It is advisable to arrange the numbers of the first hundred arranged in this way in the form of a classroom wall chart, on which children, at the teacher's instruction, practice quickly finding certain numbers, for example, numbers: 7, 17, 27, 37... 97; 94, 84, 74... etc. This table can be used later when studying operations with numbers of the first hundred.

Here are some examples of the corresponding exercises: "Add 4 to each of the numbers in the third vertical row (third column), subtract 4 from each of the numbers in the sixth column, divide the numbers in the fifth column by 5 (count in fives)," etc.

Following the numeration, operations on round tens are studied. Operations on round tens can be performed in two ways, depending on whether we are considering the composition of these numbers from simple units or from tens. Thus, to perform the operation 40 + 20, we can: a) add 20 units to 40 units and b) add 2 tens to 4 tens. The second method is obviously easier than the first and should be used primarily when performing operations on round tens.

In order to facilitate the assimilation of this technique, it is advisable to consider similar examples in the limits of 10 and 100 in parallel when explaining each action, for example:

4 + 2	4 dec. + 2 dec.	40 + 20
8 — 2	8 dec. — 2 des.	80 — 20
2 x 3	2 dec. x 3	20 x 3
8 : 2	8 dec. : 2	80 : 2 etc.

Of the operations involving round tens, multiplication and division are more difficult, especially the latter. To prepare students for learning these operations, it is useful to practice group counting exercises when learning addition and subtraction, for example:

Add 20 to 20 until you get 100.

Subtract 80 from 90 until you get 0.

When working with round tens, especially when working with multiplication and division, it is useful to use visual aids (sticks, cubes, etc.).

The first grade program ends with the study of operations with round tens.

In the 2nd grade, after a thorough review of what was covered in the 1st grade, children begin to study

addition and **subtraction** in **the limit** of 100. Addition and subtraction in the limit of 100 cover the following cases, which differ in the methods of calculation:

1. Addition and subtraction without going through tens:

1) Adding a single—digit number to round tens, for example: 50 + 6; 6 + 50.

The addition technique is based on knowledge of numbering and consists of combining the data of decimal groups (50 and 6) into one number.

2) Subtraction of units or tens from a full two-digit number: 56 — 6; 56 — 50.

The subtraction technique consists of decomposing the minuend (56) into decimal groups (50 and 6), from which one of the groups is subtracted.

3) Addition of a full two-digit number to a single-digit number when the sum of the units of the addends is less than or equal to 10, for example: 26 + 2; 2 + 26; 26 + 4; 4 + 26.

When performing addition in this case, the units of the terms are added together and the resulting sum is added to the tens of the two-digit term, for example:

$$26 + 2 = 20 + (6 + 2) = 20 + 8 = 28;$$
$$26 + 4 = 20 + (6 + 4) = 20 + 10 = 30.$$

4) Addition of a full two-digit number with round tens, for example: 56 + 20; 20 + 56.

In this case, the tens (50 and 20) are added and the units (6) are added to the resulting sum (70).

5) Addition of complete two-digit numbers when the sum of the units of the addends is less than or equal to 10, for example: 26 + 32; 26 + 34.

In this case, the basic addition technique is to successively add the tens and units of the second term to the first term, for example:

$$26 + 32 = 26 + 30 + 2 = 56 + 2 = 58;$$
$$26 + 34 = 26 + 30 + 4 = 56 + 4 = 60.$$

6) Subtraction of a single-digit number from a full two-digit number, when the units of the minuend are greater than the units of the subtrahend, for example: $38 - 2$.

In this case, the units of the minuend (8) are subtracted from the units of the subtrahend (2) and the resulting remainder (6) is added to the tens of the minuend (30).

7) Subtracting round tens from a full two-digit number, for example: $56 - 20$.

When performing subtraction in this case, the subtrahend (20) is subtracted from the tens of the minuend (50) and the units of the minuend (6) are added to the resulting remainder (30).

8) Subtraction of full two-digit numbers from full two-digit numbers, when the units of the minuend are greater than the units of the subtrahend or equal to them, for example: $56 - 24$; $56 - 26$. In this case, the tens and units of the subtrahend are successively subtracted from the minuend, for example:

$$56 - 24 = 56 - 20 - 4 = 36 - 4 = 32;$$
$$56 - 26 = 56 - 20 - 6 = 36 - 6 = 30.$$

9) Subtracting a single-digit number from round tens, for example: $60 - 2$.

In this case, one ten is taken from the minuend (60), the subtrahend (2) is subtracted from it, and the resulting remainder (8) is added to the remaining part of the minuend (50).

10) Subtracting a full two-digit number from round tens, for example 60 — 32.

In this case, tens and units of the subtrahend are successively subtracted from the minuend, for example:

$$60 - 32 = 60 - 30 - 2 = 30 - 2 = 28.$$

2. Addition and subtraction with transition through tens .

1) Addition of a single-digit number to a two-digit number, for example: 28 + 6; 6 + 28.

In this case, the two-digit addend is completed to round tens, to which the remaining units of the one-digit addend are then added, for example:

$$28 + 6 = (28 + 2) + 4 = 30 + 4 = 34;$$
$$6 + 28 = (28 + 2) + 4 = 30 + 4 = 34.$$

2) Addition of two-digit numbers, for example: 28 + 24.

The main technique in this case is to successively add the tens and units of the second term to the first term, for example:

$$28 + 24 = 28 + 20 + 4 = 48 + 4 = 52.$$

3) Subtracting a single-digit number from a two-digit number, for example: 44 — 6.

In this case, a part of the subtrahend equal to the units of the subtrahend is subtracted from the minuend, and the remaining unsubtracted units of the subtrahend

are subtracted from the resulting remainder, for example:

$$44 - 6 = (44 - 4) - 2 = 40 - 2 = 38.$$

4) Subtraction of two-digit numbers, for example:
42 — 24.

In this case, tens and units of the subtrahend are successively subtracted from the minuend, for example:

$$42 - 24 = 42 - 20 - 4 = 22 - 4 = 18.$$

When adding and subtracting within the limit of 100, after mastering the above techniques, it is useful to use the following techniques:

1) Bitwise addition, for example:

$$28 + 6 = 20 + (8 + 6) = 20 + 14 = 34;$$
$$28 + 24 = (20 + 20) + (8 + 4) = 40 + 12 = 52.$$

2) Rounding off one of the terms, for example:

$$35 + 19 = 35 + 20 - 1 = 55 - 1 = 54.$$

3) Rounding off the subtrahend, for example:

$$53 - 19 = (53 - 20) + 1 = 33 + 1 = 34.$$

Some of the above techniques for addition and subtraction in the limit of 100 are similar to the techniques used for addition and subtraction in the limit of 20. Thus, the action 56 + 3 is performed **basically in** the same way as 16 + 3, 80 — 4, as 20 — 4; 42 — 9, as 12 — 9; 60 — 12, as 20 — 12, etc. The consideration of

individual cases of addition and subtraction in the limit of 100 should therefore be preceded by a repetition of the corresponding actions in the limit of 20, then transferring the technique familiar to children to a new range of numbers.

In addition to relying on previously learned techniques, successful learning of addition and subtraction in the limit of 100 is facilitated by a detailed oral explanation of the techniques used when solving the first examples for each new case of these actions, as well as a detailed record of the intermediate calculations that make up the execution of a given action.

When studying addition and subtraction in the limit of 100, special attention should be paid to exercises that, while simultaneously reinforcing these actions, help prepare for studying multiplication and division in this limit, for example:

a) add 8 to 8 until you get 80;

6) 49 + 49; 27 + 27 + 27; 23 + 23 + 23 + 23;

c) subtract 8 from 80 until you get 0.

Table multiplication in the limit of 100 is performed using the same techniques as multiplication in the limit of 20, and, as in the limit of the second ten, the main technique is the successive addition of equal terms, and the remaining techniques (decomposition of the factor into terms and rearrangement of the factors) are used as additional ones.

For the conscious acquisition of the basic technique of table multiplication, exercises in group counting (in counting by twos, threes, fours, fives, etc.) are useful. The acquisition of this technique is also facilitated by the use of visual aids and a detailed record of the calculations performed, for example:

6 + 6 = 12..... 6 x 2 = 12
6 + 6 + 6 = 18..... 6 x 3 = 18
6 + 6 + 6 + 6 = 24..... 6 x 4 = 24 etc.

The use of visual aids can be recommended when considering other methods of table multiplication.

It is advisable to use the decomposition of a factor into terms as a multiplication technique when the factor is greater than 5 and when, as a result, it is easy to make a mistake when using the technique of successive addition.

The permutation of factors should be used very widely: when going through each multiplication table, it is necessary to find out which equalities of this table can be reduced to those encountered earlier by permuting factors. By using this technique, the number of equalities to be remembered is reduced almost in half.

Additional multiplication techniques include rounding off one of the factors. Thus, when multiplying 6 by 9, you can multiply 6 by 10 and subtract 6 from the resulting product (60). When multiplying 9 by 4, you can multiply 10 by 4 and subtract 4 from the resulting product (40).

While caring about the conscious acquisition of multiplication techniques, one should at the same time ensure that students know the multiplication table firmly by heart and resort to finding results using the studied techniques of this action only in those cases when they forget one or another result.

Tabular division in the limit of 100 is studied in parallel with the corresponding cases of multiplication. When dividing in the limit of 100, the same techniques

can be used as when dividing in the limit of 20. Of these techniques, the main one is the selection of a number that, when multiplied by the divisor, would yield the dividend in the product.

The study of each case of division should be started only after a thorough mastery of the corresponding case of multiplication.

When explaining division, each example of division should first be preceded by a corresponding example of multiplication: 6 X 4; 24: 6.

In the following, division examples are offered without preparatory multiplication examples, but when solving them, you should practice checking the obtained result using multiplication as often as possible.

When teaching children non-table multiplication, two cases of multiplication are distinguished: a) by a single-digit number and 6) by a two-digit number. In the first case, the tens of the multiplicand are multiplied by the single-digit number, then its units and the resulting products are added. In the second case, the single-digit number is first multiplied by the tens of the factor, then by its units and the resulting products are added. It is easy to see that in both cases, essentially the same technique is used, especially since by rearranging the factors, the second case can be reduced to the first.

When going through non-table multiplication, it is necessary to observe a strict sequence in the transition from easy to difficult examples, first taking examples in which the product of the units of the multiplicand by the units of the factor is less than 10 (for example: 11 x 9, 12 x 4, 32 x 3; or 9 x 11, 4 x 12, 3 x 32), then examples in which this product is equal to 10 or another "round" number [1] (for example: 12 x 5, 25 x 4; or: 5 x 12, 4 x 25) and, finally, examples in which this product

is a complete two-digit number (for example: 12 x 6, 25 x 3; or: 6 x 12, 3 x 25).

Non-tabular **division** includes division: a) by a single-digit number and 6) by a two-digit number.

The basic method of non-tabular division by a single-digit number consists of decomposing the dividend into terms, each of which is divided by the divisor, for example:

85 : 5 = (50+35) : 5 = (50 : 5)+(35 : 5) = 10+7 = 17.

For better assimilation of this technique, it is useful to first take such examples in which the units of each digit of the dividend are divided without remainder by the divisor, for example: 26 : 2, 48: 4, 55: 5, and only then move on to solving examples in which the tens of the dividend are not divided without remainder by the divisor, for example: 64: 4, 96: 6. When solving the first examples of division, especially in more difficult cases, it is useful to use clarity, dividing a given number of sticks, cubes, etc. into the required number of equal parts.

The basic method of dividing by a two-digit number consists of finding the quotient by multiplying it by the divisor. This method, which is also used in tabular division, is not new to children; but its use in non-tabular division is difficult for many children: they often succeed in finding the quotient only after many trials. To make it easier to find the quotient in non-tabular division by a two-digit number, it is advisable to start with examples that give small numbers in the quotient,

[1] A number ending in one or more zeros is usually called “round”.

for example: 32:16; 72:24. In this case, few trials are required to find the quotient, even if all the numbers are tried in order, starting with 2.

Then you should move on to solving examples with large quotients, for example: 84: 12; 95: 19; 98: 14, etc., and at first it is recommended to solve small groups of similar examples with the same divisor, for example: 42: 14; 70: 14; 98: 14; 84: 14 or: 60: 12; 96: 12; 72: 12. Sometimes it is useful to precede the solution of a given group of division examples with the solution of the corresponding multiplication examples. For example, before solving the above examples with a divisor of 14, you can do the following preparatory exercises: “Add 14 to 14 until you get 98. How much do you get if 14 x 2? 14 x 3? 14 x 4? 14 x 5? 14 x 6? 14 x 7?”

When repeating division within the limit of 100, students practice finding multiples and divisors of a given number by completing the following tasks: “Name the numbers (no more than 100) that are divisible without remainder by 11, 12, 13, 14, etc. What numbers are 45? 28? 60?” etc. divisible without remainder by.

When studying division, it is necessary to introduce children to division with a remainder of [1].

Division with remainder is widely used in written division of multi-digit numbers. Success in learning the latter therefore depends to a large extent on mastering division with remainder within the limit of 100.

When going through division with remainder, it is advisable to first solve examples with one divisor, say, several examples with divisor 2, then several examples with divisor 3, etc. In this case, it is sometimes useful to

[1] According to the current program, vie tabular division with remainder is studied in grade 3.

precede an example of division with remainder with a corresponding example of division without remainder, for example: 16: 2; 17: 2 or: 27: 3; 29: 3.

During exercises in solving problems at this stage of training, the following new types of simple problems are introduced: when studying addition and subtraction - problems on difference comparison and problems in which, given a subtrahend and remainder, it is necessary to find the minuend. When studying multiplication and division - problems on division by content, on increasing and decreasing a given number several times, on finding a part of a number, on multiple comparison. In addition, compound problems are solved, which are various combinations of types of simple problems familiar to children .

When explaining a problem that requires finding the minuend given a subtrahend and a remainder (for example: “A brother gave his sister 2 feathers and after that he had 5 feathers left. How many feathers did the brother have at first?”), it is useful to dramatize the condition, achieving a clear understanding from the children that at first the brother had both the 2 feathers that he gave to his sister and the 5 feathers that were left to him.

When considering new types of simple problems, much attention should be paid to the conscious assimilation by students of the meaning of the corresponding mathematical concepts (difference and multiple comparison, increase and decrease by several times, etc.). The achievement of this goal can be facilitated by the wide use of visual aids when solving the first problems. Thus, it is advisable to begin the consideration of problems on difference comparison with a comparison of the number of cubes in two

adjacent columns, with a comparison of the length of two sticks, two strips of paper, strings, etc.

When considering the case of division by content, it is useful to solve practical problems, for example: by superposition, find out how many times a smaller stick (or strip of paper) will fit into a larger one; find out how many times a given box contains 2 pencils, how many times it contains 5 pens; divide 12 pencils (sticks) between several students so that each gets 2 pencils.

At first, such problems are solved orally, without notes, then it is found out by what action they were solved, and a record of the solution is entered.

After mastering division by content, a comparison of this case of division with the case of division into equal parts is carried out, for which it is useful to solve problems with the same subject matter, selected so that one problem includes a case of division into equal parts, and the other — a case of division by content or vice versa. For example: “18 sheets of paper were divided equally between 3 students. How many sheets of paper did each student receive?”, “18 notebooks were distributed to several students, each with 3 notebooks. How many students were given notebooks?”

To help students better understand the difference between the two division cases, it is helpful to role-play these problems by actually doing what each problem describes.

The other types of simple problems mentioned above should also be explained with great care.

In the first hundred, compound problems in 2—3 steps are solved, mainly reduced ones. Some attention should also be paid to unreduced problems. To facilitate the solution of the latter, it is sometimes advisable to solve a reduced and unreduced problem with a similar

structure side by side, for example: “3 boys were fishing. The first caught 19 fish, the second caught 5 fish more than the first, and the third 2 times less than the second. How many fish did the third boy catch?” (reduced problem).

“3 girls were picking mushrooms. The second girl found 7 more porcini mushrooms than the first, and the third girl found 2 times less than the second. How many porcini mushrooms did the third girl find if the first girl found 25 porcini mushrooms?” (problem not given).

The number of compound problems solved within the first hundred includes problems of reduction to unity, for example:

“They paid 75 rubles for 5 meters of fabric. How much should I pay for 3 meters of this fabric?”

“3 cups cost 15 rubles. How many of these cups can you buy for 40 rubles?”

For a better understanding of the method of solving these, as well as other complex problems, it is advisable to conduct an analytical analysis of them, finding out whether it is possible to immediately solve the main question of the problem, what data is missing for this, and how to obtain this data.

Sometimes it can be useful to dramatize the conditions of the problem, to depict its content in person. Thus, when considering the conditions of the above problem, you can call two students to the board, one of whom portrays a buyer who bought 5 m of fabric for 75 rubles, and the other — a buyer who wants to buy 3 m of such fabric. With the help of questions addressed to the “buyers”, it is found out how many meters of fabric the first of them bought, how much money he paid for the purchased fabric, how many meters of

fabric the second “buyer” wants to buy, what he needs to count.

Numbering and actions within 1000

In the limit of 1000, as in the limit of 100, oral numeration is studied first, then written. It is advisable to precede the study of oral numeration in the limit of 1000 with a repetition of numeration in the limit of the first hundred. Students are asked to count a given number of sticks (cubes), for example: 60. First, they count them one by one. Then it turns out that it is much easier to count the sticks if every 10 pieces are combined into a ten. When subsequently counting by tens, it turns out that a ten is also a unit, only not a simple one, but a composite one, and that tens are counted in the same way as units.

In approximately the same way, students are introduced to a new unit of counting—a hundred. Students are asked to count a given number of sticks, connected in bundles (tens), for example: 40 tens. At first, the count is done in tens (ten, twenty, thirty, forty, etc.). Then it turns out that it is easier to count the sticks if every 10 tens are combined into a hundred. When counting in hundreds, students’ attention is drawn to the fact that a hundred, like a ten, is a composite unit and that hundreds are counted in the same way as units.

In order to make the concept of a hundred as a unit of counting as clear as possible, exercises should be given simultaneously in counting 5 units, 5 tens and 5 hundreds of sticks; 3 units, 3 tens and 3 hundreds; 7 units, 7 tens and 7 hundreds, etc.

Following exercises in counting by round hundreds, there are exercises in counting first by hundreds and tens, then by hundreds, tens and units, and it is important to strictly ensure that when illustrating these numbers with the help of visual counting material, units are invariably placed in the first place on the right hand, tens in the second and hundreds in the third.

The counting is done first on sticks (cubes), then abstractly. In both cases, the decimal composition of the numbers under consideration is clarified as often as possible (how many hundreds, tens and units they consist of). When teaching numeration, a tape measure can be used, on which children find a certain number of centimetres indicated by the teacher, for example: 145 cm, 270 cm, etc.

When studying written numeration, these numbers are made up of sticks or cubes and written in a numbering table and in parallel outside of it.

Hundreds	Tens	Units
2	3	6

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In the future, illustrating these numbers with the help of visual counting material and writing them in a numbering table is practiced only in cases where students have difficulty mastering written numbering.

In addition to the numbering table, an abacus can be used when going through written numbering.

Better assimilation of written numbers is also facilitated by students exercising in depicting these numbers on an abacus (class or trade).

When teaching written numeration, numbers are first taken that are represented by significant digits, for example: 236, 428, etc.; then numbers that have one or two zeros in their representation.

Addition and subtraction of three-digit numbers is performed partly using oral techniques and partly using written techniques.

Oral addition and subtraction. It is advisable to perform all cases of addition and subtraction of three-digit numbers without going over a hundred using oral techniques, for example: 105+38; 315+207; 356—26; 632—210, and easier cases of these actions with going over a hundred.

When adding and subtracting in the limit of 1000 without going over a hundred, basically the same calculation techniques are used as in the limit of 10, 20 or 100.

The methods of mental addition and subtraction in the limit of 1000 with the transition through a hundred are also to a certain extent similar to the methods of these actions in the limit of 100. Thus, when solving the example 80+60, the number 80 is usually supplemented to 100 and the remaining units of the second term are added to the resulting hundred. It is easy to see that the computational method used here has much in common with the method used when solving the example 8+6, where the first term is also supplemented to the nearest round number, to which the remaining units of the second term are then added.

Example: 80+60 can also be solved as follows: 8 decimals + 6 decimals = 14 decimals, or 140. In this case, its solution is reduced directly to addition in the limit of 20.

To successfully complete **written addition,** one must be able to: a) correctly and fluently add place units and 6) transform units of one place into units of the next, higher place. Therefore, written addition should be preceded by exercises in adding units of individual places, for example: 5+3+7+0+4+9; 20+60+30+70; 300+200+400, while it is useful to reduce the addition of tens and hundreds to the addition of units. For example: 20+60+30+70 = 2 decimals + 6 decimals + 3 decimals + 7 decimals = 18 decimals = 1 hundredth, 8 decimals = 180.

It is advisable to complete written addition in the following sequence: a) the sum of the place value units of the addends is less than 10, for example: 231+458; 6) the sum of the place value units is equal to or greater than 10 in one case, for example: 238+312; 437+382; c) the sum of the place value units is equal to or greater than 10 in two cases, for example: 178+562; 458+347.

When explaining written addition, one should start with adding hundreds, and then solve the example starting with ones, and it turns out that it is more convenient to start written addition with ones.

In **written subtraction,** students are required to be able to: a) perform tabular subtraction accurately and fluently, and 6) break down units of a higher rank into units of the next lower rank. Therefore, to prepare for written subtraction, it is appropriate to practice tabular subtraction of units of different ranks, for example: 13—9; 7—0; 150—60; 1100—700, while subtraction of tens and hundreds is useful to reduce to subtraction of simple units, for example: 150—60 = 15 decimals — 6 decimals = 9 decimals, or 90; 1100—700 = 11 hundredths — 7 hundredths = 4 hundredths, or 400.

Written subtraction is studied in the following sequence:

1. When subtracting, there is no need to occupy units of any digit of the number being minuend, for example: 685—324; 487—403; 756—326.

2. When performing subtraction, it is necessary to borrow units of the adjacent higher digit of the number being subtracted, first in one and then in two cases, for example: 356—238, 360—275.

3. Due to the absence of units of a particular digit of the minuend, a takes up a unit of the next higher digit, for example: 901—75.

When going through the first stage of written subtraction, you should pay attention to examples in which there are zeros in the subtrahend, for example: 579—203, or zeros in the remainder, for example: 876—256.

This concentration **includes multiplication and division of a three-digit number by a single-digit number,** with easy cases of these actions performed orally and more difficult ones in writing [1].

Oral multiplication and division. Oral multiplication and division covers the following cases:

a) Multiplication of round hundreds by a single-digit number, for example; 200 x 3, and corresponding division cases, for example: 600: 3.

6) Multiplication of two-digit numbers ending in zero by a single-digit number, for example: 80 X 7, and corresponding division cases, for example: 560:7.

[1] According to the current program, operations on round hundreds and tens in the limit of 1000 are studied in grade I, the remaining cases are studied in grade III .

c) Multiplication of three-digit numbers ending in zero by a single-digit number, for example: 120 x 6, and corresponding division cases, for example: 720:6.

Written multiplication. Written multiplication requires knowledge of the multiplication table, the ability to accurately and fluently perform addition within the limit of 100, and convert units of a lower rank into units of the next higher rank. The multiplication table must be known in application not only to simple units, but also to composite ones (tens, hundreds). Therefore, written multiplication should be preceded by preparatory exercises in mental arithmetic.

It is advisable to go through written multiplication in the following sequence:

1. Multiplication without going through ten, for example: 123 X 3; 102 X 4.

2. Multiplication with transition through ten:

a) in one case, for example: 128 X 3;

6) in two cases, for example: 236 X 4.

In order for students to better understand the meaning of multiplication as an action that replaces the addition of equal terms, exercises should be given in finding the sum of several equal terms: a) by addition and 6) by multiplication, for example:

```
  246        246
  246      X   4
+ 246
  246
```

Solving similar examples in parallel helps to better understand the written multiplication technique.

In order to successfully master written division, children must be able to accurately and fluently

perform tabular division, and they must be able to divide not only simple units, but also units of higher ranks. Therefore, it is necessary to pay much attention to exercises in tabular division, especially in division with remainder. Such exercises should be given over a relatively long period of time so that by the time of studying written division, students are able to correctly and fluently perform the computational operations required to perform this action.

It is advisable to carry out written division by a single-digit number in the following sequence:

1) Individual digit units of the dividend are divided evenly by the divisor, for example: 864:2; 963:3.

2) The units of the 3rd digit of the dividend are divided by the divisor, but the units of the 2nd and 1st digits separately are not divided evenly by the divisor, for example: 872:4; 570:5.

3) Units of the 3rd and 2nd digits are not divisible separately, but taken together they are divisible evenly by the divisor, for example: 368:4; 142:2.

4) The units of each digit of the dividend are not evenly divisible by the divisor, for example: 736:3.

5) The units of the 3rd digit of the dividend are divided evenly by the divisor, and the units of the 2nd digit are less than the divisor, as a result of which a zero is obtained in the middle of the quotient, for example: 728:7.

It is advisable to explain written division, especially its more difficult cases, using visual counting material. Thus, when explaining the fourth of the above cases of division, you can set the students the task of dividing 350 sticks (“pencils”) into two equal parts (equally between 2 schools). First, 3 hundreds of sticks are divided between 2 schools, which results in 1 hundred

for each school. The remaining hundred is divided into tens, which results in 10 tens, and there were also 5 tens, a total of 15 tens. Divide 15 tens by 2, which results in 7 tens. The remaining ten is divided into units, the resulting 10 units are divided by 2, which results in 5. Thus, for each part (each school) there is 1 hundred 7 tens 5 units, or 175 units ("pencils").

Written division. Students should first explain in detail, something like this: 375:5. 3 hundreds divided by 5 do not produce hundreds. Break 3 hundreds into tens, get 30 tens; 30 tens plus 7 tens equal 37 tens; 37 tens divided by 5, get 7 tens. Multiply 7 tens by 5, get 35 tens. Subtract 35 tens from 37 tens, get 2 tens. Break them into ones, get 20 plus 5, for a total of 25 ones. Divide 25 by 5, get 5 ones. So, a total of 7 tens and 5 ones, or 75.

Later we can move on to a more brief explanation of division.

In the 1000 limit, students' skills in solving previously encountered types of problems are consolidated and some new types are introduced.

In the "First Thousand" concentration, special attention should be paid to the so-called unaddressed problems, for example:

"They paid 380 rubles for 3 meters of woollen material and 2 meters of cloth. A meter of woollen material cost 60 rubles. How much did a meter of cloth cost?"

When solving problems in the 1,000 limit, some students have trouble choosing actions even when solving relatively easy problems. This is due to the unusualness of the numerical data that is new to them. Overcoming these difficulties can be facilitated by first examining a similar problem with small numerical data

so that after solving it, the children proceed to solving the problem with larger numbers that has made them difficult.

Of the new types of problems in the limit of 1000, it is useful to solve those that can help prepare students to solve typical problems included in the 3rd grade curriculum. Thus, to prepare for solving problems on proportional division, problems like the following are solved: “350 rubles were paid for 2 pieces of identical fabric. The first piece was 3 meters, the second - 2 meters. How many rubles did a meter of this fabric cost?”

In a similar way, preparation for solving other types of problems included in the third grade program should be carried out.

SAMPLE LESSON PLANS

I CLASS

Topic. Number and figure 5.

Lesson 1.

The purpose of the lesson is to clarify students' understanding of the number 5. To introduce them to the number 5.

Progress of the lesson. 1. Perception of the number 5.

a) Formation of this number on classroom aids. Four cubes are placed on a shelf, then one more. It is found out how many cubes are obtained in total. (The same is done on the abacus.)

6) Formation of this number on individual benefits.

Students put aside 4 sticks, then add one more to them and say how many sticks they have in total.

c) Introduction to the number figure 5.

The teacher draws the number 4 on the board (in the form of circles), then adds 1 more circle.

Students draw this figure in their notebooks.

d) Measurement in steps.

The called student takes 4 steps, then 1 more. It is determined how many steps he took in total.

2. Familiarisation with number 5.

The students show the numbers 1, 2, 3, 4. Then the teacher shows the children a card with the number 5. To check whether the children have learned the numbers well, the teacher shows them cards with numbers, requiring them to show as many sticks (or fingers) each time as there are units in the number they are showing.

3. Letter of the number 5.

The teacher shows the children how to write the number 5.

At the teacher's instruction, students write this number in their notebooks.

4. Homework: draw 5 apples; write the number 5 (1 line).

Lesson 2.

The purpose of the lesson is to consolidate the knowledge gained in the previous lesson.

Progress of the lesson.

1. Review of what was covered in the previous lesson. Writing the number 5.

2. Consideration of the composition of the number 5.

Decomposition of the number 5 into 4 and 1 on class and individual manuals. Sketching 5 berries in notebooks. Breaking them down into 4 and 1 using a vertical line (standing stick).

In a similar way, the number 5 is broken down into 1 and 4, into 3 and 2, and into 2 and 3, after which a conclusion is made as to which two numbers the number 5 can be broken down into.

3. Oral solution of problems on addition and subtraction in the limit of 5.

4. Homework: 5 sticks are to be placed into 2 piles, as was done in class. Remember how you can place 5 sticks into 2 piles in different ways. Write the number 5 (two lines).

Topic: Subtraction of single-digit numbers in the limit of 20 without going over ten.

The purpose of the lesson is to familiarize students with the technique of performing this action, and to ensure that they correctly apply this technique when solving easy examples.

Lesson progress. To prepare students for learning a new action, the teacher precedes the explanation of the new material with an oral solution to examples of subtraction within the limit of 10, of finding out the decimal composition of numbers in the second ten, and of adding single-digit numbers within the limit of 20 without going over a ten.

Sample examples:

a) 8—6; 7—5: 9—6; 8—3.

6) How many tens and units are there in the numbers 12? 16? 18? 19?

c) 12+4; 13+3; 14+4; 12+6; 15+3.

The first two groups of examples directly prepare for the new action to be studied, since they include computational operations that are reconciled when subtracting single-digit numbers without going over ten. The third group of examples is useful in that the computational technique used in solving them has much in common with the technique that should be communicated to students in this lesson.

The above preparatory exercises are followed by an explanation of the new technique, which is carried out using the easiest examples, say: 14—2; 16—2; 15—1; and the solution of each of these examples is preceded by the solution of the corresponding example on subtraction in the limit of 10, namely:

4—2	6—2	5—1
14—2	16—2	15—1

When solving the first examples, visual aids (sticks or abacus) are used. These examples are solved by the whole class under the guidance of the teacher, who writes the solution on the board, then asking the students to write the examples in their notebooks.

After the explanation, the teacher, in order to check the assimilation and initial consolidation of the new action, calls several students to the board in turn, gives each of them an example to solve and requires a detailed explanation of the

action being performed. After solving one of these examples, it is useful to ask the students to compose a problem for the solution of which the same action must be performed so that the children understand when solving which problems this action is used.

In order to check whether the new principle has been mastered by **all** students, the teacher, after questioning individual students, offers the class several examples for oral solution, and then the students solve these same examples in writing in their notebooks. In this way, the students solve these examples semi-independently.

After checking the completed work, the teacher orally solves several examples with the children, planned for homework, in order to prepare the students for the independent solution of the examples that they will have to solve at home.

At the end of the lesson, in a conversation with the children, it becomes clear what they have learned today.

II CLASS

Topic: Solving problems by reducing to one.

Lesson 1.

The goal of the lesson is to lead children to an understanding of the method of solving problems by reducing to one, based on solving simple problems.

Lesson progress. In order to prepare students to solve the above-mentioned type of problems, the teacher, during oral arithmetic classes conducted after checking homework, offers students approximately the following problems:

"4 feathers cost 12 kopecks. How much does 1 feather cost? And how much do I have to pay for 6 such feathers?"

To help children better understand what data is needed to solve a problem using the reduction to unity method, they are given the following preparatory problem in one step, with the condition of the problem illustrated by drawings on the board:

"A melon costs 4 rubles. How much should I pay for 5 of these melons?"

Then the students are offered the following task in 2 steps, the condition of which is also illustrated by a picture:

"2 watermelons cost 8 rubles. How much should I pay for 5 of these watermelons?"

When analysing the last task in a conversation with children, it becomes clear whether it is possible to immediately find out how much one needs to pay for 5 watermelons, why it was possible to immediately find out how much 5 melons cost in the previous task, why it is impossible to immediately find out how much 5 watermelons cost. From the analysis of the task, it becomes clear that before finding out how much 5 watermelons cost, one must first find out how much 1 watermelon cost. After this, a plan for the task is outlined collectively and its solution is carried out, which is written on the board and in notebooks.

When considering the following problem: "4 spoons cost 12 rubles. How much do 6 such spoons cost?", its condition is written on the board without illustrations, namely:

4 spoons—12 rubles.
6 spoons—?

When analysing this problem, it turns out that it is impossible to find out right away how much 6 spoons cost, that first you need to find out how much 1 spoon costs. Following the analysis of the problem, a plan is made collectively and its solution is given orally, after which the children are asked to write down this solution in their notebooks. Thus, unlike the previous problem, which was

solved collectively, the children solve this problem semi-independently.

In order for children to better understand the structure of problems of this type, they are asked to compose problems similar to the solved one. One of the problems composed by the students is offered to them for independent solution (oral or written).

A problem of this type and examples are assigned for homework.

When summing up the lesson in a conversation, it turns out that the children learned to solve problems in the lesson that required finding out how much several items cost, that it was impossible to find out right away, they had to first find out how much 1 item costs, and only then how much several items cost.

Lesson 2.

The purpose of the lesson is to consolidate the students' knowledge gained in the previous lesson.

Lesson progress. In order to identify the extent to which the students have understood the structure of the new problems, and at the same time to help consolidate their knowledge, the teacher asks the children to compose problems similar to one of those that were solved at home. Several problems composed by the children are heard, and one (or two) of them are offered for oral solution.

After this, children are given a more complex task of this type, for example:

"One housewife bought 2 cups for 12 rubles. Another bought 3 of the same cups and gave 20 rubles to the cashier. How much change does she need?" (The teacher writes the problem statement briefly on the board.)

After repeating the condition, an analytical oral analysis of the problem is carried out. Then the students are asked to

write down its solution in their notebooks. This problem is thus solved by the children semi-independently.

After most of the students have finished their work, the teacher, in an extended conversation, finds out how the students solved the problem, and the solution is written on the board.

Then the children are given a more complex problem of this type to solve on their own, for example:

"3 glasses of berries cost 6 rubles. The girl bought 7 glasses of berries and gave 15 rubles in payment. How much change should she get?"

At the end of the lesson, in a conversation, it becomes clear what you did and what you learned in the lesson.

Children are given 1—8 complicated problems of this type from the problem book for homework.

Theme, I understand, "so many times more."

Lesson 1.

Objective of the lesson. Using visual aids, reveal the meaning of the expression "several times more" and identify the action by which a given number increases several times.

Lesson progress. 1. After checking homework and completing a series of exercises in increasing the number by several units, the teacher informs the children of the goal of the upcoming lesson in approximately the following form:

— Children, you know what "several more" means, and you can increase a number by several units. Today you will learn what "several times more" means and how to increase a number several times. You need to know this because it often occurs in life. For example, two girls are going to school. One of them says: "Yesterday I solved 3 problems." And the other says: "And I solved 2 times more examples." What does 2 times more mean? How many examples are there? Now we will learn how to recognize this.

2. Then the teacher says: — I will now place the girls in pairs in front of the class on the right and on the left. Both of you come out from behind your desks and stand on the right. And on the left let there be one pair, a second pair and a third pair. (The girls come out from behind their desks in pairs and stand on the left.)

—Where are there more girls—on the right or on the left?

— There are more on the left. On the right there is 1 pair, and on the left there are 3 pairs.

—In that case, they say—there are 3 times more girls on the left than on the right. How many times more girls are there on the left than on the right? (There are 3 times more girls on the left than on the right.)

— Now let there be another pair on the left — the fourth. How many pairs are now on the right and how many on the left?

—How many times more girls are there on the left than on the right? (There are 4 times more girls on the left than on the right.)

—Why do you say 4 times more? (Because on the right there is one pair, and on the left there are 4 pairs.)

—I put 4 pairs of balls on the top wire of the abacus, and one pair of balls on the bottom. On which wire are there more balls and how many times more?

— Now I will count another — the fifth — pair of balls on the upper wire. How many times more balls are there now?

—Why 65 times more? (Because there is only one pair on the lower wire, and five such pairs on the upper wire.)

—I drew three circles on the bottom line of the blackboard. What should I do to have four times as many circles on the top line? (The teacher draws four times three circles on the top line.)

— I give Valya 4 pencils. What should I do so that Olya has 2 times more pencils than Valya?

—Take out your sticks. Put 3 sticks on the right and 2 times more on the left.

—Put 4 sticks on the right and 3 times more on the left.

—Now, put two sticks on the left and 6 times more on the right.

— Do you understand what “several times more” means? Now we will increase the numbers several times, calculate and write down the calculations. Here, on the bottom line, there are 3 circles. And on the top line, there are 4 times more circles. How many circles are on the top line? How did you know there were 12 circles? (We took 4 times 3 circles and got 12 circles.)

—Let’s write this: 3 circles x 4 = 12 circles.

Here, on the bottom wire, we have 2 balls. And on the top wire, 5 times more. How many balls are on the top wire? How did you know there were 10 balls?

—Let’s write this: 2 balls x 5 = 10 balls.

—So what needs to be done to increase the number several times?

3. — Now let’s learn how to solve problems in which we need to find a number that must be several times greater than a given number. Let’s take a problem similar to the one I talked about at the beginning of the lesson.

“One girl solved 4 problems, and the other 3 times more. How many problems did the other girl solve?”

Children solve the problem.

— Listen to the second problem: “One worker made 4 parts in an hour, and another — a Stakhanovite — made 5 times more. How many parts did the Stakhanovite worker make in an hour?”

After solving the problem and receiving the answer, the teacher asks questions: “How do you get a number 5 times greater than 4? How do you write down the calculation?”

4. Now come up with a problem in one step in which you need to find a number that is several times greater than another given number.

Children compose problems.

The teacher surveys as many students as possible who have completed his assignment.

Correctly composing the tasks will show that the goal of the lesson has been achieved: the students have developed the concept of increasing a number several times.

Homework: solve 2—3 one-step problems from the problem book that contain the concept of "so many times more."

Lesson 2.

Objective of the lesson. By solving problems, to demonstrate the unambiguity of terms: "more, more expensive, longer, heavier, etc. by so many times."

The second lesson continues to develop the concept of increasing a number several times by solving problems first in one, then in two steps. Various quantities are introduced into the problems: cost, weight, time, length, height, width of objects, etc. For example:

"A pencil costs 6 kopecks, and a ruler costs 3 times more. How much does a ruler cost?"

"The shovel weighs 2 kilograms, and the crowbar is 4 times heavier. How much does the crowbar weigh?" "The son is 5 years old, and the father is 6 times older than the son. How old is the father?" etc.

In such tasks it turns out that the terms "More expensive, heavier, older, higher, deeper, wider, longer by several times" have the meaning of the term "larger by several times".

This lesson increases the level of independence in the students' work: children come up with their own problems based on a specific assignment from the teacher ("Come up with a problem in which one object is several times wider (taller, longer, heavier) than another"); about 10 minutes are allocated for independent work, during which the children solve ready-made problems from the problem book.

Thus, this lesson is structured approximately according to the following plan:

1. Checking homework.

2. Exercises in mental arithmetic: repetition of the covered part of the multiplication and division table.

3. Solving one-step problems that contain the expressions: “several times more expensive”, “several times older”, “several times harder”. The problems are solved orally with subsequent recording of the action (multiplication).

4. Independent work of students: solving two similar problems from the problem book. Checking independent work.

5. Students themselves come up with problems that would contain the words (concepts): “several times longer”, “several times wider”, “several times higher”. Solving the problems they come up with.

6. Homework: solve two problems (Nos. ...). Review the multiplication table by 4.

Lesson 3.

Objective of the lesson. To consolidate the knowledge gained in previous lessons and to deepen the concept of increasing a number several times by comparing and contrasting these problems with problems on increasing a number by several units.

Lesson progress. 1. Checking homework.

2. Exercises in mental arithmetic: repetition of the multiplication and division tables.

3. Formulating the objective of the lesson (“Let’s compare problems that talk about an increase by several units with problems in which the number increases several times. Let’s establish how their solutions differ”).

4. Solving two pairs of problems and recording their solutions:

a) “Vasya has 8 kopecks, and Vanya has 2 kopecks more. How much money does Vanya have?”

8 kopecks + 2 kopecks = 10 kopecks;

6) “Vasya has 8 kopecks, and Vanya has twice as much. How much money does Vanya have?”

8 kopecks x 2 = 16 kopecks.

a) “A pencil costs 7 kopecks, and a pen costs 5 kopecks more. How much does a pen cost?”

7 kopecks + 5 kopecks = 12 beds;

6) “A pencil costs 7 kopecks, but a notebook costs 5 times more. How much does a notebook cost?”

7 kopecks x 5 = 35 kopecks.

5. Each student comes up with two problems, one of which requires increasing a number by several units, and the other requires increasing a number several times. Recording the solutions to the problems in the students’ notebooks.

6. Generalisation: a) what action is used to solve problems of increasing a number by several units; 6) what action is used to solve problems of increasing a number several times.

7. Homework: solve 2 problems that involve an increase by several units and several times.

ORAL COUNTING IN GRADES III AND IV

Mental arithmetic is important for the practical training of students, as it is widely used in life. It is also of great importance for preparing students for written calculations, the successful study of which is possible only with strong skills in mental calculations.

Meanwhile, teaching mental arithmetic, which occupies a relatively large place in the work plan of the first and second grades of primary school, often does not receive the necessary continuation and development in the third and fourth grades. As a result, some students, as shown by numerous observations and examination data, finish the fourth grade with poor preparation in the field of mental arithmetic, which hinders their successful study of arithmetic and subsequent sections of mathematics in senior grades.

It is therefore necessary to significantly improve the teaching of mental calculations in senior grades of primary school.

The purpose of oral arithmetic classes in grades 3 and 4 is to: a) consolidate and improve children's arithmetic skills acquired in the two younger grades; 6) extend these skills to larger numbers; and c) introduce students to some new computational techniques.

As is known, all actions studied in grades I and II, namely: 4 actions in the limit of 100 and easy cases of 4 actions in the limit of 1000, are performed using oral computational techniques.

Many second-grade students do not master these techniques well enough. Mental arithmetic classes in the third and fourth grades should therefore be devoted

to a certain extent to **consolidating** and **improving** the counting skills acquired by children in the lower grades.

Since we are talking about repeating what has been covered, it is necessary to offer students mixed simple or compound examples of the indicated actions relatively often, for example: 80 — 24; 18 X 3; 26 + 48; 84: 6; 410—50; 30 X 8; 120+180; 320:4; 96: 6 X 4—42+18, etc.

But when solving such examples, individual actions are not sufficiently consolidated. It is therefore advisable to devote some oral arithmetic lessons to repeating one action or even a specific case of some action, for example, a case of division with remainder, a case of non-tabular division by a two-digit number, etc. Such a limitation of the content of oral arithmetic lessons is especially appropriate in the third grade, in particular in the first half of the school year, since the students' counting skills, which they brought with them from the second grade, are often not yet sufficiently strong at this time.

When planning oral arithmetic lessons, it is necessary to take into account the students' knowledge so that more time can be allocated to repeating poorly mastered actions so that these lessons can contribute to the maximum consolidation of children's arithmetic skills.

When repeating the actions studied in the 2nd grade, special attention should be paid to actions in the limit of 100, in particular division, due to the difficulty of this action. It is necessary to remember that without a solid knowledge of division in the limit of 100, successful study of the further course of arithmetic is impossible, especially the sections of this course

“Divisibility of Numbers” and “Ordinary Fractions”, studied in the 5th grade.

It is necessary to ensure that students in grades 3 and 4 not only know how to absolutely correctly and fluently solve any division problems within the first hundred, but also know what numbers any number within this limit is divisible by (for example, what numbers 45, 68, 75, etc. are divisible without a remainder), and also what two numbers can be multiplied to get a given number (for example, what two numbers can be multiplied to get 56? 84? 36? etc.).

Oral arithmetic classes in senior grades of primary school should be conducted in close connection with the teaching of written calculations and should prepare students for the latter. Along with the methods of written execution of each action, children should be introduced to various methods of oral execution.

When performing any arithmetic operation in writing, a whole series of calculation operations must be performed orally.

Before moving on to a new case of written calculations, the teacher should:

a) carefully analyse what mental arithmetic skills are required to successfully master a new action;

6) check the extent to which students possess these skills;

c) if necessary, give them appropriate exercises to develop these skills.

In addition to oral exercises, which should precede the study of individual cases of written calculations, it is necessary to train students in the oral solution of examples and problems on this action in parallel with the study of the written method of producing each action (whether it is an action on whole abstract or

composite named numbers). For oral exercises, it is necessary to choose comparatively easy cases of the studied action, mainly on round numbers, the actions on which are easily reduced to actions in the limit of 100.

In addition to reinforcing **the computational techniques** studied in the second grade and extending these techniques to a larger range of numbers, students in the third and fourth grades should be introduced to some new computational techniques, primarily specific ones.

Let's consider computational techniques that are appropriate to introduce in grades 3 and 4 in addition to those studied in the lower grades.

Oral addition techniques:

a) Rounding off terms, for example:

38 + 59 = 40 + 60 — (2 + 1) = 97;

69 + 71 = 70 + 70 = 140.

6) Grouping of terms, for example:

28 + 65 + 72 = (28 + 72) + 65 = 100 + 65 = 165.

Oral subtraction techniques:

a) Rounding of given numbers, for example:

80 — 49 = (80 — 50) + 1 = 31;

101 — 35 = (100 — 35) + 1 = 65 + 1 = 66;

175 — 99 = (175 — 100 + 1 = 75 + 1 = 76.

6) Replacing subtraction with addition (the addition technique), for example:

215 — 86; 86 + 14 = 100; 100 + 115 = 215.

Hence the remainder is 14 + 115 = 129.

Oral multiplication techniques:

a) Grouping of factors, for example:

25 X 79 X 4 = (25 X 4) X 79 = 100 X 79 = 7900.

6) Rounding off one of the factors by increasing it by 1, for example:

35 X 9 = 35 X 10 — 35 = 315;

28 x 19 = 28 x 20—28 = 532.

c) Rounding off one of the factors by increasing it several times, for example:

76 X 5 = 76 X 10 : 2 = 760 : 2 = 380, or

(76 : 2) X 10 = 38 X 10 = 380;

68 X 50 = 68 X 100 : 2 = 3400, or

(68:2) X 100 = 3400;

36 X 25 = 36 X 100 : 4 = 3600 : 4 = 900, or

(36:4) X 100 = 9 X 100 = 900.

d) Decomposition of one of the given numbers into factors and successive multiplication by these factors, for example:

35 X 12 = 35 X (2 X 6) = 35 X 2 X 6 = 70 X 6 = 420.

Oral division techniques.

a) Rounding off the divisor by increasing it several times, for example:

720:5 = 720:10 X 2 = 72 X 2 = 144.

6) Rounding off the dividend by increasing it by several units, for example:

784 : 8 = (800 — 16) : 8 = 800 : 8 — 16:8 = 100 —2 = 98

c) Factorisation of the divisor and successive division of the dividend by these factors, for example:

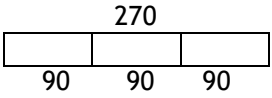
540 : 45 = 540 : (9 X 5) = 540 : 9 : 5 = 60 : 5 = 12.

The easier of the above-mentioned methods of mental calculations are introduced in the 3rd grade, the rest in the 4th.

In order to make it easier for students to learn a newly introduced method of mental arithmetic, it is sometimes useful to explain it using problems. Thus, when explaining the method of rounding off a multiplier by increasing it by several units, you can take the problem:

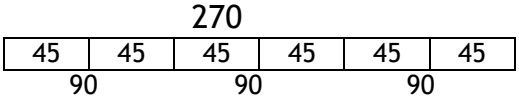
“One meter of fabric costs 35 rubles. How much should I pay for 9 meters of such fabric?”

When mentally multiplying 35 rubles by 9, to make the operation easier, we multiply 35 rubles by 10, as if finding out how much 10 meters of such fabric cost, then subtract the excess 35 rubles from the result (350 rubles). The result is 315 rubles.



Better assimilation of a new technique can also be facilitated by the use of graphic illustrations. Let’s take the technique of successive division. Let’s say we need to divide 270 by 6. Having decomposed the divisor 6 into 3.ЖХ2, we first divide 270 by 3, while illustrating the action as shown in the figure.

The division of 270 by 6 is also illustrated.



When explaining some of the above-mentioned techniques of oral calculations, it is useful to rely on the relationship between data and the results of the corresponding action that children have learned.

Mental arithmetic lessons should include solving not only examples, but also problems.

When solving problems in writing, the student has to expend a relatively large amount of mental energy on performing calculations, due to which he sometimes does not delve into the method of solving it sufficiently.

Another matter is the oral solution of a problem, in which the calculations are usually not difficult, so that the student can devote himself almost entirely to understanding the course of its solution.

It should also be noted that solving a problem orally takes significantly less time than solving it in writing. Thanks to this, it is possible to solve significantly more problems orally in the same amount of time than in writing.

In order for the oral solution to give the maximum effect, the tasks should be selected so that in some cases they serve to prepare students for the written solution of similar tasks, and in others they help to consolidate the children's skills and abilities in solving previously encountered types of tasks.

Mental arithmetic lessons should, if possible, be conducted at the beginning of each lesson. It is also necessary to ensure that throughout the lesson, students in grades III. and IV perform all calculations in their heads that can be performed orally. In problems solved even in grade IV. small numerical data are often encountered, the operations on which can easily be performed orally.

Let's say that in a problem solved in the 4th grade, it is required, among other questions, to find out the cost of 6 meters of fabric, one meter of which costs 28 rubles 75 kopecks. At first glance, it may seem that this action is too difficult to perform orally. Meanwhile, when rounding the multiplicand, orally multiplying 28 rubles 75 kopecks by 6 does not present any particular difficulties. Indeed, multiplying 30 rubles by 6, we get 180 rubles. But we took an extra 6 times 1 ruble 25 kopecks, which is 7 rubles 50 kopecks. Subtracting 7

rubles 50 kopecks from 180 rubles, we get 172 rubles 50 kopecks.

Pupils of grades III—IV should perform as many oral operations as possible not only when solving problems in writing, but also when solving examples in writing. Let's take division of multi-digit numbers as an example. In the case of division by a single-digit number, pupils of grade IV must perform all auxiliary calculations orally, first writing down only the quotient and remainders, and then only the quotient. Such a notation can be used in a number of cases when dividing by some two-digit and multi-digit divisors (for example, when dividing by 12, 30, 300, etc.). A number of calculations can be performed orally when breaking down and converting named numbers, when working with fractions, etc.

During oral arithmetic lessons, the teacher most often dictates assignments so that children are forced to perceive numerical data by ear. This form of assignment has undoubted advantages, contributing to the development of students' attention and memory, since in practice it is most often necessary to conduct oral counting over numbers perceived by ear.

However, this form of assignment should not be overused, since it requires a great deal of mental effort from students, and therefore tires them out relatively quickly. Along with assignments that require students to perceive numerical data auditorily, children should sometimes be given the opportunity to perceive these data visually as well. The latter form of assignment is appropriate when students are required to perform operations on numbers that are difficult to retain in memory.

Recording numerical data can sometimes be replaced by showing them on a mental arithmetic table

or by using rows of numbers, which consist of 10 vertical stripes, each of which has 10 numbers printed in large font [1].

In order for students in grades 3 and 4 to improve their mental arithmetic skills, it is useful to include relevant exercises in their homework.

For homework, you can use the exercises from the Oral Examples and Problems sections of the problem books for grades 3 and 4. You can also assign students the following exercises: "Write what numbers the number 30 is divisible by without a remainder? the number 31? the number 32? the number 33? the number 34?" (Students should complete this task approximately as follows: 30 is divisible by 1, 2, 3, 5, 6, 10, 15, 30; 31 is divisible by 1 and by 31, etc.). "Write all the numbers up to 100 that are divisible by 13 without a remainder." "Divide the number 45 by each of the numbers in the first ten (by numbers from to 10)." "Divide the number 70 by each of the numbers in the second ten (by numbers from 11 to 20)," etc.

Oral arithmetic classes are usually conducted in the form of a frontal survey of students. This form of survey, in which the teacher addresses his questions to the entire class each time and then surveys some students, helps to activate the attention of the class. But such a survey does not provide an opportunity to sufficiently identify the counting skills of individual students. As a result, students' answers during oral arithmetic classes are assessed very rarely. Thus, taking into account students' knowledge as a factor in

[1] See G. B. Polyak, Counting table. Rows of numbers, M. 1945.

improving academic performance is clearly not used enough here.

In order for the accounting of academic performance to contribute to the development of students' skills in mental arithmetic, along with the frontal survey, individual surveys of individual students should be practiced so that after frontal lessons in mental arithmetic, the teacher, if possible, daily surveys 1—2 students in more detail, asking each of them several questions and evaluating their answers accordingly.

Unlike a frontal survey, in which a single student usually has to answer one question, in an individual survey the called student must answer several questions from the teacher. Therefore, the teacher has more grounds for assessing his knowledge. The students' skills can also be taken into account when checking the mental arithmetic exercises included in homework. Rational organisation of mental arithmetic classes can ensure a significant improvement in students' knowledge in this area and can contribute to raising the culture of mental arithmetic in school.

NUMBERING MULTI—DIGIT NUMBERS

The task of teaching numeration is to teach children to read and write multi-digit numbers. But these skills and abilities are based on a number of concepts related to the features of the decimal number system: the concepts of ranks and classes, the recording of an infinite series of numbers using 10 digits, the naming of numbers using a small number of words, the local principle, etc. When studying numeration, these concepts are formed, and thus children are brought to an understanding of the basics of the decimal number system, to an understanding of the structure and composition of a natural number.

A distinction is made between **oral and written** numeration. The goal of oral numeration is to show students how units of a set are grouped into decimal and then thousandth groups, how numbers are formed from these decimal groups, how the name of numbers is composed, and how counting is done by units and groups of units. The goal of studying **written** numeration is to teach a child to write down any number and read the written number based on the concept of the local value of numbers.

Thus, each of the two types of numeration has its own tasks and its own content. First, oral numeration is studied, then written. In the concentric study of the numeration of numbers of any size, the same order is observed in each concentration.

The difficulties of studying the composition of a number, reading and writing numbers, and their transformation increase as the significance of the number increases and its structure becomes more

complex. To ease these difficulties, numeration can be studied in concentrations, namely: first study the oral and written numeration of numbers of the second class — the class of thousands, then study the oral and written numeration of numbers of the class of millions and, finally, the class of billions.

The class abacus, abacus (class and individual) and numbering table should be used as visual aids when studying the numeration of multi-digit numbers. In order to more clearly distinguish the classes, it is useful to remove the fourth and eighth wires on the class abacus; in addition, it is useful to make inscriptions on the left bar: “class of units” opposite the first three wires, “class of thousands” opposite the 4th, 5th and 6th wires and “class of millions” opposite the 7th, 8th and 9th wires. Teaching the numeration of multi-digit numbers can be carried out in the 3rd grade according to the following plan.

Oral and written numeration of six-digit numbers (up to a million)

Introduction to counting units. Having counted 10 units on the class abacus on the first wire in units , the children replace them **with one ten** on the second wire. Then they count in tens and, having counted 10 **tens**, they replace them with one hundred on the third wire. Then they count in hundreds and, having counted 10 **hundreds** , they replace them **with one thousand** on the fourth wire.

Up to 1 million people do this.

As each new unit of measurement is received, the teacher writes its name on the blackboard, resulting in the following table:

10 units make up 1 ten
10 tens make up 1 hundred
10 hundred is 1 thousand
10 thousand is 1 ten thousand
10 tens of thousands make up 1 hundred thousand
10 hundred thousand is a thousand thousand, or
1 million.

The compilation of this table and its assimilation are of great importance: children assimilate the name of the counting units (ranks), the order of their arrangement, the unit **ratio** of two adjacent ranks, the conventional **representation** of counting units on the abacus. The number 10 as the basis of the decimal number system appears here clearly. Students copy this table into their notebooks, read it several times and memorize it at home.

Composing multi-digit numbers and decomposing them into decimal groups. At this stage of the lessons, children learn **to compose and name** numbers composed of various counting units. To make this task easier, firstly, **the class abacus should be used in this work** (while naming the counting units that make up the number, they should be put aside on the class abacus; this helps the student remember the digits named by the teacher) and, secondly, the digit numbers should be given first for only one class, and then for two classes. The first tasks can be something like this: “What number do you get if you take 3 hundreds of thousands and 9 tens of thousands? What number will be 7

hundreds of thousands and 9 thousands? 1 hundreds of thousands and 1 tens of thousands and 8 thousands?"

Following **the composition** of numbers from these place units, there are exercises in the reverse operation — in **the decomposition** of a given number into place addends. "645 thousand — how many hundreds of thousands, tens of thousands and units of thousands are in this number separately?" "480 thousand — how many hundreds of thousands and tens of thousands are in this number? Put this number on the abacus!" A more complex task of this kind: "Name what units of account the number 215 thousand 102 (units) is made up of. Put this number on the class abacus!"

The concept of digits. The term "digit" is introduced as follows: "There are many counting units; in order to avoid mistakes, each of them is designated by its own number or digit, and each of them takes its own place when writing down numbers." And then the well-known definitions are given: "Simple units are called units of the 1st digit; they are set aside on the class abacus on **the first** wire from the bottom; they are written in **the first** place on the right. Tens are called units of the 2nd digit; they are set aside on **the second** wire; they are written in **the second** place on the right." And so on up to the 7th digit. The students repeat the definitions after the teacher. In conclusion, the teacher draws a digit table on the blackboard**.**

Hundred thousands	Ten thousands	Units thousands	Hundreds	Tens	Units
6	5	4	3	2	1

Explaining this table, the teacher establishes a connection with the table that the students already

know (see p. 453), here the same units of counting are only in that table they are arranged in a column, and here in a long row. Each digit has its own number: units—1st digit, tens —2nd digit, hundreds—3rd digit, etc. The children read this table several times and re-draw it in their notebooks.

Exercises are conducted according to the table to memorize the place of each digit. This is facilitated by questions of three types:

1. “In what place on the right are the simple units? thousands? tens of thousands? hundreds? hundreds of thousands?” etc.

2. “What digits are in third place from the right? in sixth place? in fourth place? in fifth place?” etc.

3. “In third place from the right hand is the number 8; what number does it represent? In sixth place is the number 5; what number does it represent?” etc.

The table of ranks is given to students at home to memorize the names of rank units in their natural sequence and to firmly grasp the place of each rank.

The concept of classes. In previous lessons, children learned the composition of a number from **decimal** groups (orders). Now they need to be given the concept of **a thousandth** grouping of units of a set, of **a class** .

A starting point could be reading the numbers put aside on the class abacus.

The teacher puts aside on the abacus:

On the 3rd wire there are 3 balls
on the 2nd » —8 balls
on the 1st » —5 balls

“What number is set aside?” — “385”. The teacher adds the word “units” and writes on the board: “385 units”.

“How many hundreds of units are there in this number? tens of units? prime units? That means,” the teacher explains, “the count is in **units.”**

Then the teacher puts it aside:

On the 6th wire there are 3 balls
on the 5th » 8 balls
on the 4th » 5 balls

“What number is set aside here?”—“385 **thousand** .”

The teacher writes this number on the blackboard, underlining the word “thousands.” “How many hundreds of thousands are there in this number? tens of thousands? units of thousands? So,” the teacher explains, “we’re counting in **thousands here**.”

Thus, the following entry appears on the blackboard:

385 units
385 thousand

Referring to this entry, the teacher says: “You can count by units, making up tens of units and hundreds of units; you can count up to 999 units in this way. But you can **also count by thousands** , making up **tens of** thousands and **hundreds of** thousands; you can count up to 999 thousand in this way.

When counting by units, we get 3 digits: **simple** units, tens of units, and **hundreds of** units. These 3 digits make up the class **of units** . This is the first class.

When counting by **thousands,** there are also 3 digits: **units of thousands, tens of** thousands, **hundreds of** thousands. These three digits, occupying 4th, 5th and 6th places, make up **the class of thousands** . This is the second class.”

Then the teacher draws a rank table on the blackboard and above the first three ranks draws a column (rectangle) in which he writes “1st class — class

of units". On the class abacus on the left side of the frame opposite the first three wires a paper strip with the inscription "1st class" is attached.

Next, the teacher asks the students to name the next three ranks in order and establishes that here the count is conducted in thousands from to 999 thousand. "Thousands are called units of the second class. This means that the 2nd class is the class of thousands. It includes 3 ranks: 4th, 5th and 6th. Let us mark this on our table." Above the 4th, 5th and 6th ranks, a column appears uniting them with the inscription "2nd class—class of thousands." On the class abacus, opposite the 4th, 5th and 6th wires, a paper strip with the inscription "2nd class" is attached to the side.

2nd class—class of thousands			1st class—class of units		
hundred of thousands	tens of thousands	units of thousands	hundreds	tens	units
6	5	4	3	2	1

Based on this table, students learn the names of classes, their order, and what categories are included in each class.

Exercises in writing and reading numbers are also conducted on this table. First, the numbers of **one** class (658 thousand; 325 units) are read and written down, and then the numbers **of two** classes. Then the composition of the number is analysed by ear; for example, the number 306 thousand 517 units is named. The teacher asks what classes are given here, what ranks are there, and in which ranks there are no units. Then a printed numbering table with a device for inserting cut-out numbers and a class abacus are hung

up; the students, at the teacher's request, practice depicting various numbers. The homework assignment is to draw (using a textbook) a numbering table and to learn what was studied in class about ranks and classes.

Teaching how to write and read multi-digit numbers.

This is the main question of studying the numeration of multi-digit numbers. All previous work had as its main task to prepare children for meaningful recording and reading of multi-digit numbers, to create a basis for the successful formation of this skill. The work includes the following two stages:

Stage 1. Writing round numbers of the thousands class, for example: 485000; 508000; 700000; 420000.

Before writing down a number, it is analysed, it is established by ear, what class of units it is made up of, what class is missing; this number is put on the abacus and then written down on the board and in notebooks. When writing down the number 485 thousand, the teacher asks the question: "In what places should the numbers 4, 8, 5 be so that they denote thousands?" (Answer: in the 4th, 5th and 6th.) "What should the first, second and third places be?" (With zeros.) The entry appears: 485,000. "What do the zeros indicate here?" (Answer: that there are no first class units here — the first, second and third digits.) "Compare the entries of the numbers "485" (units) and "485000" and tell me what the difference is in writing these numbers?" (The difference is that 3 zeros are added to the number thousands (485).

Based on the examples analysed in this way (at least three), the following conclusion is made: "To write a number made up of thousands, first write the number of thousands, and then add three zeros to the right of it."

The second rule is reported: “In order to make it easier to write and read large numbers, when writing them, you need to separate the class from the class with small intervals.”

This is followed by exercises in reading numbers consisting of round thousands, for example: 28000, 106000, 900000, 530000, 725000, etc. Reading the first examples is accompanied by an explanation of why this particular number should be read this way. Answering this question, the student says: “2 is in the 5th place and denotes 2 tens of thousands, 8 is in the 4th place and denotes 8 thousand, and a total of 28 thousand.”

Stage 2. Writing and reading any multi-digit numbers consisting of two classes—the class of thousands and the class of units. Before practicing writing multi-digit numbers, students review the following basic principles of written numeration:

1. When writing a number, units are placed in the 1st place from the right, tens in the 2nd, hundreds in the 3rd, thousands in the 4th, etc.

2. The same digit can represent the number of units of any digit, depending on the place it occupies.

When pronouncing a number, we say the names of its classes. For example, when naming the number 495382, we clearly hear **thousands, units:** 495 thousand 382 units. It is necessary to draw children’s attention to this circumstance and teach them to, when writing a number, break it down into classes and write down each class, starting with the highest. When dictating a number, it is necessary to make a short pause after the name of the class and ensure that the students separate the class from the class with a small gap. Following the recording, there are exercises in **reading** numbers. Before reading the written number, it is divided into

classes, starting with the first. For the exercise in reading numbers, it is necessary to give numbers written **without gaps** between classes so as to force the students to divide the number into classes; the students separate **the class** from the class with a stroke placed on top. For example: 604 753.

In order to develop the skill of quickly and correctly dividing numbers into classes, it is necessary to do more of these exercises. At this stage of studying numeration, the students' attention is also drawn to the fact that sometimes the units of some digits may be missing in numbers. The places of such digits should be occupied by zeros. For example, in the number 25 thousand 35 units, there are no hundreds. A zero is put in place of the hundreds digit and the number is written as follows: 25035.

In the number "four hundred thousand one hundred eighty-five" there are no tens of thousands or thousands; zeros are put in place of these digits: 400185.

The greatest difficulty is in writing down numbers in which one class ends and the other begins with zeros, for example: 50048; 800020, etc. We need to spend a little more time writing down such numbers.

Oral and written numbering within the billion limit

The study of numbering in this limit is carried out according to the same plan as the study of numbering of six-digit numbers, namely:

1) familiarisation with new units of measurement: million, ten million, hundred million;

2) composing numbers from different place units and naming them;

3) decomposition of a given number into place numbers;

4) counting in thousands and millions: 990 thousand — add 5 thousand to this number, 5 thousand again to the resulting sum, etc. (990 thousand, 995 thousand; 1000 thousand, or 1 million; 1 million 5 thousand, 1 million 10 thousand, etc.); to 96 thousand, add one thousand (96 thousand, 97 thousand, 98 thousand, 99 thousand, 100 thousand, 101 thousand, 102 thousand, etc.);

5) compiling a rank table and memorising the place of each rank.

Hundred million	Ten million	Units of millions.	Hundred thousands	Ten thousands	Units of thousands	Hundreds	Tens	Units
9	8	7	6	5	4	3	2	1

6) drawing up a class table:

3rd class—millions			2nd class — thousands			1st grade — units		
Hundred million	Ten million	Units of millions.	Hundred thousands	Ten thousands	Units of thousands	Hundreds	Tens	Units
9	8	7	6	5	4	3	2	1
5 8	3 9	6 1	4 5	8 2	2 8			

7) writing and reading multi-digit numbers in the limit of the class of millions:

a) writing and reading numbers consisting of round millions;

565 million, 308 million, 720 million 565,000,000 308,000,000 720,000,000

6) writing and reading any nine-digit numbers with all significant digits;

c) writing and reading numbers with missing classes and missing units of some digits.

Before starting to practice writing such numbers, students review the following points:

1. The class combines 3 digits, so each class must have 3 digits, but the highest class can have one or two digits.

2. Sometimes the numbers are missing units of some digits. The places of such digits should be filled with zeros. If an entire class is “empty”, its place is filled with three zeros. Examples of numbers with missing digits: 600 800 200; 504 308 705; 8 088 008; 70 060 060; 30 100 050, etc.

Exercises in writing numbers are accompanied by exercises in reading numbers with missing classes and ranks.

In order to create a correct and more or less concrete idea of large numbers in children, it is necessary to give several examples characterising the magnitude of large numbers, for example, one million. “Can a person live for a million hours? What distance would a million people occupy if they were lined up in one line, with two people placed one meter apart? How much time would it take to write a million letters if it takes one second to write one letter?”

It is highly desirable for the teacher to show the students a sheet of graph paper measuring 1 m x 1 m. On this sheet the children will “see” a million (a million millimetre cells).

Showing the sheet and answering the questions above will make the child’s imagination work hard and

create the prerequisites for the correct attitude of children to the magnitude of large numbers.

Decomposition of numbers into place value addends. By practicing decomposition of numbers into place value addends, students will learn to correctly understand the meaning of each digit of the number. The task can be given in the following form: “Decompose the number into place value addends” or “Represent the given number as a sum of place values.” Before starting to write down the addends, the composition of the given number from places is analysed. Let’s say that the number 6,025,380 is given to be decomposed. Starting with the decomposition operation, the student indicates that the given number consists of 6 million 25 thousand 380 units. In the class of millions, there are only units of millions; in the class of thousands, there are tens and units of thousands, in the class of units, hundreds and tens. Therefore, 6,025,380 = 6,000,000 + 20,000 + 5,000 + 300 + 80.

Of no less importance for understanding the composition of a number is the reverse operation: writing a number given as a sum of place values, according to the rules of numbering. For example: 400,000 + 9,000 + 30 + 8 = 409,038.

A good variation of this type of exercise is exercises in writing **numbers** and such numbers in which the name of the classes is written in words, for example: 60 million 50 thousand = 60,050,000.

Exercise in Counting Exercise in counting by units is appropriate to put at the end of this topic. Counting by units in the limit of large numbers is cumbersome and verbose; it is difficult for a child to keep in mind a large number of words obtained during counting. To make such counting easier, it is necessary to combine it with

fixing numbers, with recording the results of the count. And this is possible after written numeration has been completed.

It is useful to count by units only in those sections of the natural series where there is a transition from one rank to another, or from one class to another.

"99,997—continue counting, adding one by one."

"99,990—continue counting by fives and write down the numbers you get."

"999 998 — continue counting in units; count only 4 units and write down the numbers you get."

Students count to themselves and write down: 999,998, 999,999, 1,000,000, 1,000,001.

"Name a number that is 2 units less than 1 million; name a number that is 3 units less than 100 thousand," etc.

Increasing and decreasing a number by 10, 100, 1000 times. Increasing and decreasing a number by 10, 100, and 1000 times by adding and dropping zeros on the right helps to understand the principle of the place value of a digit in decimal numbering. This will be needed when studying the fragmentation and transformation of place units of a number.

The explanation of this question is carried out in the following order: a) one zero is added to the right of the given number; 6) the resulting number is compared with the given one and it is established that it has increased by 10 times; c) it is found that from adding one zero the number increases by 10 times because after adding zero each digit has moved to the left by one place; d) the rule is derived: "To increase a number by 10 times, it is enough to add one zero to it on the right."

In the same way, it turns out that when two zeros are added to a number on the right, each digit moves to

the left by two places and receives a value 100 times greater; when three zeros are added, each digit moves to the left by three places and receives a value 1,000 times greater.

The reverse operation—reducing a number by 10, 100 times, etc.—is explained using similar methodological techniques: 1) discarding zeros, 2) comparing the received and given numbers, 3) finding out the reason for the decrease in the number, and 4) deriving a rule (how to reduce a number by 10, 100, 1000 times).

Transformation of the composition of a number. The student must be able to represent each given number in various combinations of its parts (for example: 35,675 is 356 hundreds + 75 units; it is 3,567 tens + 5 units; it is 35 thousands + 675 units, etc.). This skill will be of invaluable service to the student when he approaches the study of multiplication and division, especially division, which is impossible without breaking down the dividend into its parts.

The transformation of a number comes down to two operations: breaking up and converting some digit numbers into others.

The division consists in expressing a given number of tens, hundreds, thousands, etc. in units.

Conversion, on the contrary, consists in the fact that a given number of units is converted into larger digit units—tens, hundreds, thousands, etc.

Division. There are 45 tens. Let's divide this number into units. How many units are in 45 tens? 10 tens = 100 units, 40 tens = 400 units, and 5 tens = 50 units, or 450 units in total. So, 45 tens = 450.

We also establish that 32 hundreds = 3,200; 564 hundreds = 56,400.

Transformation. The number 700 is given. Let us express this number in tens, in other words, let us find out how many tens are in 700, or in 7 hundreds? There are 10 tens in one hundred. There are 70 tens in 7 hundreds. This means that 700 = 70 tens. There are 90 tens in the number 900. This means that to find out how many tens are in the number 720, we drop the zero and get 72. Thus, 720 = 72 decimals; 930 = 93 decimals; 6,840 = 684 decimals, etc.

We will also show that 200 is 2 hundreds; 600 is 6 hundreds; 1,500 is 15 hundreds. Therefore, 8,600 = 36 hundreds; 8,900 = 89 hundreds; 14,600 = 146 hundreds.

But how many tens will there be in a number that ends not in zero, but in some significant digit? For example, how many tens will there be in the numbers 75? 456? 1,238? 25,815?

It is clear that there will be no tens only in the ones place. There are tens in all other places. We discard the ones, and the rest of the number will be the number of tens. Therefore, 75 = 7 decimals + 5 units; 456 = 45 decimals + poor; 1,238 = 123 decimals + 8 units.

How **many hundreds** are there in the numbers: 638? 3,149? 63,714? There are no hundreds only in the units and tens: they are less than a hundred. But there are hundreds in all other places. Therefore, by discarding the units and tens in a number, we get the number of hundreds. Thus: 638 is 6 hundreds + 38 units; 3,649 = 36 hundreds + 49 units; 63,714 = 637 hundreds + 14 units.

All thousands and other digits are also highlighted in numbers.

Oral and written numbering of billion class numbers

This section of numeration is studied in the 4th grade. The study is carried out in the same way as the study of numeration within the class of thousands and class of millions.

As a result of studying the numeration of multi-digit numbers in grades III and IV, students should acquire the following knowledge:

When counting, simple units are grouped into composite units of counting—tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, etc., which are counted in the same way as units.

Each unit of account constitutes a special category and, when written down, occupies a strictly defined place.

The ranks are grouped into classes—3 ranks in each class.

Each composite unit of counting of a higher rank contains 10 units of the next lower rank; therefore, our number system is called decimal.

There are 10 digits for writing numbers—nine significant ones and a tenth—zero, which is placed in place of the missing units in the digit.

The same digit can represent the number of units of any rank, depending on where it stands.

Large numbers interest and fascinate children. They learn from the teacher that the series of numbers is infinite; there is no largest number. To any number, no matter how large it is, you can add another unit, and you get an even larger number. And these large numbers are written according to the same numbering rules as the numbers denoting millions and billions.

The teacher will do well to write and read several large multi-digit numbers that go beyond the 12-digit numbers.

Let's take, for example, the 15-valued number 209 318 427 536 708.

How to read this number? Let's break it down into classes, we'll get 5 classes. The fifth class is called trillions. We read: 209 trillion 318 billion 427 million 536 thousand 708 units.

Let's take the 18-digit number 912,803,734,645,701,856 and read it: 912 quadrillion 803 trillion 734 billion 645 million 701 thousand 856 units.

With such examples, children will more strongly feel the regularity of the construction of the decimal number system and the possibility of an infinite continuation of the system, part of which they have studied.

WRITTEN CALCULATIONS IN III AND IV CLASSES

One of the main tasks of teaching arithmetic in elementary school is to teach children written calculations. Written calculations are a necessary component of mathematical literacy. Along with solving problems, they constitute the main content of the 3rd grade program. In addition, written calculations are also included in the 4th grade program, where they are repeated.

A student completing the 4th grade must have good written calculation skills.

Each calculation as a whole and each calculation operation separately must be performed consciously. The rules must be clear and justified for the student.

Written calculations must always lead to **correct, error-free** results. Errors in calculations are intolerable to the same extent as spelling errors in writing.

Written calculation skills must be **firm and stable**. Once acquired, they must not be lost. A skill that is quickly lost is an incomplete skill.

Written calculations must be performed not only correctly, but also **quickly and confidently.** Calculations performed slowly and incoherently indicate immaturity of the skill, insufficient work on its development and consolidation.

Written calculations should always be performed **in rational ways** , i.e. in ways in which the calculations do not contain anything superfluous, are performed with a minimum expenditure of time and on the basis of reasonably used laws of arithmetic operations underlying the computational techniques.

The calculation records should be brief, convenient, and easy to review. They should be arranged on the notebook page in a certain **order and symmetrically.**

A thorough explanation of the rule for the written performance of each arithmetic operation is achieved by: a) skilfully breaking down a complex skill into its component elements—into separate "cases," b) arranging these cases in the order of their gradual increase in complexity, c) highlighting **what is special and** unique in each new case, d) emphasising the elements **of similarity between** the new case and those previously studied, and e) using visual aids. With the help **of reasoning** accompanying the calculations, each operation separately and the rule as a whole are substantiated. A clear understanding of the mechanism of written calculations is facilitated by the use of an analytical technique in the explanation (see examples on p. 466), in which a complex whole is broken down into its component elements. Generally accepted, standard forms of recording calculations are in some cases introduced not immediately, but gradually, through intermediate forms.

The first exercises in the skill, aimed at deepening the understanding of computational operations, are performed by the student with detailed reasoning with the direct assistance of the teacher. In subsequent exercises, aimed at automating the skill, a gradual transition occurs: a) from detailed reasoning to brief, schematic ones, with the admission of accepted conventions in the language; 6) from exercises with direct assistance of the teacher to completely independent exercises; c) from easy examples to difficult ones; d) from a slow pace of calculations to a faster one. Exercises are conducted on such a number of

examples that ensure the acquisition of a skill that meets the above requirements. An objective indicator of the sufficiency or insufficiency of the exercises are the results of the written test. Errors made in the test are recorded and classified, their causes are established, and then work is carried out to eliminate them. If the cause of the errors lies in an insufficiently clear understanding of the computational operation, an explanation is given that deepens this understanding; if it lies in an unsteady skill, additional training exercises are given.

The acquired skill is reinforced through systematic repetition, with much attention paid to difficult computational operations.

Addition of multi-digit numbers

The study of the technique of written addition must be combined with the expansion of the concept of this arithmetic operation. Therefore, familiarisation with written addition begins with an explanation of the terms **addends and sum** . These terms are introduced into the student's speech and are reinforced through their constant use.

Written addition is done according to a certain rule. Compliance with this rule has great practical significance: computational work is facilitated and the probability of errors is reduced. Therefore, when explaining the mechanism of written addition, the students' attention is focused on the elements of this rule, and students are required to follow it precisely.

In the exercises, students are offered various variants of examples: without transition and with

transition through digits, without zeros and with zeros in the terms, with a smaller and larger number of digits and terms, with a larger number of digits first in the first term, then in the second, etc. The more diverse the variants, the more deeply the students understand the single principle of their solution.

Particular attention is paid to solving examples with three, four and five terms. These examples reinforce the skills of not only written addition of multi-digit numbers, but also oral addition of single-digit and double-digit numbers with single-digit numbers.

First, examples are solved in which the terms are already arranged in a column, and then examples are offered in which the terms are written in a row, and the students have to write them down themselves to calculate in a column.

For example: 48639 + 5796 + 647958 + 16078 =

48639		647958
5796		48639
+ 647958	or +	16078
16078		5796

In such cases, the placement of terms under each other forces the student to match the same-named digits, and this in itself is a very useful exercise that reinforces the knowledge of the composition of a number.

These examples also give students practice in the practical application of **the commutative** property of addition.

The explanation that accompanies the solution of examples on the blackboard can be detailed or brief. With a detailed explanation, the student **names the ranks** of the numbers he adds.

Such explanations are required from the student during the initial exercises during the first 2—3 lessons.

Later, children are accustomed to brief explanations—to the names of only the results of addition. For example:

$$\begin{array}{rr} & 1896 \\ + & 3748 \\ \hline & 4896 \end{array}$$

"6 plus 8 = 14, 14 plus 6 = 20. 0 I write, 2 in my mind. 2 plus 9 = 11 plus 4 = 15 plus 9 = 24. 4 I write, 2 in my mind," etc. Or even shorter: "6, 14, 20. 0 I write, 2 in my mind. 2, 11, 15, 24. 4 I write, 2 in my mind," etc.

The addition is checked by performing the operation again, with the calculation performed in a different order (from bottom to top).

Students must learn to perform written addition without errors. The written test on addition must include the main cases of written addition: 1) the general case of addition (18756 + 4897); 2) the case with zeros in the addends (40007 + 60093); 3) addition of 3 addends (38658 + 4976 + 647958).

When processing the results of the test, the nature of the errors made and their causes are established. Depending on the number and nature of the errors, additional exercises are conducted to eliminate these errors.

In the 4th grade, the skill of written addition **is improved** . Thanks to the exercises, children perform

addition more confidently, quickly and without errors. The exercises provide examples with a large number of terms, in solving which students use not only the commutative but also **the associative** property of addition.

Subtraction of multi-digit numbers

The study of subtraction, like addition, begins in grade 3 with the study **of the terminology** of this operation: minuend, subtrahend, remainder or difference, the sign of subtraction—minus. Clarification of these terms helps to form the concept of subtraction as an arithmetic operation. This is done on problems. The terms are written on the blackboard and in the students' notebooks, memorized and introduced into the student's speech during arithmetic lessons. The notes are usually given the following form:

6589 — minuend
− 4157 — subtrahend
2432 — the remainder, or difference

The rule of written subtraction is formulated by analogy with the rule of written addition. When explaining the solution of the first examples, the main points of this rule are emphasized: strictly digit-by-digit signature of the subtrahend under the minuend and the beginning of the subtraction process from units.

At the next stage of the work—in explaining those cases of subtraction when some digits of the subtrahend are greater than the corresponding digits of the minuend—the process of **borrowing** and subsequent

division of the borrowed unit is explained specifically and consistently.

Before subtracting any number from a minuend expressed as a unit with zeros (1000–856; 1,000,000–765,472, etc.), it is shown on visual aids (on a classroom abacus) how such a minuend can be transformed, namely:

1000 is 9 hundreds, 9 tens and 10 units;

1,000,000—9 hundreds of thousands, 9 tens of thousands, 9 thousands, 9 hundreds, 9 tens and 10 units.

Using individual examples like 0—8 (subtract 8 from zero) and real-life facts, the teacher explains that you can't subtract from zero. Using examples like 4—0 (subtract zero from four), it is explained that when you subtract 0 from a number, the same number remains. Thanks to such explanations, made in time, possible mistakes are prevented and the level of awareness in mastering written subtraction increases.

During the initial exercises, detailed explanations are given, for example:

$$\begin{array}{r} 4362 \\ -- \quad \underline{1895} \\ 2467 \end{array}$$

"You can't subtract 5 from 2. We take 1 ten (the student puts a dot above the number 6) and break it down into units. 10 plus 2 = 12. Subtract 5 from 12 equals 7. You can't subtract 9 tens from 5 tens. We take one hundred and break it down into tens. Subtract 9 tens from 15 tens equals 6 tens," etc.

As students gain mastery of subtraction, they move to brief explanations during exercises.

The exercises should include not only simple, easy cases of subtraction, but also difficult ones. The latter include cases of subtraction with zeros in the minuend and those general cases when it is necessary to resort to a whole series of exercises. Examples:

26483	1000000	100102
− 18694	− 987164	− 96278

Checking subtraction in the 3rd grade is done by **adding** the remainder obtained to the given subtrahend. Such checking, in addition to its direct purpose of detecting an error, helps children to understand the relationship between addition and subtraction. Checking can be done based on the record made, without rewriting the example again. It is useful to offer students to check examples with ready answers: 37824—25938=12886; among such examples there may be 1—2 examples with incorrect answers. Finding an error increases interest in the work.

The written test must include all cases typical for subtraction. The test may also include a check of the action (check: 4623—1872=2751) and a task to name the numbers in subtraction. When processing the results of the test, it is important to consider the solution of each example separately.

In the 4th grade, the concept of subtraction is clarified and formalized. Here, the relationship between the members of this action and the change in the difference depending on the change in the minuend and subtrahend are considered and formulated. The skill of written subtraction in this grade is improved and automated. Here, complete error-freeness must be

achieved in solving examples not only for general, but also for specific cases of subtraction.

Checking subtraction in this class is done not only by addition, but also by subtraction. Based on the relationship between the minuend, subtrahend and remainder ("the subtrahend is equal to the minuend without remainder"), the student, checking subtraction, subtracts the difference from the minuend and, having obtained the subtrahend, is convinced of the correctness of the subtraction.

Multiplication of multi-digit numbers

Multiplication by a single-digit number. The study of multiplication begins with establishing the connection between multiplication and addition and with clarifying the terminology of multiplication: multiplicand, factor, product, factors.

These terms should be heard in the speech of the children and the teacher throughout this topic. (It is important to remember that the multiplicand is written first and the multiplier is written second, but when reading the multiplication table, the multiplier is sometimes called first; thus, the example "6 X 5 = 30" is read "five times six = 30".)

When starting written multiplication, you need to review the multiplication table and numbering (breaking down a number into decimal numbers and selecting the required decimal units from the number; for example: 52 hundreds are 2 hundreds and 5 thousand), since the success of learning multiplication depends on this.

When starting to explain written multiplication, the teacher starts from the addition of equal terms.

The first examples for each case of multiplication are solved with detailed explanations. But in the process of exercises, detailed explanations give way to short ones, in which only a few and most necessary words are pronounced. For example: “six times nine = 54. 4 I write, 5 — in my mind. Six times eight = 48 and 5 = 53. 3 I write, 5 — in my mind” etc.

Before moving on to multiplying numbers with zeros in the middle, we need to show how zero is multiplied by a number.

0 X 4 = 0 + 0 + 0 + 0 = 0. If you take zero 4 times, you get zero. Zero is multiplied like any other number. For example:

$$\begin{array}{rr} & 730406 \\ \times & \underline{\quad\quad 4} \\ & 2921624 \end{array}$$

“Four times six = 24. I write 4, 2 in my mind. Four times zero = 0 and 2 = 2. Four times four = 16. I write 6, 1 in my mind,” etc.

When multiplying a single-digit number by a multi-digit multiplicand, the multiplier is multiplied by each digit of the multiplier.

$$\begin{array}{rr} & 8 \\ \times & \underline{4732} \\ & 37856 \end{array}$$

However, such a notation is inconvenient, and the order of multiplication is unusual. Therefore, having solved several examples in this way, it is necessary to teach the students the method of rearranging the

factors as a more convenient method of multiplication in cases where the multiplicand is a single-digit number.

$$\begin{array}{rr} & 4732 \\ \times & \underline{\quad\ 8} \\ & 37856 \end{array}$$

Before this, we need to repeat the commutative property of multiplication (the product does not change when the factors are rearranged).

Multiplication by one with zeros and by round numbers.

1. 325 x 10 = 3,250. If you multiply one unit by 10, you get one ten; if you multiply 325 units by 10, you get 325 tens, or 3,250.

2. 84 X 100 = 8,400. Multiplying one unit by 100 yields one hundred; multiplying 84 units by 100 yields 84 hundreds, or 8400.

3. 964 x 1000 = 964,000. One unit multiplied by 1000 gives one thousand; 964 units multiplied by a thousand gives 964 thousand, or 964,000.

Let us compare the multiplicands and factors with the products in all these examples and derive the rule: “To multiply a number by one with zeros, it is enough to add as many zeros to the right of the multiplicand as there are in the factor.”

Multiplication by one with zeros is always written in the line: 827 X 1000 = 827,000.

Here, examples in which the multiplicand is a unit with zeros are solved on the basis of the commutative property of multiplication:

100 X 76, 1000 X 485, 10,000 X 63, etc.

Multiplication by round numbers can be explained with the following examples:

1. 725 X 30 = (725 X 3) X 10 = 2175 X 10 = 21750. Or:

$$\begin{array}{r} 725 \\ \times \quad 30 \\ \hline 21750 \end{array}$$

2. 486 X 400 = (486 X 4) X 100 = 1944 X 100 = 194400. Or:

$$\begin{array}{r} 486 \\ \times \quad 400 \\ \hline 194400 \end{array}$$

Thus, multiplying a number by round tens comes down to multiplying this number by the number of tens and by 10. Multiplying by round hundreds comes down to multiplying—first by the number of hundreds, then by 100, etc. This rule is not communicated to children dogmatically, but is first shown with a visual example. “There are 300 nickels in the cash register. What is the quickest and easiest way to calculate how much money this is?” You need to take 300 times 5. This can be done like this: take 100 times 5, another 100 times 5, and another 100 times 5. Let’s write this down: 5 X 300 = (5 X 100) + (5 X 100) + (5 X 100) or (5 X 100) X 3. You can also count differently: you can take 3 nickels and repeat them 100 times, i.e. (5 X 3) X 100 = 1500 (kopecks).

The form of writing column multiplication follows naturally from the reasoning. Students already know that multiplication by a unit with zeros comes down to adding the corresponding number of zeros. Therefore,

to multiply, for example, 567 by 800, it is enough to multiply 567 by 8, subtract the factor 8 under the units, and then add two zeros to the resulting product. Students should clearly understand that by adding two zeros, they multiply the number by 100.

It is useful to do with children several exercises in decomposing round numbers:

600 = 6 X 100 300 = 3 X 100

8000 = 8 X 1000 80 = 8 X 10

Multiplication by two-digit and three-digit numbers.

Multiplying a number by a two-digit factor, for example, the number 875 by 37, comes down to multiplying the given number first by 30, then by 7, and adding the resulting products. It is customary to multiply the number first by **the units** of the factor, and then by **the tens** :

```
   875
X   37
  6125
+2695
 32375
```

With this notation, the student is required to be able to correctly sign the second partial product (5 tens under two tens) and understand that the second partial product means 2625 **tens**, or 26250.

To make the meaning of the second product clearer to children, it is useful to use an analytical technique when explaining the method of multiplication, breaking down the multiplication operation into its component parts, namely:

```
  875        875        6125
X   7      X  30    +  26250
 6125       26250      32375
```

Multiplication by a three-digit factor, for example, multiplying the number 468 by 349, comes down to multiplying the given number first by 9, then by 40, then by 300 and adding the resulting products (the order of multiplication could also be different, i.e. it would be possible to multiply first by 300, then by 40 and then by 9)

```
   468
X  349
  4212
 1872
+1404
163332
```

In such a notation, the student must clearly imagine that the second incomplete product means 1872 tens, or 18720, and the third incomplete product means hundreds—1404 hundreds, or 140400. In order for these products to appear with precisely this meaning, it is useful to apply the above analytical technique when explaining, multiplying by each digit of the factor separately.

Multiplication by numbers with zeros in the middle and at the end. If the multiplier lacks units of any digit and there are zeros in their place, then multiplication by zeros is not performed. The entry for the product

obtained from multiplication by units of the digit following the zero is moved one place to the left.

$$\begin{array}{r} 4832 \\ \times\ 608 \\ \hline 38656 \\ +\ 28992 \\ \hline 2987856 \end{array}$$

The second incomplete product is read as follows: 928992 hundreds, or 2899 200. In order for the meaning of the second incomplete product to be clear to children, you can show separately the multiplication by units (8) and by hundreds (600).

When multiplying numbers ending in zeros, for example, 37600 X 40, the following rule must be observed: “Numbers ending in zeros are signed during multiplication so that the significant digits are under the significant digits. The significant digits are multiplied and as many zeros are added to the resulting product as there are in both factors”:

$$\begin{array}{r} 37600 \\ \times\ \ 40 \\ \hline 1504000 \end{array}$$

This rule contains two parts: one tells how factors **are signed**, and the other tells how factors **are multiplied.** When explaining this case of multiplication and formulating the rule, it is necessary to draw the students’ attention to each part of this rule separately. The explanation is given in the following sequence:

37600 is 376 hundreds; 40 is 4 tens. To multiply 376 hundreds by any number, multiply 376 by that number

and add two zeros to the resulting product. To multiply any number by 4 tens, multiply that number by 4 and add one zero to the resulting product. Therefore, to multiply 376 hundreds by 4 tens, you can first multiply 376 by 4, ignoring the zeros for now, and then add 3 zeros to the resulting product.

The **test** to check knowledge of multiplication should include all the main cases of multiplication: 467

1) multiplication of numbers with all significant digits (857 X 396);

2) multiplication with zeros in the multiplicand (2008 X 47);

3) multiplication with zeros in the multiplier (564 X 308);

4) multiplication of numbers ending in zeros (280 x 540).

In the IV. grade, the concept of multiplication is clarified and formalised. Here, the relationship between the components of multiplication and the change in the product depending on the change in the factors are studied. The previously repeatedly used permutation of factors is now recognised by students as the main property of multiplication. Students illustrate this property with examples (12 X 5 = 5 X 12) and widely use it in mental arithmetic (4 x 13 x 25 = 25 x 4 x 13), as well as in written calculations when solving examples and problems, when checking an operation.

The associative property of multiplication is used by children in oral calculations (25 x 16 = 25 x 4 x 4 = 400).

Some rationalisation is introduced into the methods of written calculations. Students are taught to write the multiplication in a line when multiplying a multi-digit number by a single-digit number:

73846 X 4 = 295384

The same when multiplying by round tens:

5263 X 60 = 5263 X 6 X 10 = 315780

In the 4th grade, written multiplication skills are consolidated in their most difficult cases; this includes multiplication of three— and four-digit numbers with a digital composition of factors — 6, 7, 8, 9, cases of multiplication of numbers with zeros in the middle and at the end.

In the 4th grade, in multiplication, complete error-free calculations must be achieved while performing them quickly enough.

Division of multi-digit numbers

Division by a single-digit number—is the first step in learning the complex mechanism of written division. At this step, children learn all the basic elements of the division process and its order; they learn that division begins with the highest digits of the dividend, and that dividing each digit in the quotient yields units of the corresponding digits; if any digit of the dividend is not divisible entirely and the quotient does not yield units of the corresponding digit, then zero is put; having found the digit of the quotient, they multiply it by the divisor and find out what number they have divided; they find the remainder by subtraction; the remainder must not be greater than the divisor; having broken up the remainder and added to it the units of the next digit of the dividend, they form a new incomplete dividend,

which they treat in the same way as the first incomplete dividend, etc. All this is learned in a practical way, gradually, by solving examples.

Sample explanation and recording of the solution to example 2736: 6.

```
   2736    L 6
− 24       456
− 33
− 30
  36
  36
   0
```

“The highest digit in the dividend is thousands, 2 thousands. 2 thousands cannot be divided by 6 to get at least one thousand. We separate two digits in the dividend, we get 27 hundreds. We divide 27 hundreds by 6. The quotient is 4 hundreds. We multiply 4 by 6 to find out how many hundreds we divided 468 into. Four times 6 = 24. We subtract 24 from 27. The remainder is 3 hundreds. We split 3 hundreds into tens, we get 30 tens, we add 3 tens of the dividend to them, we get a total of 33 tens. We divide them by 6, the quotient is 5 tens. Five times 6 = 30”, etc.

Later, when children understand the meaning and sense of each separate calculation operation quite clearly, some simplifications are introduced into this reasoning scheme, making it more concise. Thus, a student, looking at the divisor, can immediately say: “We separate 2 digits in the dividend and divide 27 hundreds by 6.” Then, having received 3 hundreds as a

remainder, the student can say: “We bring down 3, we get 33 tens.”

But you should **always** require the student to name the ranks of the incomplete dividends and the ranks of the quotient: “We divide 27 hundreds, the quotient will be 4 hundreds.”

The division notation should be detailed at first, as indicated above. The digits of the dividend are carried strictly vertically. The products are written down. Subtraction signs are not required. When the teacher is convinced that the children understand the division method and moves on to exercises, he transfers the children to a shorter division notation, in which the products are not written down, but subtracted orally from the incomplete dividends. For example:

```
2736 L 6
 33    456
 36
  0
```

In the future, you can use an even shorter notation in a line, when neither the products nor the remainders are written down: 2736: 6 = 456.

Among the examples there should also be some in which division results in a remainder, for example:

```
7629 L 8
 42    953
 29
  5
```

The remainder is not added to the quotient.

The teacher draws the children's attention to the fact that the remainders must be less than the divisor; if the remainder is greater than the divisor, this means that the wrong digit was taken in the quotient.

Particular attention should be paid to those division cases in which zeros are written in the quotient — in the middle and at the end:

a) 5642 | 7
42 | 806
0

6) 32 0160 | 4
16 | 80040

c) 5241 | 2
12 | 2620
4
1

In order to avoid making mistakes related to missing zeros in the quotient, children must firmly understand that if any digit of the dividend is not divisible by the divisor, then there will be no units of that digit in the quotient and, therefore, a zero must be put in place of that digit in the quotient.

To prevent such errors, there are a number of methodological techniques and tools, namely:

a) the ability to determine from the first digit of the quotient how many digits there should be in the quotient;

6) designation by dots of the places of each category in the quotient;

c) the ability to determine the approximate value of the quotient before actually dividing the given number;

d) checking the division by multiplying the quotient by the divisor.

The teacher should use, if not all, then some of these techniques.

When dividing 5241 by 2, it is useful to first determine the approximate value of the quotient. This

should prevent an error—missing a zero at the end of the quotient.

Thus, by studying division by a single digit, students learn most of the concepts associated with the mechanism of written division.

Division by one with zeros. Let's take two examples: 4680/10 and 7324/10. To divide 4680 by 10 means to find out how many times 10 is contained in 4680 or how many tens are in this number. There are 468 tens in 4680. So, 4680/10 = 468.

When dividing 7324/10, we reason in the same way and come to the conclusion that dividing 7324 by 10 gives 732 as a quotient and 4 as a remainder. 7324/10 = 732 (remainder 4).

Having considered several more examples of division by 100 and by 1,000, we can derive the rule: "In order to divide a number by one with zeros, it is sufficient to separate from the dividend on the right as many digits as there are zeros in the divisor; then the remaining digits of the dividend will represent the quotient, and the separated ones will represent the remainder."

Division of a number by one with zeros is always written only in a line.

Division into round tens, hundreds, thousands. In cases where the dividend and divisor end in zeros, the latter are not crossed out or reduced, the numbers are divided as they are given. It is important that students learn to find the quotient digit based on the relationship between multiplication and division: 560 divided by 70 will give a quotient of 8, because 8 times 70 is 560. 3,200 divided by 800 will give a quotient of 4, because 4 times 800 is 3,200, etc.

When dividing into round tens, two cases are distinguished:

1. The first two digits of the dividend represent the number divided by the divisor, for example: 960 : 80.

This is where the teaching of division into round tens begins.

2. The first two digits are not divisible by the divisor and you have to separate three digits in the dividend, for example: 2150: 50.

Division into round hundreds is studied according to the same plan as division into round tens: first, cases are considered when the first 3 digits of the dividend make up a number divisible by the divisor (round hundreds), 800: 200; 9600: 300; 8470: 400.

Then, examples are solved in which 4 digits have to be separated from the dividend to find the first digit of the quotient: 4320: 600; 211500: 500.

When solving these examples, there is no need to resort to simplified methods for finding the quotient digit; the quotient is found by trial and error based on the relationship between division and multiplication; thus, when dividing 4320 by 600, the student takes into account that there are 43 hundreds in the dividend and 6 hundreds in the divisor. How many times should you take 6 hundreds to get 43 hundreds? 7 times (7 times 6 hundreds—42 hundreds).

Division by a two-digit number. To be able to divide any multi-digit number by a two-digit number, you need to be able to divide a two-digit number by a two-digit number and a three-digit number by a two-digit number with a one-digit quotient. Indeed, to divide 7840 by 32, you have to divide: a) 78 by 32, b) 144 by 32 and c) 160 by 32.

The division of a two-digit number by a two-digit number has been covered when studying hundreds. Now we need to explain to children the method of written division of a three-digit number by a two-digit number with a single-digit quotient. When explaining, special attention is paid to the question of how to quickly and correctly find the digit of the quotient in a simplified way (divide the tens of the dividend by the tens of the divisor and test the digit of the quotient, making appropriate adjustments to it).

Explanation and exercises can be carried out on the following group of examples:

1) 560 L 70
   560 8
   0

In this example, to find the quotient digit, it is enough to divide the number of tens of the dividend by the tens of the divisor (56:7==8).

2) 428 L 60
   420 7
   8

Examples of this type show that even if there are units in the Dividend, the method for finding the quotient digit remains the same: the tens of the dividend are divided by the tens digit of the divisor (42: 6 = 7).

3) 328 L 41
   328 8
   0

The divisor 41 is close to the round number 40, so in order to find the quotient in this example more easily and quickly, we divide 38 tens by 4 tens. We test the resulting number “8” to see if it will be correct when dividing 328 by 41. We mentally multiply 41 by 8. We get 328. This means that the number 8 will be the quotient when dividing these numbers. We write it in the quotient.

There should be as many examples as possible where the divisor is an easily rounded number (31; 62; 73; 51; 82; 24, etc.) so that children can master the technique of quickly finding the quotient digit.

$$\begin{array}{rl} 4)\ 245 & \text{L}\ 35 \\ \underline{245} & \quad 7 \\ 0 & \end{array}$$

Examples of this type show that the quotient digit is not always found immediately by dividing the tens of the dividend by the tens of the divisor; sometimes it is necessary to reduce this digit by one after testing it; thus 24 divided by 3 gives 8; however, testing shows that the digit 8 is not suitable: it is large, it must be reduced by one.

$$\begin{array}{rl} 5)\ 273 & \text{L}\ 39 \\ \underline{273} & \quad 7 \\ 0 & \end{array}$$

Examples of this type show that in cases where the units digits of the divisor are 7, 8 or 9, it is useful to round the divisor to the highest round number, in other words, to increase the tens digit by one.

Indeed, dividing 27 by 3, we get the number 9, which then has to be reduced by 2 units, while dividing 27 by 4, we immediately find the quotient number.

After children have mastered the skill of dividing a three-digit number by a two-digit number, you need to move on to an exercise in dividing any multi-digit number by a two-digit number. Among the examples with a two-digit divisor, examples in which the quotient is obtained with zeros in the middle and at the end should occupy a prominent place:

a) 26112 L 64		6) 268280 L 38		c) 8514 L 17	
256	408	266	7060	85	500
512		228		14	
512		228			
0		0			

Division by a three-digit number. The division of a multi-digit number by a three-digit number ultimately comes down to a series of divisions of three- and four-digit numbers by a three-digit number. For example, the process of dividing the number 107,442 by 254 breaks down into the following series of divisions:

a) 1074 L 254 6) 584 L 254 c) 762 L 254

Therefore, in order to correctly divide a multi-digit number by a three-digit number, you need to be able to divide:

a) a three-digit number by a three-digit number and

6) a four-digit number divided by a three-digit number with a one-digit quotient.

When dividing a three-digit number by a three-digit number, you need to find the quotient digit based on

the relationship between division and multiplication, asking the question: “how many times do you need to take the divisor to get the dividend?”

For example, let’s say we need to divide 804 by 268:

$$\begin{array}{rl} 804 & \mid 268 \\ \underline{804} & \quad 3 \\ 0 & \end{array}$$

To divide 804 by 268 means to find out how many times 268 is contained in the number 804. Maybe 2 times? No, because 268 times 2 gives a number slightly greater than 500. Maybe 3 times? Yes, 268 times 3 gives 804. How can we make it easier to find the quotient digit? We need to draw students’ attention to the fact that the highest digits are of primary importance when finding the quotient digit, in this case the hundreds—8 hundreds in the dividend and 2 hundreds in the divisor. 2 hundreds are contained in 8 hundreds 4 times. However, the number “4” is not suitable as a quotient, because in the dividend, in addition to 8 hundreds, there are only 4 ones, and in the divisor, in addition to 2 hundreds, there are 68 ones. We decrease the number 4 by one and get the correct quotient—3.

With such examples, children learn to find the quotient in a simplified way and check it orally; the latter is very important for successfully dividing any multi-digit number by a three-digit number.

The ability to find the quotient digit in a simplified way is very important when dividing a four-digit number by a three-digit number. For example, let’s say we are given 1384 divided by 346. How many times do we need to take 346 to get 1384? It’s hard to say. But how many times do we need to take 3 hundreds to get 13 hundreds? It’s easy to determine: 4 times. However, let’s not rush to put this digit in the quotient. Let’s

check it first: it may turn out to be correct when dividing by 346, or it may turn out to be unsuitable. Let's check it orally, so that we don't have to correct it later or cross out this digit if it's unsuitable. It's hard to multiply 346 by 4 orally. To make the verbal check easier, let's take only the tens of the divisor (34), multiply them by 4 and compare the result with the tens of the dividend (138). 4 times 34 == 136. Comparing 136 and 138, we see that the number 4 can be taken as a quotient.

Thus, a simplified method for finding the quotient digit when dividing by a three-digit number is as follows: a) divide the hundreds of the dividend by the number of hundreds of the divisor and test the found digit by verbally multiplying the tens of the divisor by this digit.

However, when using this technique, children need to be taught to pay attention to the divisor as a whole, so that individual numbers do not obscure the entire number.

Let's say we need to solve the following example:

$$\begin{array}{ll} 2415 & L\,^{396} \\ \underline{2376} & \quad 6 \\ \quad 39 & \end{array}$$

It would be inappropriate to try the number 8 here on the grounds that 24 : 3 = 8. After all, the number 396 is almost 400; 8 times 400 gives 32 hundreds, or 3200. So, 8 is a lot. The number 7 is also large. 7 times 400 = 2800. Let's try the number 6. 6 is good. This number could have been found immediately by rounding 396 to 400 and dividing 24 hundreds by 4 hundreds.

After such preparatory work, an exercise is carried out in dividing a multi-digit number by a three-digit number, for example: 51243: 589; 410675: 175, etc.

When solving such examples, it is necessary to emphasize that the remainders must not be greater than the divisor; in the process of division, it is necessary to monitor the remainders and compare them with the divisor; if the remainder is equal to the divisor or greater than it, then this means that the wrong digit was taken in the quotient.

As with dividing by a two-digit number, here you need to give a fairly large number of such examples where you get a quotient with zeros in the middle and at the end; for example:

а) 2564135 L427	6) 1493500 L725	в) 648154 L 216.
2562 6005	1450 2060	648 3000
2135	4350	154
2135	4350	
0	0	

The division tests include all the main cases of division by a single-digit number:

a) general case of division (76428: 6);

6) a special case—a quotient with zero in the middle (48312: 8, quotient—6039);

c) a special case—a quotient with zeros in the middle and at the end (490630: 7, quotient—70090); d) division with remainder (51087: 9);

d) division with remainder, where the quotient ends with zero (3154:5, quotient—630, remainder 4).

The main cases of division by two-digit and three-digit numbers:

a) the general case of division by a two-digit number (37952: 64);

6) the general case of division by a three-digit number (216221: 463);

c) the case of division with zeros in the middle and at the end of the quotient (3673160: 458, quotient— 8020);

d) the case of division with remainder, when zeros must be placed at the end of the quotient (7012:14). In the 4th grade, the skills of written division acquired in the 3rd grade must be consolidated and improved. The improvement of the skill must find its expression in the complete infallibility of calculations, in a more conscious, confident and rapid execution of the action.

When dividing by a single-digit number and by some double-digit numbers, intermediate computational operations of multiplication and subtraction can be performed mentally.

When dividing numbers ending in zeros, it is possible to use the abbreviated division method, for example:

$$\begin{array}{l|l} 292400 & 8600 \\ \underline{258} & 34 \\ 344 & \\ \underline{344} & \\ 0 & \end{array}$$

In explaining this case of division, one must rely on the well-known property of the quotient: the quotient will not change if the dividend and divisor are reduced by the same number of times. In this case, one must carefully explain the change in the remainder if the division has a remainder.

Simultaneously with the consolidation of skills in the 4th grade, **the concept of division** as an arithmetic operation is expanded and formalized.

The relationship between the dividend, divisor, and quotient is studied. Knowledge of this relationship is used to verify division. The change in the quotient with a change in data is studied; the immutability of the quotient with a simultaneous increase or decrease in the dividend and divisor is used to justify the technique of abbreviated division of numbers ending in zeros.

SAMPLE LESSONS

III CLASS

Topic: Dividing a four-digit number by a three-digit number

Lesson 1.

The purpose of the lesson is to explain to children the method of finding the quotient when dividing a four-digit number by a three-digit number (with a one-digit quotient).

Lesson progress. 1. The following knowledge and skills are the basis and starting point for this lesson: a) the ability to divide multi-digit numbers by round hundreds; 6) the ability to mentally multiply three-digit numbers by single-digit numbers; c) understanding the connection between division and multiplication and the ability to use this connection to check the quotient digit.

Based on this, the teacher gives the following examples and problems for repetition in the form of oral counting:

a) Division into round hundreds: 600:300; 1200:400; 2500:500, etc. The purpose of these exercises is to

emphasize that to find the quotient it is sufficient to divide the number of hundreds of the dividend (6, 12, 25) by the number of hundreds of the divisor (3, 4, 5).

6) Oral multiplication of three-digit numbers by one-digit numbers: 4802; 5304; 780 X 3.

c) Checking division using multiplication: “Divide 48 by 16. Check the quotient. Divide 84 by 12. Check the division is correct!” etc.

Tasks:

“The plane flies at a speed of 400 km per hour. How many hours will it take to fly a distance of 1200 km?”

“A meter of cloth costs 120 rubles. How much do 7 meters of this cloth cost?”

2. After the review, the teacher tells the children the goal of the current lesson (to learn how to correctly and quickly divide multi-digit numbers by three-digit numbers) and begins to explain the new material. The explanation is given using examples that are selected so that the following pattern can be easily discovered: when dividing a three— or four-digit number by a three-digit number, the quotient is the same as when dividing the hundreds of the dividend by the hundreds digit of the divisor.

The following examples should serve to establish this pattern and obtain such a conclusion:

1) 1248:318; 2) 9580:516; 3) 1392:232; 4) 2905: 415.

The starting point in explaining new material should be the reproduction of what will serve as the basis for this new material. In this case, the starting point is the division of a four-digit number into round hundreds. Therefore, the explanation begins with solving an example like:

$$\begin{array}{ll} 1825 & \mid 300 \\ \underline{1800} & \quad 6 \\ 25 & \end{array}$$

This division case is familiar to children—they solve it on their own and, when explaining, emphasize that to find the

result, it was enough to divide 18 by 3 (18 hundreds by 3 hundreds).

After this, the first example from the new material is offered:

$$1248 \text{ L } {}^{312}$$

The element of novelty in this example is emphasized: in the previously solved examples, the divisor was round hundreds, and in this example, the divisor is a three-digit number, which contains not only hundreds , but also tens and units (312). How can we find the digit of the quotient in this case? The teacher resorts to analogy. Is it possible to use the same technique here that we used when dividing by round hundreds? There is a basis for the analogy: 312 is not much different from 300, 312 is close to 300. Therefore, let's try to divide not by 312, but by 300; Let's find out how much we get from dividing 1248 by 300, and then check the found digit of the quotient, maybe it will also work for dividing 1248 by 312. So, we divide 1248 by 300 and get 4 as the quotient. Can we, having received 4, immediately write this four as the quotient? No, we can't: it was obtained by dividing by 300, but we needed to divide by 312. Since 300 and 312 are close numbers, it is possible that dividing by 312 will also give us 4, but this needs to be checked. How to check? Students know the method of checking: you need to multiply the divisor (312) by the quotient (4). We multiply 312 by 4, we get the dividend (1248). So, the number 4 is the quotient of 1248 divided by 312. Now we can put 4 in the quotient; the solution will have the following form:

$$\begin{array}{l} 1248 \text{ L } {}^{312} \\ \underline{1248\ } 4 \\ 0 \end{array}$$

At the end of the division, the children's attention is once again focused on the fact that we easily found the

quotient because we tried to divide not by the entire divisor (312), but only by its round hundreds.

The children have so far perceived only a separate, concrete fact. In order for them to perceive this method as general, the teacher offers one after another three more examples, indicated above. Perceiving the explanation of these examples, the students will notice their differences (different numbers) and their similarities; in all examples, the dividend is a four-digit number, the divisor is a three-digit number. And most importantly: in all such cases, the quotient is found in the same way. Thus, in the student's mind , **a generalisation is formed,** a transition from the concrete and individual to the general, the abstract.

This generalisation-conclusion is given in the following formulation: "When you need to divide a four-digit number by a three-digit number, you should divide the hundreds of the dividend by the hundreds of the divisor and check the resulting number."

3. In order to find out whether the students understood the explanation correctly and deeply enough, and on the other hand, to give the children themselves the opportunity to make sure that they are correctly assimilating new knowledge, after the explanation, the children are asked to solve independently first those examples on which the teacher gave the explanation, and then three examples of the same type: 1) 1668:417; 2) 4368:624; 3) 3632:718.

After solving these examples, one of the students is called to the board and shows how he solved the example, and the others check the solution of the examples in their notebooks. Giving explanations for the solution, the student exactly reproduces the reasoning scheme that the teacher used.

The first to be called to the board is the stronger student, who quickly perceives new things and has a good command of speech; the last to be called is the less successful student. And if this student gives a satisfactory

explanation, one can be sure that the explanation has been understood correctly.

4. At the previous stage of the lesson, based on specific and isolated examples, the children came to the conclusion of the rule by generalisation. Now, for a deeper understanding of the method of dividing a four-digit number by a three-digit number, it is necessary to move from a general, abstract conclusion to a specific, isolated one. To do this, the teacher offers the students two or three examples one after another. The examples are solved on the board and in notebooks. For homework, they are given 5—6 examples to solve, similar to those that were explained in class, and a problem in which a case of dividing a four-digit number by a three-digit number is encountered.

Lesson 2.

The purpose of the lesson is to deepen children's understanding of the basic technique of dividing a four-digit number by a three-digit number, showing its modification using new versions of examples; to show the practical application of this division skill when solving problems.

Lesson progress. 1. After checking homework, the teacher exercises children in oral arithmetic. The oral arithmetic exercise is conducted using examples and problems that are part of written division (multiplication of two-digit numbers by a one-digit number; division of two-digit numbers by a one-digit number with remainder):

a) 65X4 ; 86X9 ; 38X6 ; 6) 28 dec. X 5; 45 dec. X 8; 34 dec. X 7.

c) 75:8; 34:6; 51:7, 46:9.

d) Tasks: 1) “The car travelled 40 km in an hour, and the plane flew 8 times more in this time. How many kilometres did the plane fly in one hour?” 475

2) “I divided 38 pencils equally into 4 boxes. How many pencils were put into each box and how many were left over?”

2. Then the teacher moves on to the exercise in written division. The exercises in this lesson have two goals: on the one hand, to a certain extent, to automate the skill of division, and on the other, to deepen the understanding of the method of division. To achieve the first goal, it is necessary to teach children to perform division more confidently, quickly and independently, without detailed reasoning. To achieve the second goal, it is necessary to introduce various variants of examples into the exercises, requiring modification of the basic division method. Accordingly, the exercises are divided into two parts: the first is the frontal work of the class under the direct supervision of the teacher, and the second is the independent work of the students. At each stage, examples of various variants are introduced, some of which will require additional explanations from the teacher.

Part 1—frontal work of the class under the direct supervision of the teacher.

Solution of examples: 1) 5615: 922; 2) 3708: 645; 3) 2715: 396. In solving the first example, the technique of dividing a four-digit number by a three-digit number is repeated. The solution of the second example has the following peculiarity: when testing the digit of the quotient obtained by dividing the hundreds of the dividend (37) by the hundreds of the divisor (6), it turns out that the resulting digit is large and must be reduced by one. When solving the third example, the digit of the

quotient after the first test must be reduced by two units. The question arises of how to quickly find the infra quotient without unnecessary checks, why in this case the first digit of the quotient 9 turned out to be large and it was necessary to reduce it by two times. The answer to this question is given by the analysis of the divisor. The divisor 396 is close to 400, not 300; therefore, when rounding, it is advantageous to round 396 to 400 and find the first digit of the quotient by dividing 27 by 4, and not by 3.

Having offered the students two more examples to analyse: 1716:286; 3212:485, the teacher adds a clarification to the rule for finding the quotient digit known to the children: “If the tens digit of the divisor is greater than 5, then it is useful to round the divisor to a larger round number” (or in other words: it is useful to increase the hundreds digit of the divisor by one).

Solution to the problem: “At a sewing factory, 289 dresses were sewn from 1,156 m of material. How many of these dresses can be sewn from 800 m of material?” When solving this problem, the division case just considered will find its practical application and will be consolidated.

Part 2—independent work of students.

Independent solution of examples of different options for the rule under consideration: 5033:719; 4108:586; 1845:293.

During independent work of the class, the teacher helps weaker students by giving them additional explanations.

Checking independent work. When checking, students are required to provide a detailed explanation of the method for finding the quotient number in each example.

3. **For homework you are given** 5 problems to solve on dividing a four-digit number by a one-digit number and one problem with written questions.

PROBLEM SOLVING TRAINING

The main goals of teaching arithmetic in primary school are achieved to a large extent through solving arithmetic problems. The knowledge, skills and abilities acquired by students in solving problems form the basis of their further mathematical education. All activities related to solving problems to a certain extent contribute to the formation of the child's character - his will, persistence and tenacity in achieving the intended goal, the ability to make efforts to overcome difficulties. The role of solving problems as a factor in the "development of students' logical thinking, their ability to establish relationships between quantities, and make correct conclusions" is very important [1] . Solving problems is a powerful means of mental development of a child, the development of his thinking, attention and creative imagination.

But not every problem solving contributes to the development of children's logical thinking. If the solution to a problem is aimed only at getting the right

[1] Primary school programs. Arithmetic. Explanatory note, 1949 edition.

answer, and in any way, even by trial and error, by adjusting the result “to the answer”, then such exercises are of little use. The fact is that to achieve the stated goals, not only the results of the problem solution are important, but mainly those thinking processes that led to the solution and helped to find the right answer. Not only the methods of solution are important, but also the methods of reasoning that accompany the solution.

The main point in working on a problem is the method of reasoning that leads to drawing up a solution plan and to the solution of the problem itself.

The initial logical operations on which reasoning is based are, as is well known, analysis and synthesis. Analysis in problem solving implies a process of thinking that goes from the problem question to the numerical data needed to solve it. Synthesis implies an act of thinking that goes from the numerical data of the problem to the question that follows from them.

In reasoning when solving problems, one has to use both analysis and synthesis. Solving a problem as a whole is an analytical-synthetic process. In it, the solver’s thought constantly moves from the problem question to the numerical data and from the numerical data to the question. Having posed a question arising from a given situation, the student considers whether the problem statement contains numerical data on the basis of which the question can be solved. Conversely, by examining the numerical data and combining them, the student determines which question can be solved on the basis of this data.

If you follow the train of thought of a student pondering a solution to a problem, it turns out that in

this process of "thinking" about a problem, the student goes through the following stages:

"I need to solve such and such a question. On what data can I solve it? Let's look at the problem statement." Next: "I have such and such numerical data. On their basis, I can solve such and such a question. But **is it necessary** to solve it? Let's look at what the problem asks?"

In the first case, the student, proceeding from the problem question and using analysis, at the same time addresses the problem condition, the problem as a whole, and only on the basis of a holistic representation of the problem, as a set of data and the question, finds the correct solution. In the second case, proceeding from the problem data and using synthesis, the student at the same time addresses the problem question in order to use this question to check the expediency of performing a particular operation on a given pair of numbers, and if the problem question confirms the need to combine a given pair of numbers, stops at it and finds the correct solution to the problem.

Thus, when searching for ways to solve a given problem, the analysing and synthesising aspects of thinking are in constant interaction, complementing and checking each other.

When analysing a problem, there can be two starting positions: either **the question of the problem or the numerical data** in the problem. When the problem solver starts from the question, analysis comes to the forefront and plays a leading role; synthesis accompanies it. When numerical data are taken as the starting point, synthesis comes to the forefront: it gives movement to thought, plays a leading role, is predominant; analysis accompanies it, serving as a

means of checking the appropriateness of the movement of thought and a guide for its direction.

Given the existence of two methods of logical work on a problem—analysis and synthesis—it is natural to raise the question of their comparative value, and the conditions for applying each of them in analysing a specific problem.

In advanced pedagogical practice, analysis, as a method of logical work on a task, is given preference over synthesis. And indeed, this method of analysis is more valuable, since it contributes to the development of logical thinking of students to a much greater extent than synthesis.

This is explained by the fact that it fixes the children's primary attention on the question of the task, it emphasizes the great importance of the question. The students' reasoning, the construction of a chain of logical conclusions, begins with the question of the task, and each link of this chain, in turn, contains smaller questions.

Synthesis is inferior to analysis in its influence on the development of students' thinking and speech. This is explained by the fact that it does not focus students' attention on the question of the problem to the extent that this is done when analysing a problem. It focuses children's attention mainly on the numerical data of the problem, on the actions that can be performed on this data. It often leads to a separation of the content of the problem from its question: the student begins to solve the problem without taking its question into account, without subordinating the course of his thoughts to what is ultimately asked in the problem.

Hence, there is a lot of randomness and irrationality in the search for the correct ways to solve the problem.

Among the conditions that make analysis accessible to students, the first and foremost place is occupied by the observance of strict gradualness in the increase in the complexity of the forms of analysis and its difficulty for students.

Methods and techniques for teaching the analysis of arithmetic problems

Already at the initial introduction of children to a compound problem, in the first grade, it is advisable to use an analytical method, leading children from a ready-made problem in two steps to its constituent simple problems. The specificity of such a problem is that it cannot be solved immediately, in one step, as is done when solving a simple problem. If at the first introduction to a compound problem you start from a ready-made problem in two steps, then the difference between a simple problem and a complex one appears with complete clarity.

To begin with, you can take a task that is convenient to illustrate with objects:

“Six red pencils and four blue pencils were put into a box. Then seven pencils were taken from the box. How many pencils were left in the box?”

The teacher shows the children first 6 red pencils, then 4 blue ones and **writes these numerical data on the board.** Then he puts the pencils in a box. The children do not see how many there are in total. After this, the teacher takes out 7 pencils. **He also writes this third data on the board** , and the main thing sought remains hidden in the box.

The students repeat the problem, highlight the question, and solve the problem in their heads, as they are accustomed to doing with simple problems. Everyone gets the correct answer: 3 pencils.

Teacher : How did we know that there were 3 pencils left in the box?

Student: From 10 pencils we subtracted 7 pencils and got 3 pencils.

Teacher (turning to the board where the numerical data is written): Where did you get 10? After all, there is no such number in the problem?

Student: You need to add 4 pencils to 6 pencils and you get 10 pencils.

Teacher: What did we learn by adding 6 pencils and 4 pencils?

Student: We found out how many pencils there were in the box.

Teacher: You see, first we found out how many pencils there were in the box; to do this, we added 6 pencils and 4 pencils:

1) 6 ct. + 4 kar. = 10 kar.

This is the first action. Then we found out how many pencils were left; to do this, we subtracted 7 pencils from 10 pencils:

2) 10 ct. — 7 kar. = 3 kar.

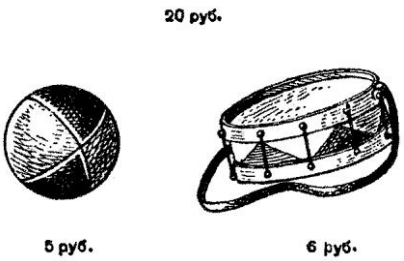
This is the second action. So, how many actions do you have to perform to solve this problem?

Student. Two acts.

Teacher: What did we learn in the first act? In the second act?

In conclusion, the teacher emphasizes that some problems can be solved **immediately** , in one action. But there are problems that cannot be solved immediately—you have to do one action first, then another; for example, first add, then subtract.

From now on, before solving each problem, children establish whether it can be solved immediately. If it turns out that it cannot, then the teacher asks, what can be learned immediately?



This is how children first begin to apply the past analysis. The question “is it possible to solve the problem immediately?” encourages children to match the data to the question; this is the analytical moment. The next question “what can be found out immediately?” encourages children to match the question to the data; this is the synthetic moment. The leading role here is played by analysis.

This simple analysis is further deepened by introducing an additional question “why” (“why can’t we immediately find out how many pencils are left?”).

We solve the problem: “A mother bought her children a ball for 5 rubles and a drum for 6 rubles. How much change did she get from 20 rubles?”

The picture drawn by the teacher on the board explains the text of the problem, but does not illustrate the numbers themselves.

In analysing this problem, the teacher asks the following questions:

Teacher: Is it possible to find out right away how much change I have received? Student: No, it is not possible.

Teacher: Why can't we find out right away?

Student: Because we don't know how much we have to pay for the ball and the drum.

Teacher: And how much do I need to pay for the ball and drum? Can I find out right away?

Student: Yes, you can find out right away.

Having become accustomed to the question "why", children begin to analyse the problem without expecting this question from the teacher. They reason like this: "It is impossible to find out right away how much change the mother received, because we do not know how much in total needs to be paid for the ball and the drum. But how much in total needs to be paid for the ball and the drum, this can be found out right away. Therefore, we will solve the problem like this: first, we find out how much in total the mother paid for the ball and the drum; then we find out how much change the mother received from the 20 rubles."

Having solved a compound problem, children make up simple problems into which they had to dissect the compound. Thus, having solved the above problem, children write down its solution:

1) 5 rubles + 6 rubles = 11 rubles.
2) 20 rubles - 11 rubles = 9 rubles.

Answer: 9 rubles.

Now the teacher asks the children to create problems for each line.

1st task: “The ball costs 5 rubles, and the drum costs 6 rubles. How much do the ball and the drum cost together?”

2nd task: “The ball and the drum cost 11 rubles. The cashier gave 90 rubles. How much change should I get?”

Thus, already in the first grade, children learn to break down a complex problem into two simple ones.

In the second grade, children begin to solve problems in three steps. The simplest problems of this type are those that require multiplication and then subtraction or addition. For example: “We bought 2 plates for 4 rubles and 5 spoons for 2 rubles. How much did the entire purchase cost?”

When analysing problems of this kind, we need to teach children to indicate both numbers that we do not know and that we need to know to solve the problem at once. Usually, children name only one unknown quantity. Then the teacher asks additionally: “And why else?” This question is enough for children to name both unknown quantities—in this case, the cost of the plates and the cost of the spoons.

When breaking down more difficult 3-step problems, one of the most common mistakes is skipping the intermediate logical link.

Let's take the problem: "There were 40 apples in one basket, and 10 apples more in the other. All these apples were put into boxes of 30 apples each. How many boxes were these apples put into?"

The analysis is usually carried out as follows:

Teacher: Is it possible to immediately find out how many boxes were needed to fit all the apples?

Student : It is impossible, because we do not know how many apples were in the second basket.

Teacher: But did they put only the apples from the second basket into the boxes?

Student: No, apples from two baskets were put into the boxes. So we can't solve the problem right away because we don't know how many apples were in the two baskets.

Teacher: Is it possible to immediately find out how many apples were in the two baskets?

Student: No, because we don't know how many apples were in the second basket.

Teacher: Is it possible to immediately find out how many apples were in the second basket?

Student. Yes, you can.

After this, an oral plan for solving this problem is made: 1) How many apples are in the second basket? 2) How many apples are in two baskets? 3) How many boxes were these apples placed in?

The solution to the problem is written as follows:

1) 40 apples + 10 apples = 50 apples.
2) 40 apples + 50 apples = 90 apples.
3) 90 apples : 30 apples = 3.

Answer: 3 boxes.

Having written down the solution to the problem, it is useful, based on the recording of the actions, to compose the simple tasks into which the given problem is divided:

1) There are 40 apples in one basket, and 10 apples more in the other. How many apples are in the second basket?

Etc.

We repeat, when using such a simple analysis, the teacher's main duty is to ensure that children do not miss: intermediate logical links when reasoning. If the teacher makes such mistakes and puts up with them, then he thereby teaches the student to be imprecise and inhibits the development of logical thinking in children.

In the third grade, the analysis of the problem becomes somewhat more complicated and more complete, taking the form in which it is used at subsequent stages of training. The complication consists in the fact that when analysing a problem, the student learns to name not one (as was the case in the first and second grades), but both components necessary for solving the question, regardless of whether they are known or unknown. In order to more clearly outline the difference in the forms of analysis in the second grades, we will give an example of analysing a specific problem.

"For 59 rubles we bought 5 cups for 7 rubles and 6 saucers. How much does one saucer cost?"

Let's use **the simplest analysis** used in the IP class:

Teacher: Is it possible to find out right away how much one saucer costs?

Student: No, we can't, because we don't know how much we paid for all the saucers.

Teacher: Is it possible to find out right away how much was paid for all the saucers?

Student: No, because we don't know how much all the cups cost.

Teacher: Can I find out the cost of the cups right away?

Student: Yes, you can find out right away.

Teacher: So what will be the first question? Etc.

In a coherent form, without leading questions from the teacher, the reasoning will be as follows: "It is impossible to find out right away how much each saucer costs, since it is not known how much they paid for all the saucers. It is also impossible to find out right away how much they paid for all the saucers, since it is not known how much all the cups cost. It is possible to find out right away how much all the cups cost, since it is known that 5 cups were bought and that each cup cost 7 rubles."

As we see, here only at the last stage of the reasoning are both components indicated, since both are given.

Let's use **a full analysis** of the same problem (using leading questions):

Teacher: What does the problem ask?

Student: The problem asks how much one saucer costs.

Teacher : What two numbers do you need to know to solve this question right away?

Student: You need to know how many saucers were bought and how much they paid for them.

Teacher: Do we know these numbers?

Student: We know how many saucers were bought (6 saucers), but we don't know how much all the saucers cost.

Teacher: So, we need to find out how much all the saucers cost. What two numbers do we need to have to solve this question at once? Etc.

Without leading questions in a student's coherent presentation, the complete analysis will have the following form:

"The problem asks how much each saucer costs. To solve this question, we need to know how much all the saucers cost and how many saucers were bought. We know how many saucers were bought—6 saucers. But we don't know how much all the saucers cost. To find out how much all the saucers cost, we need to know how much the entire purchase cost and how much the cups cost. We know how much the entire purchase cost—59 rubles, but we don't know how much the cups cost. To find out how much all the cups cost, we need to know how much one cup costs and how many cups were bought. We know both of these numbers. So, the first question of the problem is, "How much do all the cups cost?" And so on.

As we see, here at each stage of reasoning the student names the data that are necessary to solve the problem. Such a complete, harmonious and finished form of reasoning is not given to children immediately and not without difficulty. The student needs to be thoroughly prepared for it. The preparation that was carried out during the 1st and 2nd grades is not enough. New exercises are needed, in which it should be shown that a) to solve the problem it is necessary to perform one or more arithmetic operations on numbers and that b) in each case to perform the operation it is necessary

to have two numbers. This is shown in simple problems of three types, which can be encountered when analysing compound problems:

1) both numbers needed to solve the question posed are unknown;

2) one number is known, the other is unknown;

3) both numbers are known.

It is better to start the exercises with problems in which one number is known. For example: “A boy picked 8 apples from one apple tree and several apples from another. How many apples did the boy pick in total?” The problem is analysed.

Teacher : Is it possible to solve this problem?

Student: No, you can’t.

Teacher: Why can’t it be solved?

Student: Because we don’t know how many apples the boy picked from the other apple tree.

Teacher: How many numbers do you need to have to solve this problem?

Student: You need to have two numbers. Teacher: And what action solves it?

Student Addition.

Conclusion. To find out how many apples were picked in total, you need to know: how many were picked from one apple tree and how many were picked from another apple tree. After that, the teacher calls out the second number, let’s say 12 apples, and the students solve the problem.

Three more problems are solved in this way: subtraction, multiplication and division.

The second stage of preparatory exercises is working on simple problems without numerical data. For example: “There were books on the shelf. Several books were taken off the shelf. How many books are left?”

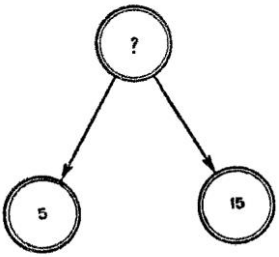
From the analysis of this problem, a conclusion is drawn, which is formulated as follows: “To find out how many books **are left**, you need to know how many books were lying there initially and how many books were removed.”

The third stage of preparatory exercises is working on simple problems in which both numbers are given.

For example: “We bought 5 kg of sugar at 15 rubles per kilogram. How much does the purchase cost?”

In this problem, the student learns to reason in the following form: “In order to solve the problem, you need to know how many kilograms of sugar were bought and how much a kilogram costs. We know both numbers. To solve the problem, you need to multiply 15 rubles by 5, which gives you 75 rubles. Answer: 75 rubles.

It is useful to illustrate the solution to this problem as shown in the figure.



Having completed the exercises in analysis on simple problems, you need to move on to **the analysis of problems in two** steps, and then in **three steps**.

Three-step problem analysis is a fairly long discussion. In order for students not to lose the thread of the discussion, it should be broken down into

separate links, illustrating them with visual images. The role of such an image can be played by a diagram consisting of circles filled with numerical data of the problem and question marks to indicate unknowns. Instead of drawing circles on the board, you can prepare them in advance from thick paper (for example, from the covers of old notebooks) of different colours.

It is useful to solve 5—6 such problems for exercises in full analysis, in which the analysis is illustrated by a diagram that has a symmetrical appearance. For example: "A store sold 40 boxes of grapes one day, 18 kg each, and 20 boxes of 12 kg the next day. How many kilograms of grapes did the store sell in total in two days?"

The first link in the analysis of this problem is prepared by working on simple problems in which both numbers are unknown. Their places will be taken by circles with question marks. The second and third links are the analysis of simple problems on multiplication with known data.

The solution written on the board will look like this:

Task.
1st day — 40 boxes, 18 *kg each*
2nd day — 20 boxes , 12 *kg each*
} How much in total?

The task is analysed using drawing (see page 484).

Plan and solution.

1) How many grapes were sold on the 1st day?
   18 *kg* X 40 = 720 *kg*
2) How many grapes were sold on the 2nd day?
   12 *kg* x 20 = 240 *kg*

3) How many grapes were sold in two days?

720 *kg* + 240 *kg* = 960 *kg*

Answer: 960 *kg* .

After the problem has been presented, the numerical data has been written down and the condition has been repeated, the teacher says: “Now we will analyse the problem.” First of all, the question of the problem is highlighted.

Teacher: Let’s mark this question with a circle and a question mark. What do you need to know to answer the question of the task?

Student: You need to know how many grapes were sold on the first day and how many on the second day.

Teacher (draws two arrows on the board and a circle for each arrow): Do we know how many grapes were sold on the first day and how many were sold on the second day?

Student: No, it is unknown.

Teacher : What do you need to know to solve each of these questions?

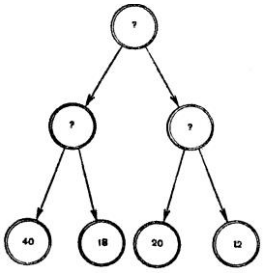
Student: You need to know how many boxes were sold and how many kilograms were in each box.

Teacher (draws two arrows from the right circle and two arrows from the left circle, one circle for each arrow): Do we know how many boxes were sold on the first day and how many kilograms were in each box?

Student: We know. We need to write 40 in the left circle and 18 in the right one.

Teacher: Do we know how many boxes were sold on the second day and how many kilograms were in each box?

Student: We know. We need to write 20 in the left circle and 12 in the right one.



The analysis is finished. Now we need to move on to making a plan. The children repeat the questions, and the teacher puts Roman numerals next to the corresponding circles. After this, the solution is written down in the notebooks.

Drawing the analysis diagram on the board is done by the teacher, but later you can teach children to draw such diagrams in notebooks, devoting a separate lesson to this issue. In order not to waste time on drawing diagrams in class, you can assign this work for homework.

In the fourth grade, the use of the analytical method of analysing problems continues. Here, the analysis is complicated in two respects: on the one hand, more difficult problems are taken, on the other hand, students are required to be more independent in formulating their reasoning. When analysing a problem, the student reasons coherently, explains the analysis of the problem

with a drawing on the board, or, if the problem is solved at home, reasons out loud, looking at the drawing in the notebook.

fourth-grade students practice analysing problems of approximately the same difficulty level as in the third grade. It is necessary to approach problems with expressions greater or lesser by so many or so many times with some caution. Many such problems are solved in the third and even in the second grade, but with the use of the simplest analysis. In the fourth grade, they are solved with the use of complete analysis. Here we mean problems of the following type, for example: “The plane flew 1940 km on the first day, 340 km more on the second. On the third day, it flew 895 km less than on the first two days together. How many kilometres did the plane fly in three days?”

When solving this problem, you need to think like this: “In order to answer the main question of the problem, you need to know: how many kilometres did the plane fly on the first day (1940 km), on the second (?)) and on the third day (?).

In order to find out how many kilometres the plane flew on the second day, you need to know how many kilometres it flew on the first day (1940 km) and how much more on the second day (by 340 km). In order to find out how many kilometres it flew on the third day, you need to know: how many kilometres it flew on the first and second days together (?) and how much less it flew on the third day (by 895 km).

In order to find out how many kilometres the plane flew on the first and second days together, you need to know how many kilometres it flew on

the first day (1940 km) and how many on the second. The last question can be answered right away. We will start making a plan with it."

When translating this reasoning into the language of a diagram, the latter turns out to be quite complex, so its use seems inappropriate.

In the fourth grade, intensive work is simultaneously carried out on the assimilation by students of the relationship between quantities, especially between those that are more often encountered in problems and in practical life: between price, cost and quantity, between distance, speed and time, between the total weight, the number of objects and the weight of each object, between the total harvest, the harvest per unit area and the size of the area, etc.

By verbally solving simple problems, children learn: what quantity can be found from two given quantities (by speed and time — distance; by price and quantity — cost; by area and total harvest — harvest per unit of area — are or hectare, etc.); what two quantities must be available as data to determine the desired quantity (to find the price, it is enough to know the cost and quantity; to find the path travelled by a body, it is enough to know the speed and time of movement; to find the total weight — the weight unit and the number of such units, etc.).

Knowledge of the relationship between quantities is a prerequisite for successful analysis. If children have a poor understanding of this issue, their reasoning will be confused and erroneous.

Thus, by putting the analysis of the problem into a certain verbal form, by getting children to

formulate precise statements first in the form of answers to leading questions, later in the form of coherent reasoning, we create conditions under which thinking is formed and developed, and this in turn is a condition for successfully solving the problem.

Typical tasks

Much of what has been said about analysis as applied to the analysis of compound arithmetic problems also applies to **clip problems.** Some types of problems lend themselves to analytical analysis, just like compound arithmetic problems. These include problems solved **by the method of reduction to one, to find the arithmetic mean, on the complex triple rule, problems on motion.** But some types of problems solved in elementary school require the use of a special kind of analysis. In other cases, the establishment of cause-and-effect relationships between changing quantities plays a major role in this analysis; such are problems of finding an unknown by the difference of the numerical values of a given quantity and problems solved by the method of eliminating one of the quantities. In other cases, the introduction of a conventional unit is of great importance in this analysis—such are problems of finding two numbers by their sum and difference, by their sum and multiple ratio.

Each type of analysis has its own verbal form and represents a certain type of reasoning that children must master. Due to the structural features of typical problems, reasoning plays an even greater

role in solving them than in solving compound arithmetic problems. Often, they are the only key to finding a way to solve a given typical problem, since through reasoning the student manages to uncover the true connections and relationships between the quantities given in the problem. It goes without saying that the clarity and concreteness of teaching retain their full strength and significance when solving most problems.

In order to facilitate students' understanding of typical problems and the assimilation of the reasoning scheme accompanying their solution, it is extremely important to introduce students to typical problems in a system that would ensure the immediate proximity of problems that have something in common in the reasoning scheme. Let us consider the form of analysis of typical problems, arranging them into groups according to the feature of commonality in the reasoning scheme.

Problems on the simple triple rule, on proportional division and on the complex triple rule. Solving problems in this group introduces students to the proportional dependence of quantities (primarily with direct proportional dependence), which is often encountered in nature and in everyday human practice. These rules combine into one group problems of varying complexity and difficulty, from easy problems on "reduction to one", solved already in the 1st grade, to problems on the complex triple rule, accessible only to students in the 4th grade. When analysing these problems, students' reasoning should emphasize the proportional dependence of the quantities given in the problem (quantity of goods

and their cost, length of the path and duration of movement, number of workers and volume of work, etc.).

Let's solve the problem: "They paid 30 rubles for B m of calico. How much does 3 m of such calico cost?"

Problem analysis: In order to answer the problem question, you need to know how much 1 m of calico costs. In order to find out how much 1 m of calico costs, you need to know how many meters were in the first piece (5 m) and how much was paid for this piece (30 rubles).

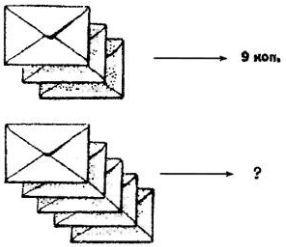
Since both data are available, we need to start the solution with them.

Reasoning: "If 5 m cost 30 rubles, then 1 m costs 5 *times less* (30 rubles : 5 = 6 rubles). If 1 m costs 6 rubles, then 3 m cost 3 *times more* (6 rubles X 3 = 18 rubles)."

These judgments are very important, since they express the direct proportional dependence of the quantity of calico and its cost. A third-grade student should have a good command of such reasoning.

For greater clarity, it is useful to write the problem statement in such a form that homogeneous quantities are brought into correspondence.

When first introduced to problems of this kind, the content of the problem should be illustrated with a drawing. For example, the problem "3 envelopes cost 9 kopecks. E How much do 5 such envelopes cost?" can be illustrated with the drawing provided here.



In terms of the form of analysis and reasoning, problems on proportional division are close to problems on the simple triple rule. “Reduction to unity” is included in these problems as a component element.

Let’s solve the problem: “360 rubles were paid for two pieces of the same material. The first piece had 5 m, the second piece 4 m. How many rubles did each piece cost?”

From the solution of this problem it is easy to see that it is reduced to a problem on a simple triple rule, solved by the method of direct reduction to unity.

When analysing this problem, one must pay attention to the question of the problem.

Teacher: How many answers will there be in this problem?

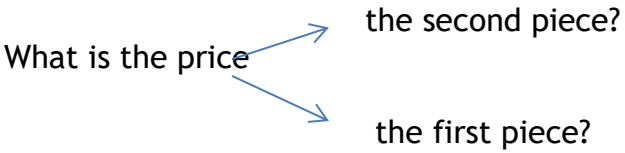
Students: Two answers.

Teacher: Why?

Student: Because the question is, how much does each piece cost? And there are two pieces, and they are not the same size.

Teacher: So, instead of the main question, the plan will have two separate questions: how much does the first piece cost and how much does the second piece cost.

When writing the problem statement on the board, the teacher writes down a split question:



Teacher: Let's find out first how much the first piece costs.

Analysis of the problem in a coherent presentation: "Let's find out how much the first piece costs. To solve this question, we need to know how many meters there were in the piece (5 m) and how much 1 m costs (?).

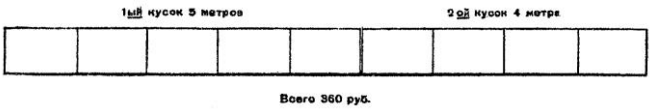
To find out how much 1 m of material costs, you need to know how many meters of material there were (?) and how much all the material cost (360 rubles).

To find out how much matter there was in total, you need to know how many meters were in the first piece (5 m) and how many meters were in the second piece (4 m).

Let's make a plan for the solution. The first question: "How many meters were in the two pieces?" Etc.

A graphic illustration can be used as a visual aid.

1st piece 5 meters 2nd piece 4 meters



Problems on the complex triple rule , as is known, are reduced to problems on the simple triple rule in their solution method. In elementary school, they are also solved by reduction to one. But in terms of semantic content, they are significantly more complex than problems on the simple triple rule, since they introduce quantities that are proportionally dependent not on one, but **on several** other quantities. For example: “On 7 machines in 5 hours, 630 parts are manufactured. How many parts will be manufactured on 10 machines in 8 hours?” Here, the desired number of parts is proportional to the number of machines and the duration of their work. The method for solving these problems is a generalisation of the method of reduction to one.

To facilitate understanding of the method of solving these problems in their more complex form, it is necessary to first practice children in solving the simplest problems of this type. For example: “On 7 machines in 5 hours 630 parts are manufactured. How many parts can be manufactured on one machine in 1 hour?” Or: “On one machine in 1 hour 18 parts can be manufactured. How many parts can be manufactured on 10 machines in 8 hours?”

At the beginning of the analysis of this problem, it is necessary to emphasize the main and fundamental thing in it: the number of parts depends on two quantities — on the number of machines and on the number of hours. The more (fewer) machines, the more (fewer) parts. The longer (shorter) the duration of work, the more

(fewer) parts. This is the essence of the proportional dependence of the number of parts on the number of machines and the duration of their work. Then it is necessary to point out that the transition from a given number of machines and hours to a unit of machines and a unit of time is not done immediately, but gradually.

7 machines in 5 hours — 630 parts
1 machine in 5 hours — 630 parts: 7
1 machine in 1 hour — 630 parts: 7:5

The transition from 1 machine and 1 hour to 10 machines and 8 hours of work is made with the same gradualness.

It is essential that the conditions of the problems on the complex triple rule are written in 2 lines:

7 machines in 5 hours — 630 parts
10 machines in 8 hours — ?

In such a record, the values of homogeneous quantities are compared, and this makes it easier to solve the problem.

Problems on finding an unknown by the difference of two numbers and problems solved by eliminating one of the quantities have some similarities with the problems of the previous group. These are also problems with proportional quantities. They also have some similarities in the methods of solution. But in the nature of the analysis and the type of reasoning, these problems, having much in common, differ sharply from the previous problems. As already indicated above, the

analysis in these problems is based on establishing a cause-and-effect relationship between the quantities given in the problem and on the conclusions arising from this relationship.

Let's look at a specific problem of this type: "36 bags of potatoes were harvested from one plot, and 29 bags from another. The first plot yielded 336 kg more than the second. How many kilograms of potatoes were harvested from each plot?"

From the question it is clear that this task requires two answers: how many kilograms of potatoes were removed from the first plot and how many kilograms were removed from the second plot. These two questions should be included in the solution plan.

Already when reading the conditions of this problem, it is necessary to establish a cause-and-effect relationship between the difference in the number of kilograms and the difference in the number of bags: "From the first section, they got 336 *kg* more potatoes precisely because they took 7 more bags from it." Next, based on these two differences, you need to make a conclusion: "There are 336 *kg* of potatoes in 7 bags." After this, the solution to this problem is reduced to solving the problem by direct reduction to one.

Thus, the analysis and reasoning that lead the student to the correct solution of this problem have their own specifics, which follow from the conditions of the problem. The student must be able to grasp this feature of the condition and draw a logical conclusion from it. In order to help the student, it is necessary to presuppose the solution of

such problems by solving simple problems in which both differences are directly given.

For example: “Kolya bought 3 more pencils than Vanya and paid 24 kopecks more. How much does one pencil cost?”, “One train was on the road 2 hours longer than the other and travelled 98 km more. What is the speed of the trains per hour (assuming they are moving at the same speed)?” etc.

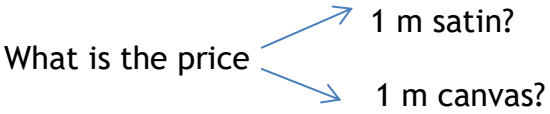
To establish a cause-and-effect relationship between quantities, which is of decisive importance in this type of problem, the teacher prompts students by asking the question “why?”

The type of problems we have considered finds its development in problems solved by excluding one of the quantities.

Let’s take the problem: “The first time we bought 5 m of satin and 12 m of canvas and paid 488 rubles for the entire purchase. The second time we paid 95 rubles for 5 m of satin and 7 m of canvas. How much does 1 m of satin and 1 m of canvas cost?”

Reading the conditions of this problem, children grasp the cause-and-effect relationship between the difference in the amount of linen and the difference in its cost. At a glance at the numerical data, it is clear that the first time more linen was bought than the second time, and more was paid, with the same amount of satin. This already determines the course of reasoning that must be used to solve this problem. Let’s write the numerical data in two lines and divide the problem question into two questions:

5 m satin and 12 m canvas — 145 rubles
5 m satin and 7 m canvas — 95 rubles



Problem analysis:

Teacher: Why did you pay less for the material the second time than the first? Does it depend on the satin?

Student: No, because they bought the same amount of satin.

Teacher. What about the canvases?

Student: They bought less canvases, that's why they paid less.

Teacher: Look at the questions written under the line. Which of them is easier to answer?

Student: It is easier to find out how much 1 m of canvas costs. Teacher: How can I find out?

Student: You need to find out how much less you paid the second time, how much less canvas you bought, and then find out how much 1 m of canvas costs.

The reasoning is synthetic in nature; it is based on the cause-and-effect relationship that has been established.

At this point you can stop and write down the plan and solution to the first part of the problem.

Plan and solution:

1) How many meters less canvas did you buy the second time?

12 m — 7 m = 5 m.

2) How much less did you pay the second time?

145 rubles — 95 rubles = 50 rubles.

3) How much does 1 m of canvas cost?

50 rub.: 5 rub. = 10 rub.

After recording, we move on to analysing the second part of the problem.

Teacher: What else is left to learn?

Student: It remains to find out how much 1 m of satin costs.

Teacher. For this we only need to take one line, for example, the second one. Let's make up a problem from which we could find out how much 1 m of satin costs.

Student. They paid 95 rubles for 5 meters of satin and 7 meters of canvas. A meter of canvas costs 10 rubles. How much does a meter of satin cost?

This problem (on mixing of the 1st kind) is analysed, a plan for its solution is drawn up and the cost of one meter of satin is found.

From the above it is clear that both tasks considered above are of great importance in the development of students' logical thinking.

Problems on finding two numbers by their sum and multiple ratio. The arithmetic basis of these problems is proportional division. In the arithmetic course of the 5th grade, these problems are included as a subtype in the problems on proportional division. But in the elementary grades, they stand out from this type of problems, occupy

an independent place and have special significance. With these problems, children develop the concept of the so-called conventional unit (part); with these problems, they learn to replace the relationships between specific numbers with the relationship of abstract parts, which presupposes preliminary acquisition of the concepts of the whole and its part.

The analysis of these problems in primary school and the type of reasoning associated with it differs sharply from the type of reasoning when solving problems on proportional division. This gives the problems "in parts" the right to an independent place.

When first introducing students to this type of problem, it is useful to take a well-known problem on proportional division as a starting point. For example: "After grinding 240 *kg* of grain, 1 bag of bran and 4 bags of flour of the same weight were obtained. How many kilograms of bran and how many kilograms of flour were obtained?"

After solving this problem, a conversation is held.

Teacher: Let's assume that a bag of bran is 1 part of the total weight of bran and flour. How many of these same parts are there for flour?

The Disciple. 4 parts.

Teacher : And how many equal x parts were there in total?

Student: There are 5 equal parts in total.

Teacher : How did you get the number 5?

Student. Add four parts to one part, you get 5 parts. Under the teacher's guidance, children change the formulation of the original problem:

“After milling 240 *kg* of grain, 1 part of bran and 4 parts of flour of the same weight were obtained. How many kilograms of bran and flour were obtained separately?”

The problem is repeated and solved by changing the first question. Now it is formulated as follows: how many equal parts were there in total?

It is necessary to ensure that children do not skip the words “equal”, since there are only two unequal parts (bran and flour) and therefore children tend to divide the total amount into two equal parts when solving such problems.

It is necessary to spend some time solving such problems in order to accustom children to the correct formulation of the first question.

After this, the same problem is taken and its formulation is changed again.

Teacher : What was more: flour or bran?

Student: There was 4 times more flour than bran.

Under the guidance of the teacher, the task is formulated in a new way:

“After grinding 240 kg of grain, 4 times more flour was obtained than bran. How many kilograms of bran and flour were obtained separately?” In the future, the analysis of such problems will consist of the following:

“There are two answers to this problem. So instead of the main question, we will have two questions: 1) how many kilograms of bran were obtained? and 2) how many kilograms of flour were obtained?

Let’s find out how many kilograms of bran we got.

The problem says that there was 4 times more flour than bran. This means that there was 1 part bran and 4 parts flour.

In order to find out how many kilograms there are in 1 part, you need to know: how many equal parts there were in total (?) and how much grain there was in total (240 *kg*).

To find out how many equal parts there were, you need to add 1 part and 4 parts. This is where you need to start solving the problem.

Let's draw up a plan for a solution."

Having formulated the first and second questions, the student adds a third one, which can now be solved on the basis of the previous one.

In the 4th grade, after the main problems, you can give a whole series of complicated problems of the same kind.

When solving such problems, children get used to making a note before starting to solve, from which it is clear how many equal parts there are for each added. After this, the reasoning is constructed in exactly the same way as was shown above.

We move on to **problems on finding numbers by their sum and difference.** By this time, students have had time to become sufficiently familiar with the concept of the conditional part, to learn to translate the relationship between specific data of the problem into this conditional language. Therefore, the transition to a new type of problems, which also require the selection of a certain conditional unit, despite the difference complicating the matter, will not cause any particular difficulties.

Let us take such a problem, first linking it with measures of length, so that we can turn to the help

of strips to illustrate the "parts" and surplus into which the given sum will have to be decomposed:

"We bought 27 m of black and red satin, and there was 3 m more red satin than black. How much black and red satin did you buy separately?"

Teacher: Which satin was less: red or black?

Student: There was less black satin.

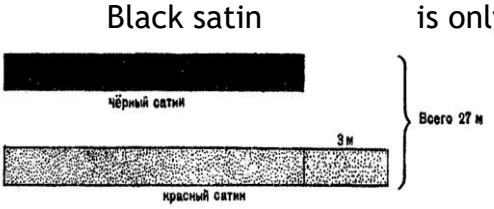
Teacher: Let's draw a piece of black satin as a strip (draws a strip on the board). Let's assume that the black satin makes up one part of all the purchased material. What can we say about the red satin in that case?

Student: There is 3 m more red satin, i.e. the same amount plus 3 m more.

Teacher : Black satin is one part. And red?

Student : One part and another 3 m.

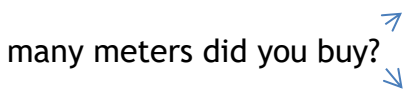
The teacher draws a strip on the board that represents red satin and writes down the numerical data.



Teacher: Let's make a table, as you did in other cases, and write down the questions of the problem.

The following entry appears under the drawing:

Black satin—1 piece
"Red" — 1 piece and 3 m.



What follows is a discussion that is first conducted on the teacher's questions and then presented by the students in a coherent form:

"Two questions cannot be solved at the same time. Let's find out first how much black satin was bought, which makes up 1 part of the entire material.

In order to find out how many meters there are per 1 part, you need to know: how many equal parts there were in total (?) and how many meters of satin there are per these parts (?).

You can find out right away how many equal parts there were: 1 h. + 1 h.

You can also find out right away how many meters there are in these equal parts: 27 m — 3 m.

Let's draw up a solution plan.

1) How many equal parts were there in total?

2) How many meters are there in these equal parts?

3) How many meters of black satin did you buy?

To these three questions we can now add a fourth, since it can be resolved on the basis of all the previous ones:

4) How many meters of red satin did you buy?

This reasoning cannot be called an ordinary complete analysis—it is an analysis of a special kind, when we have to dissect the main question of the problem, operate with conditional "parts", resort to

either a complete or an incomplete analysis ("you can find out right away").

At first glance it may seem that the first question (how many equal parts were there in total) is unnecessary. Until now it has not usually been asked. However, this question plays a significant role.

First, it explains the origin of the number 2, which later appears as a divisor and which is not given directly in the problem. The operation by which it is found was omitted only because it is very easy to add one to one. But from a logical point of view, such an omission is ignoring one of the links in the entire argument.

Secondly, having found out how many equal parts there were in total, we can formulate the second question very simply.

Thirdly, we have already had to find out how many equal parts there were in total when solving problems on finding numbers by their sum and multiple ratio. This question is no less important when solving **complicated** problems on finding terms by their sum and difference.

Let us explain this using a specific problem:

"In four days, the plane flew 3890 km, and on the second day it flew 85 km more than on the first, on the third day 35 km more than on the second, and on the fourth as much as on the first and second days together. How many kilometres did the plane fly on each of these days?"

Work on the task begins with drawing up a table:

Day 1 — 1 part.
Day 2 — 1 part and 85 km
Day 3 — 1 parts and (85 km + 35 km)

Day 4 – 2 parts and 85 km.

Then an analysis is carried out in relation to the question of how many kilometres are in 1 part.

After this, a solution plan is drawn up:

1) How many equal parts were there in total?

2) How many kilometres were there in excess? (Or: how many kilometres are in excess?)

3) How many kilometres are there in all equal parts?

4) How many kilometres did the plane fly on the first day? Etc.

Problems on motion. Problems on oncoming motion and on motion of bodies in one direction, when one body overtakes another, are singled out into a special group, despite the fact that they can be analysed as ordinary compound arithmetic problems. They do not dissolve in the mass of other problems and constitute a special type of problem on the following basis: a) in problems on oncoming motion, directional quantities with a two-way and, moreover, uniform change are given; 6) problems on motion are connected with space and time; by solving them, the student develops his spatial concepts; c) in problems on motion, three quantities are involved: path, speed and time; by solving these problems, the student learns the relationship between these quantities.

All this constitutes a very significant specificity of problems on movement. This specificity should not be ignored and these problems should not be dissolved in a mass of others solved by one formula.

When solving problems on motion, it is necessary to make extensive use of **graphics** and illustrations.

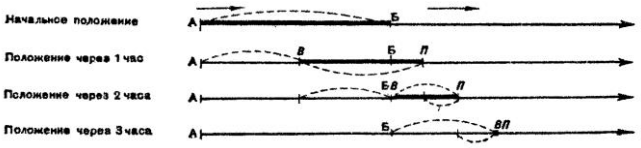
Graphics are of particular importance here. In problems on motion, a segment on a drawing depicts actual distances in a reduced form. The student must be able to show all changes in quantities on a drawing, on a segment.

Some difficulties for children may be presented by problems involving the movement of two bodies, when one body catches up with the other. In these problems, graphic illustrations are especially useful.

Let's take the problem:

"Point *A* is 24 km away from point B. A cyclist, traveling at a speed of 12 km per hour, left point *A* in the direction of point B. At the same time, a pedestrian, traveling at a speed of 4 km per hour, left point B. How many hours will it take for the cyclist to catch up with the pedestrian?" (the cyclist and the pedestrian are moving in the same direction).

A special drawing can help to understand the process of solving this problem.

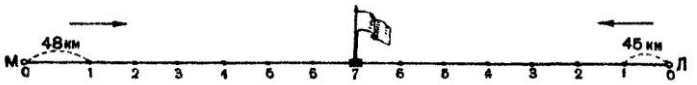


The purpose of the drawing is to show dynamically how gradually, hour by hour, the distance separating the cyclist from the pedestrian decreases from 24 km to 0 km.

It is useful to demonstrate the reduction in distance when two bodies move at different speeds

first by using the movement of two students in a class.

Drawings illustrating **oncoming** traffic are more elementary.



The nature of the drawing shows that it illustrates a problem of oncoming traffic, in which, given two speeds and the time of movement, it is necessary to find the distance from point *M* to point *L*. The drawing also suggests a solution to the problem:

1) 48 *km* X 7 = 336 *km*
2) 45 *km* X 7 = 315 *km*
3) 336 *km* + 315 *km* = 651 *km* .

Answer: 651 *km*. Another way to solve this problem:

1) 48 *km* + 45 *km* = 93 *km*
2) 93 *km* X 7 = 651 *km*.

Answer: 651 *km*.

The solution of typical problems must be carried out according to a strictly defined system with a pre-thought-out alternation of types of problems and options within a given type. Typical problems must be solved in such a sequence that the solution

of problems of the previous type helps to master the problems of the next type.

Taking into account the characteristics of the types of tasks considered, the following system of their arrangement must be recognized as the most appropriate:

III class.

1. Problems on the simple triple rule, solved by the method of reduction to unity (direct and inverse).
2. Problems on proportional division.
3. Problems on finding the unknown by the difference between two numbers.
4. Problems on finding numbers by their sum and multiple ratio.
5. Problems on finding numbers by their sum and difference.
6. Tasks on oncoming traffic.

IV class.

1. Problems on the simple rule of three, solved by the method of relations.
2. Problems on the complex rule of three.
3. Complex problems on finding numbers by their sum and multiple ratio.

4. Complex problems on finding numbers by their sum and difference.

5. Problems solved by eliminating one of the quantities.

6. Problems on the movement of two bodies in the same direction.

7. Problems on calculating the arithmetic mean.

When studying a particular type of problem, one should not introduce an excessively large number of similar problems in a row, differing only in plot and numbers, since in this case the solution of the problems turns into a solution according to a template, according to a stencil. But at the same time, one should not introduce additional conditions too early in a problem of this type, so as not to distract the children's attention from the main solution technique.

It is necessary to pay great attention to varying the structure of the problem, changing the formulation of the problem, so that children learn to recognize the same mathematical structure behind different external forms.

To eliminate the possibility of solving problems using a template, it is necessary, when solving problems of one type, to introduce such problems ("control") that do not belong to this type, but have some similar features with it. Solving such problems will contribute to a deeper understanding and more precise differentiation of the typical solution method.

It is necessary to resort more often to **comparison** and **contrast** of tasks of different types containing some similar elements in the condition.

This contributes to the successful formation of the concept of the type of task.

The concept of the type of task that children develop in this way can be consolidated in the name of the type of task.

Lessons on solving typical problems, as well as any other problems, should be structured so that the concept of problem types is formed in the process of students' **independent activity** . Such a method of work is valuable when children, under the guidance of a teacher, **themselves** find solutions to problems of a given type. This can be facilitated by: using visual aids, writing down the condition in the form of a diagram that reveals the relationship between the quantities given in the problem, comparing the given problem with other problems known to students. For the same purpose, problems for initial acquaintance with a new type should be given with small numbers, quite accessible for oral calculations, so that the calculations do not distract students' attention from the semantic side of the problem.

After solving 5—8 problems of one type, it is necessary to conduct exercises in the **independent** composition of students' own problems of this type. These exercises achieve a deeper understanding by students of both the structure of a typical problem and the method of solving it.

Solving problems is of great importance for developing students' logical thinking, attention, imagination and volitional qualities. Solving problems is also of great importance in terms of cultivating such qualities in children as independence in thinking and acting, a creative

attitude to work, initiative, accuracy in calculations and in expressing thoughts, neatness and diligence. Along with this, solving problems should also be used for the purposes of ideological and political education of students. The subject matter of the problems, their content reflects our Soviet reality: socialist construction in all areas of life, the workers' struggle to fulfil the five-year plan, the struggle of Stakhanovites and agricultural leaders to increase labour productivity, the concern of the party and the government for Soviet people (enormous material and monetary expenditures on culture, on helping mothers with many families, on helping the disabled, etc.).

In the existing problem books, not all the topics that reflect our modern times are presented with sufficient completeness. This is especially true for the post-war five-year plan for the restoration and development of the national economy of the USSR (for 1946-1950). Therefore, the teacher himself needs to take the numerical data of the plan and compose various types of problems that are understandable to elementary school students in terms of content. Here are examples of such problems:

1. "In 1913, 8,900 cars were produced. If this number is multiplied by 56 times and 1,600 is added, the number of cars that will be produced in 1950 will be obtained. How many cars will be produced by the end of 1950 ?"

Answer: 500 thousand.

2. "In 1940, 41 billion rubles were spent on cultural and everyday services for the workers of the USSR, and in 1950, it is planned to spend 106 billion

rubles. How many billion rubles more will be spent in 1950 compared to 1940?"

Answer: 65 billion rubles.

When compiling this type of problem, it is advisable to use numerical data related to the region (territory), city, village in which the school is located.

Along with using the materials of the five-year plan to draw up tasks, it is necessary to take facts from current life, which reflect the workers' struggle to increase labour productivity and improve the living conditions of broad sections of the population. Such facts are usually given in the reports of the boards of collective farms and industrial enterprises; they are published in district, regional and central newspapers. Facts taken from the life of their collective farm, plant, factory are especially interesting and convincing for students.

Thus, the following task will be interesting for children of the Komsomolets collective farm in the Kuban region: "In our collective farm, the average harvest of winter wheat is 175 poods per hectare. But the team of Komsomol member Anna Krepkaya harvested 5976 poods from 24 *hectares* . How much higher is the harvest per hectare for Anna Krepkaya?" (the fact was published in Pravda).

The students' social work can also be reflected in the tasks. Children take part in collecting ears of corn, in fighting agricultural pests, in collecting medicinal herbs, and help the collective farm in performing some agricultural work. The social

significance of this work, its benefits can be characterized by very convincing figures.

The educational influence of such tasks on children is undeniable: children are introduced to the interests that the entire country currently lives by; children are shown the best people of our country in this way, children are infected with work enthusiasm.

Such problems are a good addition to the problems solved in class and at home using the problem book.

By composing and solving problems using local material, using “live” numbers, and learning arithmetic, children also learn the most advanced Soviet work culture in the world.

SAMPLE LESSONS ON SOLVING PROBLEMS WITH WRITTEN EXPLANATION

IV CLASS

In the first lesson, devoted to developing skills in written explanation of problems on addition, subtraction, multiplication and division, the objective of the lesson can be formulated approximately as follows: "When solving a problem, you need to be able to explain the problem, explain why you add numbers in one case, divide in another, multiply in a third, etc. You can recognize and give an oral explanation. Today in the lesson we will learn to compose a written explanation. And henceforth we will solve problems not only with an oral, but also with a written method, with a written presentation of the explanation."

The teacher offers the children an elementary problem to solve orally: "A student bought ink and a pen in a store. The ink cost 36 kopecks, and the pen was 8 kopecks cheaper. The student gave 1 ruble to the cashier as payment. How much change did he get?"

After learning the conditions, an oral plan for the solution is made and a detailed explanation of each action is formulated. Having made an oral explanation of the first action, the children then, at the teacher's suggestion, write this explanation down in their notebooks. After this, they move on to a detailed oral explanation of the second action. The students then write down the oral formulation of the explanation of the second action in their notebooks. This is how work is carried out on the last-third actions. As a result, the following entry is obtained in the students' notebooks:

Task:

A student bought ink and a pen. The ink cost 36 kopecks; the pen cost 8 kopecks less. The student gave 1 ruble to the cashier as payment. How much change did the student receive?

Decision with written explanation:

1) First, let's find out how much the pen cost. Ink cost 36 kopecks, and the pen was 8 kopecks cheaper. To find out the cost of the pen, you need to reduce 36 kopecks by 8 kopecks, or subtract 8 kopecks from 36 kopecks.

36 kopecks - 8 kopecks. = 28 kopecks

2) Next, we find out how much the entire purchase cost, if the ink cost 36 kopecks and the pen 28 kopecks. In order to find out the cost of the entire purchase, you need to add 36 kopecks and 28 kopecks.

36 kopecks + 28 kopecks = 64 kopecks

3) Finally, we find out how much change the student received. The payment was 1 ruble, and the purchase cost 64 kopecks. In order to find out how much change was received, we need to subtract 64 kopecks from 1 ruble.

1 rub. — 64 kopecks. = 36 kopecks

Answer: the student received 36 kopecks in change.

Измерения на земле

Measurements on the ground

—Now let's see,—says the teacher,—how the division operation is explained. Let's solve and explain the problem: "They paid 200 rubles for 5 meters of material. How many meters of such material can be bought for 360 rubles?"

Work on this task is carried out in the same way as on the previous task, i.e. first an oral explanation of the first action is composed, and this oral explanation is then written down in the notebook. Then the same work is carried out on the second action. The teacher pays special attention to the thoroughness of the formulation of the explanation of this action (division by content): it is more difficult for children, therefore the teacher formulates this explanation in parts and repeats it to the students several times before moving on to writing it down.

As a result of working on the second task, the following entry is obtained:

Task:

We paid 200 rubles for 5 m of material. How many meters of such material can be bought for 360 rubles?

Decision with written explanation:

1) First, let's find out how much 1 m of material costs. 5 m cost 200 rubles, and Ë m costs 5 times less. To find out the cost of 1 m, you need to reduce 200 rubles by 5 times, or divide by 5.

200 rub. : 5 = 40 rub.

2) Next, we find out how many meters of material can be bought for 360 rubles. If 1 m costs 40 rubles, then for 360 rubles you can buy as many meters as 40 rubles are contained in 360 rubles. To find out how many times 40 rubles are contained in 360 rubles, you need to divide 360 rubles by 40 rubles.

360 rub. : 40 rub. = 9.

Answer: for 360 rubles you can buy 9 m of motherboard.

The children are asked the following questions (to which they give answers):

1) In what cases is addition used?
2) In what cases is subtraction used?
3) When is one number multiplied by another?
4) When is the division operation used?

"When solving a problem with a written explanation," the teacher concludes, "we will always, after verbally drawing up a plan, write the plan's question and then explain in writing the action that we perform to solve the question.

The homework assignment is to solve the problem with a written explanation: “100 litres of milk were poured equally into 5 cans. 3 cans were taken to the canteen. How many litres of milk are left?”

In the 2nd lesson, the goal of the lesson is to consolidate children’s ability to explain in writing the actions when solving a complex arithmetic problem.

Lesson progress. The teacher asks the students to find the problem in the problem book: “24 *kg* of flour were bought for an orphanage for 192 rubles. Then they also bought 175 *kg* of flour and 80 *kg* of millet for 1,880 rubles. How much is 1 *kg* of flour more expensive than 1 *kg* of millet?”, silently read its condition to themselves, think it over and outline a plan for solving it. When most of the students raise their hand, the teacher asks one of the students to read the problem out loud and say orally how they would solve it in the form of questions. The student should ask the following questions:

1) How much does 1 *kg* of flour cost? 2) How much does 175 *kg* of flour cost? 3) How much does 80 *kg* of millet cost? 4) How much does 1 *kg* of millet cost? 5) How much more expensive is 1 *kg* of flour than 1 *kg* of millet?

Based on the verbally outlined plan, a solution to the problem is made: selection of numbers, choice of actions and calculations, which are arranged in the following order:

1) 192 rubles: 24 = 8 rubles.
2) 8 rubles x 175 = 1400 rubles.
3) 1880 rubles — 1400 rubles = 480 rubles.
4) 480 rubles: 80 = 6 rubles.
5) 8 rubles — 6 rubles = 2 rubles.

Answer: 2 rubles.

Then the teacher begins to compose an oral explanation for each action, followed by recording this explanation. The explanation of each action consists of

the following points: the formulation of the question, an indication of the numerical data needed to solve the question, and an explanation of why this or that action must be performed on the names of numbers. All this is done orally first, and then the orally formulated phrases are written down. The result of such work is the following entry:

Task:

1—24 *kg* of flour costs 192 rubles.

II — 175 *kg* of flour and 80 *kg* of millet — 1,880 rubles.

How much more expensive is 1 *kg* of flour than 1 *kg* of millet?

Solution to the problem with explanation:

1) How much does 1 *kg* of flour cost?

24 *kg* of flour cost 192 rubles, and 1 *kg* of flour costs 24 times less. To find out the price of 1 *kg* of flour, you need to divide 192 rubles by 24:

192 rub. : 24 = 8 rub.

2) How much does 175 *kg* of flour cost?

1 ka of flour costs 8 rubles, and 175 *kg* costs 175 times more. To find out the cost of 175 *kg* , you need to multiply 8 rubles by 175:

8 rubles x 175 = 1400 rubles.

3) How much does 80 kg of millet cost?

All the flour in the cereals together cost 1880 rubles, the same flour costs 1400 rubles. To find out the cost of all the cereals, you need to subtract the cost of the flour from the total cost:

1880 rub. — 1400 rub. = 480 rub.

4) How much does 1 *kg* of millet cost?

80 *kg* of millet cost 480 rubles, and 1 *kg* of millet costs 80 times less. To find out the price of 1 *kg* of millet, you need to divide 480 rubles by 80.

480 : 80 = 6 rubles.

5) How much more expensive is 1 *kg of flour than 1 kg* of millet?

1 *kg* of flour costs 8 rubles, and 1 *kg* of millet costs 6 rubles. To find out how much more expensive a kilogram of flour is than 1 *kg* of millet, you need to subtract 6 rubles from 8 rubles.

8 rubles — 6 rubles = 2 rubles.

Answer: 1 *kg* of flour costs 2 rubles more than 1 *kg of millet*.

Homework assignment: solve a problem (from a problem book) with a written explanation.

ELEMENTS OF GEOMETRY IN PRIMARY EDUCATION

In the arithmetic program of the first years of study, elements of geometry are intertwined with the basic arithmetic material. However, it is in the junior grades that it is important to establish the most correct and rational methods of familiarising students with the principles of geometry and to lay a solid foundation of initial spatial concepts in the children's minds.

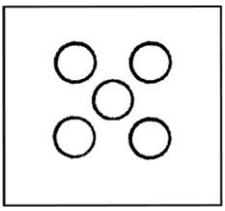
In the first grade, during the very first lessons, it is necessary to check the children's stock of initial ideas about the size of objects (big-small, high-low, wide-narrow, etc.), about their relative location (far-close, right-left, in the middle-at the edge). In September, the teacher can conduct a number of interesting observations and games with the children, allowing them to find out the children's stock of spatial ideas. Thus, during a walk on the lawn, the teacher can ask the children to gather in the middle of it, and then some run to the right, and others to the left. On the same lawn, the teacher has every opportunity to check the children's ideas about the relative position of various objects (trees, bushes), as well as their relative size. It is useful to bring a picture to the classroom and have a conversation about what objects are visible in the picture, which of them are larger, which are smaller, which of them are closer, which are further.

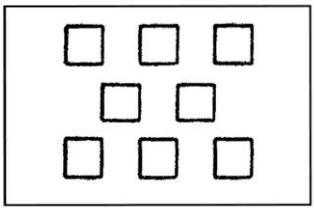
In the first grade, children need to be given basic skills of linear measurements. During the year,

children become familiar with the meter and centimetre and use them to measure various lengths and distances. It is desirable that each student makes a meter and a ruler 20-30 cm long and, using them for various measurements, forms a correct and accurate idea of the size of a meter and centimetre. Work on refining the students' eye is of great importance. Children, on the teacher's instructions, determine various lengths or distances on the ground by eye, and then, using their control measurement, establish the errors of their eye.

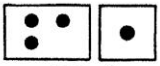
In the development of spatial concepts, it is of great importance to solve problems such as, for example: "The width of the river in winter was 24 m. In the spring, during the flood, the river overflowed its banks by 5 m on one side, and by 12 m on the other side. How wide was the river during the flood?"

Geometrical representations can be used when studying the first ten numbers. The concept of a number is formed faster and more firmly if children's spatial representations come to the aid of their numerical representations. Therefore, it is useful to illustrate the study of the first ten numbers with numerical figures, using circles, squares, and triangles. For example, the number 5 can be illustrated by a square with five circles, the number 8 — by eight squares enclosed in a rectangle.

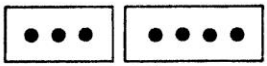




Using numerical figures, children can clearly understand the composition of numbers and the first operations with them.



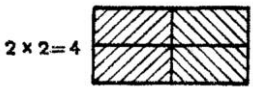
3+1=4



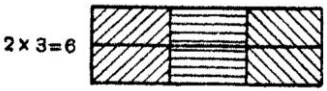
3+4=7

5−1=4

The multiplication table of numbers in the second ten is also well illustrated with the help of geometric figures.



2×2=4



2×3=6

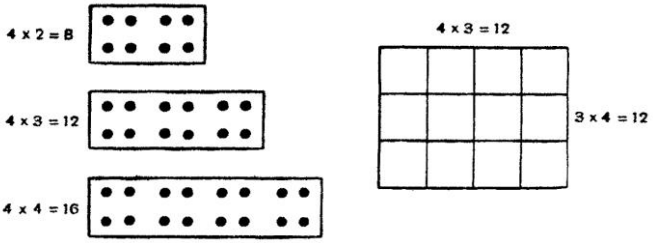
In these lessons, children not only consolidate their skills in performing operations with numbers, but also become familiar with the simplest geometric figures: squares, triangles.

In the second grade, students receive a concrete idea of the length of a kilometre. Together with the teacher, children must walk along a highway or

along a railroad track a distance of one kilometres, determine the time it takes to travel this distance, see the length of the distance travelled, i.e., in practice, form a concrete idea of the length of a kilometres.

In the 2nd grade, as in the 1st, problems are solved to develop children's spatial representations.

When studying operations on numbers of the first hundred and thousand, it is useful to use geometric illustrations. Thus, the multiplication table can be illustrated with geometric figures.



An interesting geometric representation can be obtained by the commutative law of multiplication: "The product does not change when the factors are changed."

The teacher draws a rectangle on the board, divided into 12 cells, and asks the students to count the number of cells in this rectangle, first by rows, then by columns. The children write down the two results of counting the cells: "4 X 3 = 12" and "3 X 4 = 12" and are convinced of the validity of the commutative property of multiplication .

Children's understanding of the difference and multiple comparison of two numbers is greatly

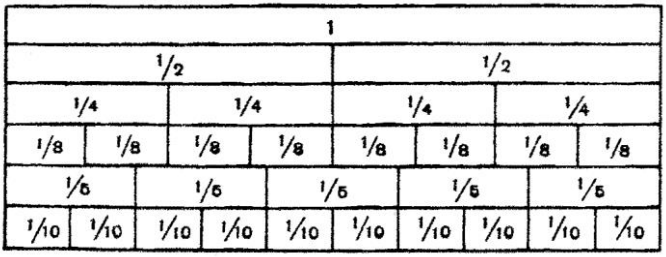
facilitated if these comparisons are made on straight line segments or rectangular strips.

In the third grade, children study new units of length measurement: decimetre and millimetre. They learn that one meter contains 10 decimetres, make rulers one decimetre long, divide these rulers into centimetres and get a concrete idea of the length of a decimetre. Children carry out practical measurements of the sizes of a variety of objects in the classroom and extracurricular environment with a decimetre: the length and width of a classroom table, a class register, a window pane, etc. It is necessary for students to express the results of measurements in meters, decimetres, and centimetres, i.e. in the form of compound named numbers. The teacher also introduces children to a millimetre. In this case, students acquire the skills of accurately measuring small objects: a matchbox, a postcard, etc.

In the 3rd grade, work continues on refining the eye.

Children should be asked to measure the length of their steps in decimetres and then learn to use their steps to determine the distances between individual points in the classroom or in the area.

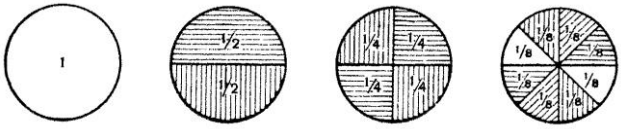
In the third grade, children become familiar with scale and how to use it to draw certain straight line segments on paper.



When introducing fractions, it is very important to use geometric concepts in children. Let's say that children are studying the fraction "half". Each child receives a paper or cardboard circle and then, at the teacher's direction, folds it in half. Children paint each half of the circle in different colours and establish that each part of the circle represents its half. Children do the same with paper strips (rectangles): they fold them in half and paint the halves in different colours. They are introduced to fractions of ¼ and 1/8 in the same way: they fold circles and strips of paper first in half, and then into four parts (and when studying the fraction of $_{1/8}$ into 8 parts paint the resulting fractions of the figures in different colours. Children study $^{1/5}$ and $_{1/10}$ fractions using strips of paper divided into the corresponding number of parts and painted in different colours.

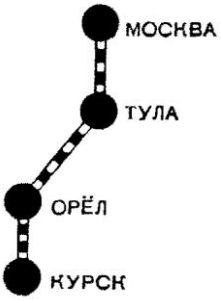
Fractions of a unit can also be studied on circles drawn on a blackboard or in a notebook using a compass. These circles are divided into a certain number of sectors depending on the fractions being

studied. When bending a circle into parts, children get a visual representation of its centre and radii.



Problem solving should be used to develop spatial representations. In this regard, problems on movement deserve special attention; their solution should be illustrated using segments. Let's assume that children are solving problems on the oncoming movement of trains or pedestrians. In this case, they mark two points *A* and *B on a straight line* , then use equal segments to mark the speeds of their oncoming movement, and finally use a symbol to mark their meeting point.

Worthy of attention. tasks on determining the relative position and distances between individual cities. Let's assume that the task being solved is: "From Moscow to Kursk via Tula and Orel 538 km. From Moscow to Orel 383 km, and from Tula to Kursk 344 km. Determine the distance from Tula to Orel." Children draw a schematic location of the cities and determine the distance between the cities.



In the third grade there are problems on rectangular diagrams; drawing rectangles of a certain height helps children visualize the relationship between the values of the problem.

In the 4th grade, students move on to studying special topics in geometry: a straight line and a segment, angles, a rectangle and a square and their areas, a cube and a rectangular parallelepiped and their volume. The study should be of a purely practical, effective nature. Children observe straight lines in the form of tightly stretched threads, a trace along the fold of paper, straight lines drawn on the blackboard and on the pages of a notebook. In the process of work, they acquire the ability to draw in different directions and measure segments of straight lines limited, for example, by points *A* and *B*. In connection with drawing straight lines and measuring their segments, they understand that a straight line is the shortest distance between two points, that only one straight line can be drawn through two points, that two straight lines intersect only at one point, and they refine their ideas about the vertical and horizontal directions of a straight line. Along with learning the basic properties of a

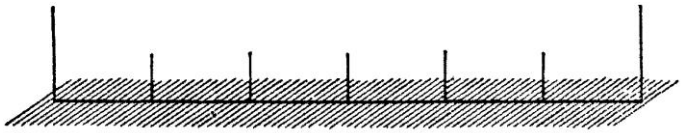
straight line, children practice beating a tightly stretched string on a chalkboard, independently measuring the length and width of a classroom, measuring the sides of a chalkboard, etc.

Measurements are taken not only on objects in the classroom environment, but also outside the classroom: in the school yard, on the street, on the lawn.

To carry out these measurements, the teacher selects a suitable location and draws up a plan for carrying out the measurements in advance: how many units the students will be divided into, what measurements each unit will perform, what measuring instruments will be needed.

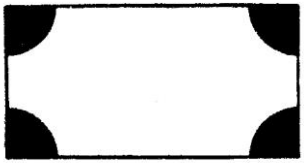
Let us dwell on the content of one of the measurement works: “Hanging a straight line on the ground”.

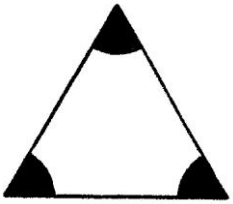
In order to involve all students in this work, the class is divided into 3—4 groups. Each group selects two points on the ground between which a straight line must be drawn and the distance measured. The selected points are marked with poles. Near each pole there are 2—3 students, who form a control group. While completing the task, the students of this group observe whether the straight line is hung correctly between the poles, i.e. whether the stakes driven in between the poles are installed correctly along a straight line.



The task of hanging a straight line is assigned to the second group of students, who place the stakes between the poles in a straight line. The third group measures the distance between the poles using a tape measure or survey chain. The fourth group records the results of the measurements in a notebook. After measuring the distance between two points, it is possible to give the children an additional task: measure this distance in steps and calculate the length of one step. After completing the measurement, the children make sketches of straight-line segments in their notebooks, corresponding in length to the measured distances on the ground. When drawing these segments, a certain scale must be used.

The study of angles should also be of an experimental nature. The teacher shows angles on objects in the classroom, on book bindings, on notebooks. Children draw rectangles and triangles on the board and in notebooks and mark their angles.





The teacher demonstrates cardboard models of various angles (preferably coloured) to the students, showing the sides and vertices of the angles.

Using angle models, the teacher also finds out which angles are called right angles and which are oblique. By superimposing coloured angle models on top of each other, the teacher finds out that some angles are larger than a right angle (such angles are called obtuse angles), while others are smaller than a right angle (they are called acute). By superimposing right angles on top of each other, their equality is established. Children, on the teacher's instructions, draw right, obtuse, and acute angles and find their vertices and sides. Homework is given to draw various angles in notebooks and shade them with coloured pencils.

Later, the teacher draws two trees on the blackboard and, through conversation, finds out that one tree is depicted growing straight, and the other is depicted growing crookedly.

In subsequent lessons, two shapes are studied: **a rectangle and a square.**

Having brought paper models of rectangles and squares to the class, the teacher distributes them to the children and asks them to observe these figures and compare them. Then the children measure the sides of the rectangles and squares and establish that the opposite sides of rectangles are equal to each other, while all sides of squares are equal. By placing squares on the corners of the models, the students are convinced that all the angles of squares and rectangles are right angles, which is why they are called “rectangles”. Next, the similarities and differences between a square and a rectangle are formulated. In conclusion, the students recognize rectangles and squares on objects from the classroom and extracurricular environment.

In the next lesson, students draw rectangles and squares with given side lengths.

The study of two figures—a rectangle and a square —is completed by constructing a rectangle on the ground (in the school yard, on the school grounds).

Before going out into the area, the teacher introduces the children to the structure and use of the ekker.

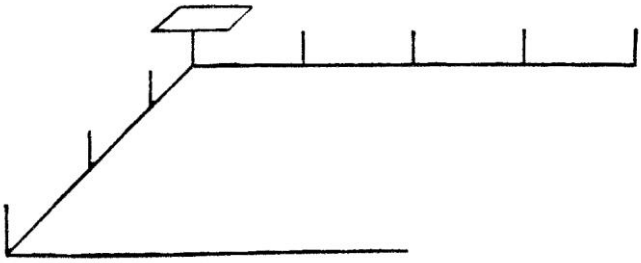
The class is divided into sections. Let’s say that it is necessary to mark out a rectangle 50 m long and 20 m wide on the ground. To do this, a point is chosen to fix one vertex of the rectangle, and the first section of students with an ekker in their hands is located at this point. This section is the control group. The children in the section look through the ekker’s slots and check the correct placement of the stakes on the sides of the right angle. The second section receives the stakes at its disposal and, at

the direction of the students in the first section, drives the stakes into the ground so that they stretch out in straight lines in the direction of the sides of the right angle.

Students in the third grade use a tape measure to measure the distances along the sides of a right angle (in this case, 50 m along one side of the angle and 20 m along the other).

Then the ekker is transferred to another vertex of the rectangle so that one pair of its slots “looks” at the already hung side of the right angle, while the third side of the rectangle is hung in the direction of the other pair of slots.

Thus, a rectangle of given dimensions is marked out on the ground. This work requires accuracy from students, since in case of incorrect construction of right angles, there may be a “discrepancy” in finding the fourth vertex of the rectangle (i.e. the end points of the first and fourth sides of the quadrangle will not coincide with each other).



The study of the topic “Measuring the area of a rectangle” is carried out in the following stages:

I. The concept of area. Direct measurement of areas of rectangles with square measures.

The teacher finds out that area is the size of the surface of a figure. At the teacher’s request, the children show the surface of a blackboard, a table top, a book binding, a window sill.

In the conversation, the teacher reminds the class that segments are measured by linear measures. Similarly, the areas of figures are measured by comparing them with some pre-selected areas. For example, the area of a table top can be measured by placing at least identical notebooks on it and counting them. But the notebooks can be different in size, and their area is unknown to us. Certain units of area measurement are established: square meter, square decimetre, square centimetre. The teacher shows samples of these measures made of cardboard or paper . The children examine them and draw a square decimetre and a square centimetre in their notebooks. Next, the area of the table and desk top is measured by placing square decimetres on them and counting them.

II . Measuring the area of a rectangle. The teacher shows how to measure, for example, the area of a rectangular strip 4 *dm long* and 1 *dm wide* . In this strip, 4 *sq. dm*, and each laid square decimetre will fall on a linear decimetre in length. It is also shown that in a strip of 5 *dm in length and 1 dm* in width, 5 sq. *dm are laid*, etc.

Then the area of a rectangle 4 *dm long* and 2 *dm wide is measured* . This rectangle is drawn on the board and divided into two strips. The lower strip is divided into four squares. The children establish that each strip of the rectangle contains 4 *sq. dm*, and the entire area of the rectangle is twice as large, i.e. 8 *sq. dm* (the figure is shown in a reduced form).

The calculation of area is written as follows:

4 sq. *dm* X 2 = 8 sq. *dm*.

After this, the children draw rectangles in their notebooks 8 *cm long* (two cells = one centimetre) and 3 *cm wide* and establish that this rectangle contains three strips with an area of 8 *square cm* each. Therefore, the entire area contains 8 *square cm* X 3 = 24 *square cm*. In this way, children are gradually and clearly brought to the understanding that the length of a rectangle always shows how many square units will be in one strip, and the width shows how many strips are contained in the rectangle. The product of the number of square units contained in one strip and the number of strips will give the value of the entire area. Therefore, if the length of a rectangle is 8 *cm* and the width is 5

cm , then its area will be equal to 8 *square cm* X 5 = 40 *square cm*.

Having understood the essence of the process of calculating the area of a rectangle, children learn the rule: “In order to calculate the area of a rectangle, you need to measure its length and width in the same units and multiply the resulting numbers.” The result of measuring areas is always expressed in square units of measurement.

After deducing the rule, children apply it to measuring the area of a wide variety of objects: the area of the classroom floor, the top of the table and desks, the blackboard, the binding of books and the class register, etc.

When studying **land area measurements** in open areas, rectangles are constructed, one with an area of one are and the other with an area of one hectare. In this case, children are placed on the sides of the rectangles obtained on the ground and receive a concrete idea of the size of an area and a hectare.

At the same time, two more works are carried out on the ground: the construction of a rectangle of a certain area and the measurement of the area of a given rectangular plot of land.

To complete the first task, children make a preliminary calculation of the length and width of a plot of a given area, and then conduct a practical construction of the plot on the ground. To complete the second task, students are divided into three groups. One group hangs the sides of a rectangular plot, another group measures these sides, and the third group performs calculations to determine the

area of the plot. Upon returning to class, students draw plans of the plots of land they measured.

Measuring the volume of a cube and a rectangular parallelepiped. Students become familiar with a cube, receive models of it, observe how many edges, faces, and faces it has. By measuring the edges of a cube and examining the angles of its faces, children are convinced that each face of a cube is a square. They establish empirically that all faces of a cube are equal to each other.

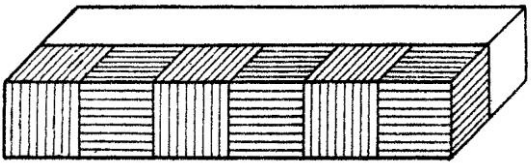
is also studied . Children establish through observation and measurement that all the lateral faces of the parallelepiped are rectangles. By outlining the faces of the parallelepiped on paper, children are convinced that the opposite faces of the parallelepiped are equal to each other. On the parallelepiped models and its drawings, they show the faces: upper, lower, right, left, front and back; they count the number of faces, edges and vertices of the parallelepiped; they establish what is the similarity and difference between a cube and a parallelepiped.

Having introduced the students to a cube and a rectangular parallelepiped, the teacher moves on to measuring volumes. It is established that objects shaped like a cube or parallelepiped have a capacity. By pouring loose bodies, the teacher shows how to measure the capacity of a small box or crate. The children are convinced that the capacity of bodies can be measured by comparing it with the capacity of other bodies that are smaller in size. The teacher shows that the capacity of a crate, box, and other bodies is most conveniently measured by

comparing them with cubes with edges 1 *cm* (cubic centimetre) or 1 *dm* (cubic decimetre) long. First, the capacity of the box is measured by **filling it with cubes** with edges 1 *cm long* and counting the number of these cubes. It is very important that several such measurements are made using cubic decimetres.

Next, the teacher brings to class a rectangular beam, for example, 8 *cm long* with a cross-section of one square centimetre. This beam is divided into 8 cubes, with each cube being one linear centimetre long and containing one cubic centimetre in volume. In this way, children are convinced that rectangular beams with a square cross-section have as many cubic units of measurement as there are linear units in their length.

In the same lesson, the teacher shows the children a rectangular parallelepiped in the form of a layer, the length and width of which are several centimetres (or, even more desirable, decimetres), and the height contains only one unit of measurement. Let's assume that the layer is 6 *cm long, 2 cm* wide and 1 *cm* high . Then this layer is divided into two bars, each with a volume of 6 cubic cm. The volume of the entire layer: 6 *cubic cm* X 2 = 12 *cubic cm*.



The teacher establishes that in each rectangular **layer** with a height of one unit of measurement there are as many bars as there are linear units in the width of the layer. In each bar there are as many **cubic** units of volume as there are linear units of measurement in length.

The volume of a layer is determined by the product of the number of cubic units of volume of one bar and the number of bars. The result of calculating the volume is always expressed in **cubic** units of measurement. In class, they solve the problem: how many cubic centimetres will fit in a box if

its length is 5 *cm* , width is 4 cm, height is 1 *cm*
» 10 *cm* » 3 *cm* » 1 *cm*
» 15 *cm* » 4 *cm* » 1 cm

Next, the parallelepiped is divided into separate layers, with one layer for each unit of measurement in height. Let this parallelepiped be 6 *cm long, 3 cm* wide and 4 *cm* high . Then it will have 4 layers, each layer with 3 bars and each bar with 6 cubic cm of volume. The total volume of the entire body will be 72 *cubic cm* . The calculation record for the volume is as follows: 6 *cubic cm* X 3 X 4 = 72 *cubic cm*. On another rectangular parallelepiped, children once again understand that to find the volume of a parallelepiped, it is necessary to measure the length, width and height of the body in the same units and multiply the result of measuring the length, expressed in cubic units, by the width and height of the body. The volume is expressed only in cubic units.

In the following lessons, students find the volumes of various objects in the environment (boxes, crates, cupboards, rooms) by making independent measurements, and find the relationship between various cubic units of measurement: a cubic meter and a cubic decimetre, a cubic decimetre and a cubic centimetre. It is desirable that students glue a cube with an edge of one decimetre out of cardboard, mold a cube with an edge of one meter out of snow, in order to have a concrete idea of the size of cubic measures. When calculating the volume of a rectangular parallelepiped, children should clearly understand that measuring the height of the body is necessary to find the number of its layers, measuring the width to find the number of bars in each layer, and measuring the length to find the number of cubic units in each bar. The product of three dimensions of a parallelepiped gives the volume of the body, necessarily expressed in cubic units.

In subsequent arithmetic lessons, children independently measure the volumes of various objects in the classroom, solve problems on calculating the volumes of bodies. It is useful to go outside the school and measure the volumes of such objects as a cellar, a barn, an earthen ditch, etc. Students must measure the length, width and height (or depth) of these objects, and then perform the necessary calculations.

Geometrical material in primary grades takes up a small number of lessons. However, its study is of great importance for the general development of children. Studying geometry develops in students an interest in independent work, in researching the

issues being studied, a desire to expand their knowledge and practical skills.

Familiarisation with elementary geometry allows children to successfully study geography, physics and a systematic course of geometry in subsequent grades. Children study visual geometry with great interest and enthusiasm; it attracts them with its concreteness, vitality; the presence of many measuring and constructive works.

TESTING STUDENTS' KNOWLEDGE OF ARITHMETIC

Checking students' knowledge of arithmetic, as well as other subjects, is necessary for both the teacher and the students. The teacher needs to check the students' knowledge before presenting new material, in order to know to what extent the previously covered material can serve as a basis for learning the new. After explaining the new material, the teacher checks how the students understood what was explained, what they remembered, and what they did not pay due attention to. After consolidating the knowledge communicated, the teacher checks how the students learned the knowledge in order to take into account the learning outcomes.

Based on these results, the teacher, if necessary, provides explanations, conducts additional exercises, and corrects any gaps and shortcomings in the knowledge of the entire class and individual students.

Testing knowledge is thus one of the means to ensure the success of the student's further studies. This is its great importance for students.

In the process of studying arithmetic, children not only gain knowledge, but also master various skills and abilities. Therefore, it is necessary to check not only the knowledge of students, but also their skills and abilities.

When checking knowledge, the teacher finds out not only how children formulate a rule, give a definition, indicate the properties of a number or

action, but also how they understand what they answer. The teacher must be convinced that children associate the correct ideas and concepts with words. Therefore, when checking knowledge, it is necessary to require students to confirm their judgments with examples, offer to apply the rule to solving problems or completing some task.

When checking, one should pay attention not only to the presence of knowledge, but also to its quality: correctness, accuracy, completeness, depth, strength and effectiveness, i.e. to the presence of a connection between knowledge and the students' experience and practice.

The knowledge, skills and abilities of students are gradually improved in the learning process. When testing, it is necessary to establish at what level the knowledge, skills and abilities of a particular student are. This is especially important to know when testing the knowledge of a weak student. In this case, it is important for the teacher to determine not only what the student does not know, but also what he knows well; only on the basis of what the student has firmly learned is it possible to further expand and deepen his knowledge. For this test, a work can be used in which a series of gradually more complex examples or problems are selected, starting with simple and easy ones.

The following methods are used to test the knowledge, skills and abilities of students: a) oral questioning; 6) observation of students' independent work in class; c) viewing and checking students' independent written work completed in class and at home; d) conducting written tests.

An oral survey is conducted at each lesson and can be combined with checking the completion of homework: the called student first tells how he solved the examples and problems assigned for homework, and then answers the questions posed by the teacher.

During independent work in class, the teacher helps children in case of difficulties, observing the work of all students. These observations, as well as subsequent verification of the results of independent work, give the teacher the opportunity to judge what the children have firmly learned, what skills and abilities they have not yet mastered, and what are the gaps in the knowledge of individual students.

To test the knowledge, skills and abilities of students for each completed section of the program, the teacher conducts tests.

Tests can be of different types: small in volume, designed for 10—15 minutes, and tests that take up an entire lesson. In terms of content, tests can be either homogeneous (consisting only of examples or only of problems), or combined, i.e. containing both a problem and examples. There can be tests of a practical nature, for example, related to children taking measurements.

When composing the text of a test, the teacher carefully considers its purpose and, in accordance with it, includes in the work certain types of examples, certain types of problems.

The test provides an opportunity to determine how well students have mastered individual elements of a particular section of knowledge, skills and abilities.

The test should cover the main and most essential aspects of the knowledge, skills and abilities acquired by students. Let's give some examples. The test on subtraction of multi-digit numbers (for grade III) may contain cases of performing this action that vary in difficulty, namely:

1) Subtraction performed without breaking up higher-order units, for example: 8759—2546.

2) Subtraction of a number with zeros; the difference is a number with zeros: 9068—7008.

3) Subtraction performed with the division of higher-order units: 12547—8259.

4) Subtraction from a number in which some digits are missing units, with the division of higher digit units, for example: 10102—8795.

A test on **multiplication** of multi-digit numbers may contain the following main cases of this action:

1) Factors with all significant digits, for example: 756 X 498.

2) Multiplicand with zeros in the middle: 6008 X 47.

3) Multiplier with zeros in the middle: 738 X 306.

4) Multipliers with zeros at the end: 580 X 4,600.

When conducting tests, conditions must be provided for students to complete these tests **completely independently .**

For this purpose, examples and problems offered for independent solution are usually compiled in two versions so that each version includes examples and problems that are identical in mathematical content and degree of difficulty, but differ from each other in numerical data and the specific design of the mathematical content.

The teacher carefully checks the completed work and evaluates the knowledge, skills and abilities of the students based on how they are revealed in the work completed by the students.

After checking and evaluating the work of each student, the teacher summarizes the overall results of the test for the class: how many students and which ones completed the work with a 5, a 4, etc., what kind of mistakes were most common, which examples and problems presented the greatest difficulties for students and caused the greatest number of mistakes in their solution.

You can write down the examples in which the children made the most mistakes, and write down typical mistakes. You can also write down the results of each student's work in a table, in which the correct solution is marked with a + sign, the wrong one with a — sign, and the absence of a solution with a 0 sign (see the sample table form on page 510).

Summarising the results of students' performance of the test makes it possible to determine which questions of the program have been poorly mastered by the whole class, and which by individual students. Based on this, the teacher plans what explanations or exercises should be done with the whole class, what additional classes should be conducted with individual students.

Table form
to summarize the results of the test in arithmetic

	How are the examples solved?	Total	

Io. Io. p/p	surnames Those who shunned	№ 1	No. 2	№ 3	No. 4	№ 5	№ 6	correct decisions	wrong decisions	Ratings
1. **2.**	…… ……									

Total:
Correct decisions.....
Wrong decisions.....

Assessment of students' knowledge

When testing mental arithmetic skills, written calculations, and problem solving, knowledge of theory is also tested: definitions, rules, tables of measures, tables of arithmetic operations.

The oral survey is conducted within the framework of a given lesson, as well as within the framework of previously covered material in one or several sections of the program.

Questions and tasks should be given to the whole class, but the student who is selected for testing answers them. The student's answer should be precise, sufficiently complete, and, where required, presented in the form of coherent speech, and supported by examples.

When performing oral calculations, the student is required to: a) have a solid knowledge of arithmetic tables; 6) be able to apply basic

calculation techniques and explain how to perform the operations; c) be sufficiently quick in calculations.

A gross error in oral calculations is considered to be an incorrect answer, which is associated with ignorance of tables of arithmetic operations, ignorance of unit ratios of measures, ignorance of the basic techniques of oral calculations. Less gross errors include irrational calculation techniques, difficulties in explaining the action being performed.

To test a student in mental arithmetic in grades I and II , 3—4 examples in one action are given, and in grades III and IV , 5—6 examples in one action. The size of the numbers and the choice of actions are determined by the program.

A score of 5 is given if the student performed the calculations correctly, quickly enough, and was able to independently explain the process of performing the actions.

A grade of 4 is given if the student is able to independently perform and explain the action, but made no more than 1—2 minor errors in solving it.

A grade of 3 is given if the student made 1—3 mistakes in solving the problem, but no more than 2 gross mistakes.

A grade of 2 is given if the student made 2—4 mistakes in solving the problem, but no more than 2 serious mistakes.

A grade of 1 is given if the student made more than 2 gross errors in calculations.

When performing calculations on the blackboard, the student is required to: a) have a solid knowledge of arithmetic tables, rules for performing arithmetic operations and transformations; 6) be able to

explain how to perform an action; c) be able to use rational methods of written calculations.

When testing knowledge and skills in calculations at the board, the student is usually given one example, the difficulty of which is determined by the program and the problem book.

A grade of 5 is given if the student has correctly solved the example, demonstrated the ability to use rational calculation methods; explained in the form of coherent speech how the solution was performed, demonstrated solid knowledge related to solving the example (for example, knowledge of numbering, unit ratios of measures, etc.).

A grade of 4 is given if the student made 1—2 mistakes in solving the example and in the explanation, of which no more than one was serious.

A grade of 3 is given if the student made 1—3 mistakes in solving the example and in the explanation, but no more than a gross mistake.

A grade of 2 is given if the student made 2—4 mistakes in solving the example and in the explanation, but no more than 2 gross mistakes.

A grade of 1 is given if the student made more than 2 serious mistakes.

To assess the student's knowledge in the field of problem solving, 1—2 oral problems or 1 written problem, which is solved on the blackboard, are given. Problems are given in accordance with the program and the problem book.

A grade of 5 is given if the student has correctly solved and explained the solution to the problem, and has also demonstrated the ability to use rational methods of solving the problem: applied the least number of actions in solving it, arranged all

calculations rationally and performed in compliance with the accepted forms of recording actions, presented the solution in the form of a coherent story. A grade of 4 is given if the student has correctly solved the problem, but made 1—2 minor errors in solving and explaining it (inaccurate formulation of the question, omission of the name).

A grade of 3 is given if the student made 1—3 mistakes in solving and explaining the problem, with no more than one gross mistake (with the solution being correct, an error was made in the calculations, or with the actions being performed correctly, the question was asked incorrectly).

A grade of 2 is given if the student made 2—4 mistakes in solving and explaining the problem, with no more than 2 gross mistakes.

A grade of 1 is given if more than three gross errors are made in the work .

Written work consisting of examples only is assessed as follows:

A grade of 5 is given if all examples are solved correctly; the most rational methods are used in the calculations; the records of the solution of the examples are done neatly and are arranged sequentially; the solution is checked in those cases where it is required.

A grade of 4 is given if the work contains 1—2 errors, but no more than one gross error.

A grade of 3 is given if the work contains 2—4 errors, but no more than two gross errors.

A grade of 2 is given if 3—6 errors were made when solving examples, but no more than three gross errors.

A grade of 1 is given if more than three gross errors are made in the work.

In written work consisting only of problems, a grade of 5 is given if the problem is solved correctly: the plan for solving the problem is drawn up correctly, the actions are chosen correctly, all questions for them are formulated accurately, the names are set correctly, all calculations are performed correctly using the most rational methods, the notes are made neatly and arranged sequentially.

If a student gives his own original, completely rational solution to a problem, a grade of 5 can be given even if there are one or two minor flaws in the work.

A score of 4 is given if the plan for solving the problem is drawn up correctly, all actions are chosen correctly, but 1—2 mistakes are made during the solution, of which no more than one is serious.

A grade of 3 is given if, while solving the problem correctly, 2—4 mistakes were made, but no more than two gross mistakes.

A score of 2 is given if the solution to the problem is incorrect.

A grade of 1 is given if the student does not start working or reduces the solution to a random combination of numbers.

In cases where the written work consists of examples and problems, one overall grade and two grades separately for both parts of the work can be given (in the case of a sharp difference in the quality of the performance of each part of the work).

In this case: a) if both parts of the work are completed equally (for example, both are excellent, good, etc.), this assessment should be common for the entire work as a whole.

If the assessment of both parts differs by one level, for example, the assessments are 5 and 4 or 4 and 3, etc., then the work as a whole is given the lower of the two assessments.

The lower of the two given grades is also given in the case where part of the work is given a grade of 5 and the other a grade of 3, but the teacher can also give such work a grade of 4 if the grade of 5 is given for the main part of the work. If one part of the work is given a grade of 5 or 4 and the other a grade of 2 or 1, then the teacher can give the grade of 3 for the entire work if the higher of the two given grades is given for the main part of the work.

The given assessment standards are approximate. When assessing a specific work, it is necessary to take into account its degree of complexity and the children's previous work.

APPROXIMATE ANNUAL PLAN OF WORK IN ARITHMETIC

The proposed sample plan for arithmetic, as well as for the Russian language, gives the teacher only an approximate distribution of time and arrangement of material. Therefore, when using the plan, the teacher will need to do a lot of additional creative work. The study of the program material must necessarily be accompanied by **a review of what has been covered.** Review is necessary both before moving on to studying a new section and when studying individual questions within a section, to connect the new with the old. Review should be the completion of work on each topic. In the sample plan, which indicates only the progressive movement in the study of the program, review is planned mainly at the beginning and at the end of the quarter and should be closely connected with tests.

When planning, it is also necessary to take into account that a solid acquisition of arithmetic knowledge is possible only as a result of the correct combination of solving examples with **solving problems.** Solving problems should accompany all the children's work on acquiring the theory and computational skills. Solving problems concretizes arithmetic knowledge, brings arithmetic closer to life. The approximate plan contains repeated indications of the types and kinds of problems, the place that problems should occupy in the general system of work on arithmetic. In the specific plan, the teacher's attention to problems should be

increased: they should be indicated not only in the main topics, but also in each individual question of the topic; each type of problem given in a certain period of time should then be repeated many times in connection with the study of other questions of the program.

Finally, for the successful acquisition of arithmetic, various **exercises are necessary**, including exercises related to the performance of tasks of a practical nature on local material. Carrying out the simplest measurement work on the ground, compiling and solving problems on "live" numbers taken from the surrounding life, carrying out some statistical work of public importance (to help, for example, a collective farm) — all this brings theory closer to practice, is the application of the knowledge obtained to the resolution of practical issues and thus contributes to the solid acquisition of arithmetic. In the approximate plan, these questions could be reflected only in individual topics (for example, in the planning of geometric material), but in the specific plan of the teacher they should be reflected more often and more systematically.

1ST CLASS

In the first quarter of the school year, the **first-**grade teacher faces the task of identifying the stock of numerical concepts with which children came to school, systematising, clarifying and filling in their gaps about the numbers of the first ten, familiarising them with addition and subtraction in the limit of 10, teaching them

to solve the easiest types of simple problems on these actions.

Experience in teaching seven-year-old children shows that in the first quarter it is impossible to manage to study all cases of addition and subtraction within the limit of 10, therefore the attached plan provides for the transfer of cases of addition and subtraction of 7, 8, 9 to the second quarter.

The educational material can be distributed approximately according to the following plan [1]:

Identifying children's numerical concepts—2 hours.

Study of numbers of the first five — 9 hours (approximately 2 hours for studying each number). Addition in the limit of 5 — 2 hours.

Study of numbers of the second heel—12 hours (approximately 2 hours for studying each number). Addition of 1 within the limit of 10—2 hours. Subtraction of 1 within the limit of 10—2 hours. Addition of 2 within the limit of 10—3 hours. Subtraction of 2 within the limit of 10—3 hours.

Test and subsequent analysis of its results — 2 hours.

Addition of 3 within 10—3 hours. Subtraction of 3 within 10—3 hours. Addition of 4 within 10-2 hours. Subtraction of 4 within 10—2 hours. Addition of 5 and subtraction of 5 within 10—3 hours. Addition and subtraction of 6 within 10—3 hours. Review of material covered—5 hours. Test and subsequent analysis of its results—2 hours.

In the 2nd quarter, the study of addition and subtraction in the limit of 10 is completed and numeration in the limit of 20 is studied, and then

[1] In the sample plan for this class, as well as for subsequent classes, the lessons allocated to each topic include time for solving not only examples, but also problems.

addition and subtraction without going through a decade. The study of each case of addition and subtraction in the limit of 20 should be preceded by appropriate preparatory exercises in addition and subtraction up to 10. In the "Second Ten" concentration, new types of simple problems on addition and subtraction are introduced (in particular, problems on increasing and decreasing by several units), as well as problems in two actions. Teaching actions alternates with the consideration of some measures (meter, kilogram). When studying measures, children should be trained in measuring with them. The work is carried out according to the following plan:

Addition and subtraction of 7 within 10—2 hours, Addition and subtraction of 8 within 10—2 hours, Addition and subtraction of 9 within 10—1 hour. Review of what has been covered—1 hour. Meter and measurement with it—2 hours. Test and subsequent analysis of its results—2 hours.

Oral and written numbering within the range of 20—4 hours.

Addition without going over a ten in the limit of 20—5 hours: adding a single-digit number to a two-digit number (2) [2] , adding a two-digit number to a single-digit number (2), repeating what has been covered (1).

Kilogram and weighing by kilogram—2 hours.

Increase by several units—4 hours.

Test: and subsequent analysis of its results—2 hours .

Subtraction of a single—digit number without passing through a dozen in the range of 20—5 hours: subtraction of a single-digit number from a full two-digit number (2), subtraction of a single-digit number from 20 (2), repetition of the past (1).

[2] The numbers in brackets indicate the number of teaching hours.

Reduction by several days—3 hours. Two—step tasks—3 hours. Two—action problems—3 hours. Subtraction of a two-digit number in the limit of 20—3 hours: subtraction of a two-digit number from a full two-digit number (2), subtraction of a two-digit number from 20 (1). Repeating what has been covered—3 hours. Test and subsequent analysis of its results—2 hours.

In the 3rd quarter, the main teaching material is actions in the limit of 20 (addition and subtraction with transition through ten, repetition of all cases of addition and subtraction, and then multiplication and division). The study of addition and subtraction with transition through ten should be preceded by exercises in decomposing the numbers of the first ten into addends (8 = 7 + 1; 8 = 5 + 2; 8 = 5 + 3; 8 = 4 + 4, etc.), since without this skill, successful study of the named actions is impossible.

Multiplication and division in the limit of 20 are studied separately: first all cases of multiplication, then all cases of division. According to the current program, in the first grade only division into equal parts is studied.

In parallel with the study of actions, problems in 1—2 actions are solved. The litre is introduced as a measure in this quarter.

The work begins with a review of what has been covered — 1 hour. Then the litre and its measurement are studied — 1 hour.

Addition with transition through ten in the limit of 20—6 hours: adding single-digit numbers to 9 (1), to 8 (1), to 7, 6 (1), to 5, 4 (1), to 3, 2 (1), repeating what has been covered (1).

Subtraction with transition through Tens in the limit of 20—6 hours: subtraction of single-digit numbers from 11 (1), from 12 (1), from 13, 14 (1), from 15, 16 (1), from 17, 18 (1), repetition of what has been covered (1).

Repetition of all cases of addition and subtraction within the limit of 20—3 hours.

Test and subsequent analysis of its results — hours. Multiplication 8 — 3 hours.

Multiplication 3—3 hours. Multiplication 4—3 hours. Multiplication 5—2 hours, Multiplication 6—2 hours. Multiplication 7, 8, 9, 10—2 hours.

Repetition of all cases of multiplication within the limit of 20—3 hours.

Test and subsequent analysis of its results—2 hours.

Division into 2—3 hours: division by 2 of the numbers 2, 4, 6, 8, 10 (1), division by 2 of the numbers 12, 14, 16, 18, 20 (1), repetition of what has been covered (1).

Divide by 3—3 hours. Divide by 4—3 hours. Divide by 5—2 hours. Division by 6—2 hours. Division into 7, 8, 9, 10—2 hours.

Repetition of all cases of multiplication and division within the limit of 20—3 hours.

Test and subsequent analysis of its results—2 hours.

In the 4th quarter, we study numbering in the limit of 100 and four operations on round tens in this limit.

Since in the first grade only division into parts is studied, then when studying division of round tens only the case of division by a single-digit number is considered.

In the fourth quarter, the centimetre is introduced as a measure of length, and the day and hour as a measure of time. Problems are solved mainly in two steps. At the end of the quarter, a review of what was covered during the year is conducted.

The work, just like in the 3rd quarter, begins with a review of what has been covered—3 hours . Then the following topics are covered:

Numbering within the limit of 100—5 hours: oral numbering (2), written numbering (2), consolidation of what has been covered (1).

Centimetre and measurement with it—2 hours.

Addition of round tens—3 hours: explanation of addition (1), reinforcement of addition and problem solving (2).

Subtraction of round tens—3 hours.

Multiplication of round tens—4 hours.

Dividing round tens by a single-digit number is 4 hours.

Test and subsequent analysis of its results—2 hours.

Time units: day, hour—2 hours.

Review of what was covered over the year—8 hours: review of addition and subtraction in the limit of 20 (2), review of multiplication and division in this limit (2), review of numbering in the limit of 100 and operations on round tens (1), solving examples and problems on all operations (3).

Test and subsequent analysis of its results—2 hours.

II CLASS

In the 1st quarter, first of all, it is necessary to review what was covered in **the 1st** grade. In this quarter, all cases of addition and subtraction in the limit of 100 are studied, after which they begin to study table multiplication and division, which in this class, unlike the first grade, are covered in parallel so that after each case of multiplication, the corresponding case of division is considered.

Of the simple problems in the first quarter, problems on division by content and on difference comparison are introduced. Of the compound problems, problems are solved mainly in two steps. The work is carried out according to the following plan:

Repetition of four actions in the limit of 20—6 hours: repetition of addition and subtraction (3), multiplication and division (3).

Repeating four actions on round tens within the limit of 100 is 3 hours.

First acquaintance with division by content—5 hours.

Test and subsequent analysis of its results—2 hours.

Addition in the limit of 100 without going through a ten—9 hours: addition of single-digit numbers with round tens (1), addition of single-digit numbers with full two-digit numbers (2), addition of full two-digit numbers with round tens (2), addition of two-digit numbers (4).

Subtraction within the limit of 100 without going over a ten—10 o'clock, a case of subtraction when the remainder or subtrahend is a round number (2), subtraction of single-digit numbers from full two-digit numbers (2), subtraction of round tens from a full two-digit number (2), subtraction of full two-digit numbers (4).

Difference comparison—5 hours: explanation and solution of simple problems (2), solution of compound problems (3).

Test and subsequent analysis of its results—2 hours .

Addition in the limit of 100 with transition through ten—6 hours: addition of single-digit numbers with double-digit numbers (2), addition of double-digit numbers (4).

Subtraction in the limit of 100 with transition through ten—6 hours: subtraction of single-digit numbers from two-digit numbers (2), subtraction of two-digit numbers (4).

Repetition of all cases of addition and subtraction in the limit of 100 — 2 hours.

Multiplication by 2 and division by 2—3 hours: multiplication by 2 and division into parts (1), division by content (1), comparison of both cases of division (1).

Multiplication 3—2 hours.

Test and subsequent analysis of its results—2 hours.

The main teaching material **in the 2nd** quarter is table multiplication and division. In this quarter, most cases of these actions are studied and problems on increasing and decreasing by several times and on multiple comparison are introduced. After examining each of these types of problems, a comparison is made with the corresponding type of problems on the difference change of numbers: an increase by several times is compared with an increase by several units, a decrease by several times — with a decrease by several units, multiple comparison — with a difference comparison.

In this quarter, the compound problems are solved in 2—3 steps, including problems solved by the method of reduction to one. The educational material is planned as follows:

Division into 3—4 hours; division into parts (1), division by content (2), comparison of both cases of division (1).

Multiplication 4—3 hours: explanation (1), consolidation and problem solving (2).

Division into 4—4 hours: division into parts (1), division by content (2), comparison of both cases of division (1).

Test and subsequent analysis of its results—2 hours.

Increase several times—4 hours: explanation and solution of simple problems (2), solution of complex problems (2).

Multiplication 5—3 hours. Division by 5—3 hours. Reduction by several times — 4 hours: explanation and solution of simple problems (2), solution of compound problems (2). Test and subsequent analysis of its results — 2 hours.

Multiplication 6—3 hours, Division by 6—4 hours. Multiple comparison — 5 hours: explanation and solution of simple problems (2), solution of compound problems (3). Review of the covered part of the multiplication

table — 2 hours. Test and subsequent analysis of its results — 2 hours.

In the 3rd quarter, the study of tabular multiplication and division is completed, after which tabular division with remainder is studied, and then non-tabular multiplication and division. When studying the last action, only the case is considered when it is performed without remainder (according to the current program, non-tabular division with remainder is studied in the 3rd grade). In this quarter, the most commonly used measures of time are studied. Problems are solved in 2—3 actions.

Review of the material covered—1 hour.

Multiplication 7—3 hours. Divide by 7—4 hours.

Multiplication 8—4 hours: explanation (1), consolidation and problem solving (3). Division by 8—4 hours. Test — 1 hour.

Multiplication 9—4 hours. Divided by 9—5 hours.

Division tables with remainder—6 hours. Review of all cases of multiplication and division tables—5 hours. Test and subsequent analysis of its results—2 hours.

Non-tabular multiplication by a single-digit and two-digit number—6 hours: Non-tabular multiplication by a single-digit number (4). Non-tabular multiplication by a two-digit number (2).

Non-tabular division by a single-digit number is 7 hours.

Divide by a two-digit number — 6 hours.

Test and subsequent analysis of its results—2 hours.

Tasks and examples for all actions within the limit of 100—8 hours.

Measures of time—5 hours, including a time table: year, month, day, hour, minute (2), determining time by the clock (1), solving problems on calculating time within a day (2).

Test and subsequent analysis of its results—2 hours.

In the 4th quarter, numeration and all operations on round hundreds and tens in the limit of 1000 are studied. The operations are performed on the basis of oral calculation techniques. Multiplication and division are performed only by single-digit numbers. At the end of the quarter, all the studied operations and main types of problems are repeated. The work is planned as follows:

Numbering within the range of 1000—4 hours.

Measures of length (kilometres) and measures of weight (grams)—2 hours.

All operations on round hundreds and tens: addition—4 hours, subtraction—4 hours. Test—1 hour.

Multiplication—5 hours, division—6 hours. Problems and examples on all operations with round tens and coitions—3 hours. Test with subsequent analysis—2 hours.

Review of material covered during the year—8 hours.

Annual test: testing problem solving skills—1 hour; testing computational skills—1 hour.

III CLASS

The main task of teaching arithmetic **in the first quarter** is to repeat everything covered in **the second** grade about hundreds and thousands, to give students the skills of **oral** multiplication and division by 10 and round tens, as well as the skills of **written** addition and subtraction of three-digit numbers, multiplication and division of three-digit numbers by a single-digit number. When repeating hundreds, much attention should be paid to oral methods of calculations within 1000 and repetition of the multiplication and division table. Along with developing computational skills, children practice solving problems. In addition to ordinary arithmetic problems, in the third grade, students learn to solve typical problems.

In the 1st quarter , according to the program, children need to be taught to solve problems on the simple triple rule, as well as problems on proportional division. Solving ordinary arithmetic problems is closely connected with teaching computational skills, so special lessons for them are not allocated in the plan; problems are solved in each lesson and, in addition, in the order of doing homework.

The plan provides 6 hours for solving typical problems. It is advisable to conduct these lessons in two stages: first, after studying addition and subtraction within the limit of 1000, devote approximately 3 hours to problems on the simple triple rule. Second, after studying division, devote 3 hours to solving problems on proportional division. These lessons provide an explanation of problems of this type and conduct exercises in solving them.

In accordance with the specified tasks of the 1st quarter, the work plan for this quarter can be presented as follows:

Repetition of the first hundred — 5 hours.

Repetition of numbering and all operations on round hundreds and tens in the limit of 1000—3 hours . Non-tabular division with remainder in the limit of 100—5 hours. Test with subsequent analysis—2 hours.

Written addition within the limit of 1000—4 hours. Written subtraction within the limit of 1000—5 hours. Solving problems by reducing to one—3 hours. Test with subsequent analysis—2 hours.

Written multiplication by a single-digit number in the limit of 1000—5 hours.

Oral multiplication by 10 and round tens—3 hours.

Written division of three-digit numbers by a single-digit number—5 hours.

Oral division of three-digit numbers by 10 and by round tens—4 hours.

Solving problems on proportional division — 3 hours.
Review of the material covered—3 hours.
Test with subsequent analysis—2 hours.

In the 2nd quarter, the study of multi-digit numbers begins. Here, the enumeration of multi-digit numbers (up to and including the class of millions), addition and subtraction of multi-digit numbers, and also multiplication of multi-digit numbers by a single-digit number are studied.

Work on computational skills is accompanied by problem solving, which is carried out in each lesson and in the order of completing homework.

Among the typical problems in this quarter, problems on finding an unknown by the difference between two numbers are solved .

The work plan for this quarter can be presented as follows:

Numeration of multi-digit numbers: a) study of numbers of the first two classes, or six-digit numbers, 5 hours: acquaintance with new counting units (1); composition and decomposition of numbers using class abacus (1); acquaintance with the terms “category” and “class”, places of digits (1); exercises in reading and writing six-digit numbers (2); 6) study of numbers of the first three classes, or nine-digit numbers — 2 hours; c) transformed numbers — fragmentation and transformation of digit units of a number — 2 hours.

Tables of measures of length (kilometres, meter, decimetre, centimetre, millimetre) and weight (ton, centner, kilogram, gram)—3 hours.

Test—1 hour.

Addition of multi-digit numbers with problem solving—6 hours: addition terminology and addition rule (1), addition exercises with verification (2), addition of several terms and problem solving (3).

Subtraction of multi-digit numbers with problem solving—8 hours, including: familiarising students with the terminology of subtraction and solving examples in which the digit units of the minuend are greater than the corresponding units of the subtrahend (1); a method for checking subtraction (1); solving examples in which some digits of the subtrahend are greater than the corresponding digits of the minuend (2), solving an example with zeros in the minuend (2), exercises in composing and solving various types of subtraction problems (2).

Multiplication of multi-digit numbers by a single-digit number—5 hours.

Test with subsequent analysis — 2 hours.

Solving problems on finding an unknown by the difference between two numbers—4 hours.

Review of the material covered—2 hours. Test with subsequent analysis—2 hours.

In the 3rd quarter, the main content of work on arithmetic is the study of multiplication and division of multi-digit numbers.

Simultaneously with the development of written multiplication and division skills, it is necessary to teach children to solve problems in which these actions are used, and to ensure that students clearly represent all the main cases of the application of these actions, especially division: division into equal parts and by content, finding a part of a number, reducing a number several times and multiple comparison.

Of exceptional importance is a solid and concrete knowledge of tables of measures and the ability to quickly and correctly perform division and transformation of named numbers; therefore, in the plan for these exercises, it is necessary to provide special lessons and, in addition, it is necessary for children to practice these

transformations in subsequent lessons in the order of oral counting.

From **the typical** problems in this quarter, students must solve problems on finding two numbers by their sum and multiple ratio, by their sum and difference.

The plan for the 3rd quarter can be drawn up as follows:

Multiplication of multi-digit numbers with problem solving (continued)—18 hours: multiplication by 10 (1), multiplication by round tens (2), multiplication by a two-digit number (4), multiplication by 100 (1), multiplication by round hundreds (1), multiplication by a three-digit number (3), by a number with zeros in the middle (2), multiplication of numbers with zeros at the end in both factors (3), splitting named numbers (1).

Test with subsequent analysis—2 hours.

1. Division of multi-digit numbers by a single-digit number, by one with zeros, and by round tens with problem solving—12 hours; mastering division terminology (1); general case of division and checking division (3); division with zeros in the quotient (2); division with remainder (1); exercises in students composing problems on division into equal parts and by content (1); division by a number expressed as one with zeros (2); division by round tens (2).

Solving typical problems on finding two numbers by their sum and multiple ratio—3 hours.

Test with analysis of results—2 hours.

2. Division into two-digit and three-digit numbers with problem solving—16 hours; division of a three-digit number by a two-digit number with a single-digit quotient (1); division of a three-digit number by a two-digit number with a two-digit quotient (1); division of multi-digit numbers by a two-digit number with a multi-digit quotient—general cases (2), division of multi-digit numbers by a two-digit number—special cases (2), division by round hundreds with a single-digit quotient

(1); division of a three-digit number by a three-digit number (1); division of a multi-digit number by a three-digit number—general case without remainder and with remainder (3); division of a multi-digit number by a three-digit number with zeros in the quotient—in the middle and at the end (3), transformation of named numbers (2).

Solving typical problems on finding two numbers by their sum and difference—4 hours.

Quarterly test—2 hours.

of the arithmetic work in **the 4th quarter** is to: a) complete the study of four operations on multi-digit numbers, 6) introduce children to two types of problems—problems involving oncoming movement and problems involving finding a part of a number, 7) repeat everything covered in the 3rd grade.

According to these tasks, the material of the 4th quarter can be planned as follows: the order of performing arithmetic operations — 4 hours. Solving examples and problems on all operations with multi-digit numbers — 6 hours, Test with subsequent analysis — 2 hours, Problems on oncoming traffic — 5 hours. Problems on finding several parts of a number (with familiarisation of students with fractions of a unit and their recording) — 5 hours. Measures of time — 2 hours. Test — 1 hour. Measurements on the ground — 3 hours. Review of what was covered during the year — 8 hours.

Annual test—2 hours: a) examination of computational skills—1 hour; 6) testing of problem-solving skills—1 hour.

IV CLASS

The main task **of the first quarter** is to repeat the numeration and four arithmetic operations covered in the

third grade and, together with the repetition, to impart to the students the knowledge of the relationship between the components of arithmetic operations. Work on each operation should begin with a repetition of what the children already know about the given operation and end with the assimilation of new material. The repetition of the numeration of multi-digit numbers should be completed by expanding the numerical domain—the study of fourth-grade numbers—the class of billions.

The repetition should include both the consolidation of computational skills and the solution of problems related to the application of each arithmetic operation in problems. Along with the solution of ordinary arithmetic problems, all types of problems covered in the 3rd grade should be repeated in the first quarter. Repetition is carried out throughout the 1st quarter; repetition lessons alternate with lessons on studying new material: Problems are introduced that can be solved by the method of relations and by the method of eliminating the unknown.

The plan for **the 1st quarter** can be drawn up approximately as follows:

Numbering of multi-digit numbers—5 hours: repetition of oral and written numbering covered in the 3rd grade (2); mastering new material related to the study of the class of billions (3).

Addition of multi-digit numbers—3 hours: review of the terminology of the action, rules for performing written addition and techniques for oral addition, solving examples and problems (2); the relationship between the terms and the sum (1).

Subtraction of multi-digit numbers—4 hours: repetition of the terminology of the action, written subtraction and oral subtraction techniques, solving examples and problems (2), the relationship between the minuend, subtrahend and difference (2).

Multiplication of multi-digit numbers—5 hours: review of the terminology of the action, solving examples and problems (3); the relationship between factors and the product (2).

Division of multi-digit numbers—6 hours. Repetition of the terminology of the action, solving examples and problems on division (4); the relationship between the dividend, the doer and the quotient (2).

Solving examples and problems for all actions—4 hours.

Test and subsequent analysis of its results—2 hours.

The system of metric measures and operations on metric measures—22 hours: tables of measures, division and transformation of named numbers (2); addition of composite named numbers (3); subtraction of composite named numbers (4); multiplication of composite named numbers (5); test work and subsequent analysis of its results (2).

Repetition of various types of problems covered in the 3rd grade (problems on the simple rule of three, on proportional division, on finding two numbers by the sum and difference, by the sum and multiple ratio, by the difference of two quantities, problems on motion, on finding a part of a number) — 11 hours. Solving problems by the method of ratios and the method of eliminating the unknown — 6 hours.

Quarterly test and subsequent analysis of its results—2 hours: solution of problems (1) and solution of examples (1) are checked.

The content of the arithmetic work in **the 2nd quarter** consists of the study of geometric material, which is divided into two parts: a) calculating areas and b) calculating volumes.

Calculation of areas is preceded by familiarisation of students with figures — a square and a rectangle. Measuring volumes is preceded by familiarisation of students with two geometric bodies — a cube and a

rectangular parallelepiped. In working on these sections, a large place should be occupied by practical exercises in direct measurement of segments, the length and width of rectangles, the length, width and height of objects in the shape of a cube and a rectangular parallelepiped, calculating the volumes of the specified objects, exercises in developing the eye.

In this quarter, children learn to solve problems involving the movement of two bodies in one direction. The plan for studying the above material is as follows:

Division of composite named numbers—6 hours.

Solving problems on all operations with composite named numbers—3 hours.

Square measures and area calculation —14 hours, including: introduction to a rectangle—properties of its sides and angles, drawing rectangles with bent sides (2); introduction to a square—properties of its sides and angles, comparison of a rectangle and a square (1): concept of area, introduction to units of area measurement: square centimetre, squares, decimetre, square meter (1); measurement of area by superimposition and by dividing the area into squares; derivation of the rule for calculating areas (2); measurement and calculation of the area of a classroom, corridor, light area of windows, area of a table, area of rectangular figures cut out of cardboard or plywood (1); going out into an open area to measure the area of a school yard, school plot; introduction to measures of land areas—are and hectare (1); area of a square; table of square measures (1); solving problems on area calculation (3); test and subsequent analysis of its results (2).

Cubic measures and volume calculations (14 hours): introduction to a cube and a rectangular parallelepiped (2); the concept of volume; specific introduction to units of volume measurement—cubic centimetre, cubic decimetre, and cubic meter (1): measuring the volume

(of a box, crate) by filling it with cubic decimetres; concepts—"cube," "beam," "layer"; making a bar out of cubes, a layer out of bars, a parallelepiped out of layers; deriving the rule for calculating the volume of a rectangular parallelepiped (2); exercises in conceiving and calculating the volumes of models shaped like rectangular parallelepipeds, the volume of a classroom, corridor, etc. (2); volume of a cube; table of cubic measures (2); solving problems on volume calculations (3); test and subsequent analysis of its results (2).

Solving complex problems of the same types that were solved in the 3rd grade—7 hours.

Solving problems on the motion of two bodies in one direction.—3 hours.

Test with analysis of its results — 2 hours.

The main content of the work **in the 3rd quarter** is learning operations with time measures and studying simple fractions.

Typical problems solved in this quarter include: calculating time, finding a number based on a given part (one), problems on the complex triple rule, and calculating the arithmetic mean.

In addition, here we are given the concept of percentage and problems are solved to find several percent of a number expressed in round hundreds.

All this material can be studied in the following way;

Operations on compound named numbers with time measures—23 hours; of which—table of time measures (1), division of named numbers (2), transformation (2), addition of named numbers (3), subtraction (3), multiplication (3), division (5), problems on all operations (2), test (2).

Solving problems on calculating time—6 hours: determining the duration of an event given its beginning and end (2), determining the end of an event given its beginning and duration (2), determining the beginning of an event given its end and duration (2).

Solving problems on the complex rule of three — 4 hours.

Solving problems on calculating the arithmetic mean—4 hours.

Test with subsequent analysis — 2 hours.

Learning about fractions—20 hours: Introduction to ½ (1), ¼ and ⅛ (1), introduction to ⅕ and 1/10 (1), numerator and denominator of fractions, reading and writing fractions (2), mixed number and its conversion to an improper fraction, excluding a whole from an improper fraction (3), addition and subtraction of fractions with the same name and multiples, solving problems (5), problems on finding a number given one of its parts (3), repetition of solving problems on finding a part of a number (2), test (2),

Introduction to percentages—6 hours: introduction to 1/100 (1), concept of percentages using specific examples (1), solution of problems on calculating one or several percentages of a given number (4).

Review of material covered in a quarter to 2 hours.

Quarterly test—2 hours.

In the 4th quarter, the change in the results of actions in connection with the change in data is studied as new material. In addition, in this quarter, everything covered in arithmetic is repeated in connection with the preparation of students for exams.

The study of the material is carried out according to the following plan.

Study of changes in the results of arithmetic operations in connection with changes in data—16 hours: change in the sum depending on the changed terms (3), change in the difference (4), change in the product (3), change in the quotient (4), test (2).

Review of the material covered—28 hours.

The specific decision on the question of how much time (lessons) to allocate for reviewing a particular section, a particular topic, depends on the state of

knowledge of the students in a given class, on what was covered in a given class during the year more thoroughly, and what less thoroughly.

The approximate number of lessons indicated can be distributed as follows:

Numbering and actions are abstract numbers — 7 hours: numbering of multi-digit numbers (1); addition and subtraction — oral and written (1); multiplication — oral and written (1); division; order of operations and brackets (1); solving compound arithmetic problems (2); test (1).

Tables of measures and operations with composite named numbers — 6 hours: tables of measures — length, weight, time; fragmentation and transformation of named numbers with metric measures and time measures (1); four arithmetic operations with composite named numbers — with metric measures and time measures (2); solving arithmetic problems with composite named numbers: solving problems on calculating time (2); test (1).

Fractions—2 hours.

Geometric material—3 hours.

Solving typical problems—8 hours (distributed evenly throughout the quarter).

Annual test: a) testing computational skills—1 hour; 6) testing problem solving skills—1 hour.

VISUAL AIDS ON ARITHMETIC

When studying arithmetic, it is necessary to widely use **visuality and concreteness** in teaching.

Let us trace how, for example, a student comes to the conclusion that 5 + 3 = 8.

At first he is convinced of this on concrete objects, which are counted, moved, felt, and are before his eyes. The student performs this operation repeatedly, counting and adding: 3 sticks to 5 sticks, 3 pebbles to 5 pebbles, 3 fingers to 6 fingers, adding 3 houses to 5 houses, etc.

Then he carries out the same operation on objects mentally, by imagination. He solves problems, for example, of this kind: “Vanya found 5 mushrooms under one bush, 3 mushrooms under another. How many mushrooms did Vanya find in total?” “Manya picked 5 carrots from one bed, and 3 carrots from another. How many carrots did Manya pick in total?” etc.

The results obtained as a result of counting and adding up specific quantities according to the idea are immediately checked on some objects that replace mushrooms, carrots, or are sketched on the board or in a notebook. As a result of such systematic preliminary work, the student, gradually freeing himself from the material basis (sticks, mushrooms, pebbles), identifies what was common to all these phenomena, and comes to the conclusion that 5 of anything plus 3 of the same thing will be 8, or 5 + 3 = 8.

If there is no preliminary stage in which the main role is played by the participation of the senses,

direct experience, children will not be able to consciously and firmly assimilate the results of the operation discussed above, which in this case boiled down to finding the sum of two numbers.

Simply memorising the result, not based on experience, would contradict the process of the student's cognitive activity, would make the work uninteresting; instead of developing, it would dull the student's abilities and would lead, at best, only to the development of mechanical memory.

Visual aids are useful when introducing children to any new concept, any new computational operation. When the student has accumulated experience and comprehended it, you can abandon the auxiliary means and move on to conclusions, generalisations, abstract concepts, to performing calculations, computations without using visual aids.

Visual aids are especially important in the first years of study. A teacher, showing initiative and a little ingenuity, can enliven the study of any section of the program with interesting and useful didactic material, which will also help to activate the students' work. Children will be able to make most of the necessary aids themselves, following the teacher's instructions.

It is necessary to pay attention to the production of aids not only for display by the teacher, but also those that should be available to each student (the so-called didactic material).

Thus, when studying the measurement of volumes, it is not enough for the teacher to have cubes, but it is extremely important for the children to have them too, so that each student has the opportunity to come to a conclusion on the basis of

their own experience about how to measure the volume of bodies.

It is not enough to have a ready-made meter in the class, but it is important that each student makes it.

In addition to visual aids, it is necessary to have in the classroom material that summarizes the class's work on a particular section of the program, for example: on multiplication tables, operations on multi-digit numbers, named numbers, etc.

These final materials in the form of posters should be periodically hung in the classroom, changing them during the work process.

All aids must be large in size so that they are clearly visible to all students without exception.

Particular attention should be paid to the blackboard. In the work of a school, a good blackboard of sufficient size, good chalk and a clean rag are of great importance.

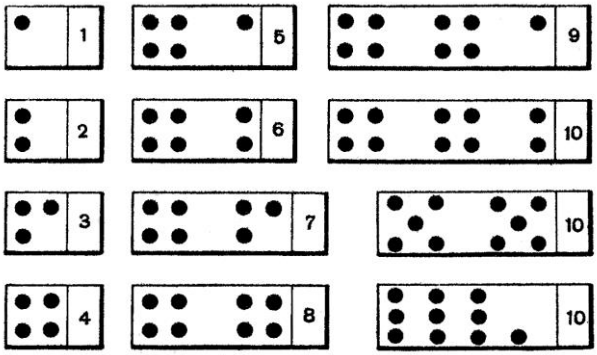
Clear and distinct writing, the ability to save the solution of the problem as a whole on the board are extremely important for the success of the work. Therefore, it is necessary to provide the school with a good, sufficiently sized blackboard and good chalk.

The skilful use of visual aids and didactic material enlivens the work, increases the activity and interest of students. However, it should be noted that visual aids are ultimately only a means, a necessary step for the development of abstract thinking, so a certain measure must be observed in their use.

Below is a brief description of visual aids and teaching materials for the first four primary grades.

1ST CLASS

By the time they enter school, many children can already count to 20, to 100, and sometimes even more. But this does not mean that children do this counting consciously. Most often, they count mechanically, and as for arithmetic operations, in the overwhelming majority of cases they do not master them at all. And sometimes there are cases when children, even after a series of exercises, cannot consciously and correctly indicate the result of a particular action within the limit of 10 or 20, regardless of whether they are given an abstract numerical example (5 + 4; 10 — 8; 8 + 3), or some real-life problem within the same numbers. Of particular importance for future success are thoughtful, unhurried lessons with children in the first days of their stay at school, in particular complete mastery of the first ten. What does this mean? This means that the student must associate a certain idea of a particular set of objects with the concept of each number (1, 2, 3, etc.), clearly imagine the place of each number in the natural series, the composition of any number within the first ten. Both of these ensure the error-free finding of the result of addition and subtraction within this limit, and this ensures success in the future when moving on to the second ten, etc.



Along with counting objects in the surrounding environment (desks, windows, pencils, notebooks, houses, trees, etc.), children count the sticks, cubes, bars, etc. they have. Numerical figures can also serve as a final moment in the formation of the concept of number.

They can be of various shapes.

In the first lesson, students select "1" from a variety of objects. This numerical figure is specified by a picture on the board or in notebooks of a house, a horse, a tree, etc. Then, as a conclusion, a card is shown with one circle and the number 1 next to it: "This is how we will write the number 1. This is how 1 is written." The children draw a circle in a notebook and write the number 1 next to it (see the picture on p. 5 21).

Numerical figures on objects. Numerical figures are a convenient way to train children in decomposing numbers into addends and in completing them to a given number.

In addition to numerical figures, it is useful to provide images of some objects that are close and familiar to students,

Drawings of various objects can be made first under the guidance of a teacher in class, then independently at home.

Drawings of vegetables, birds, domestic animals, etc. Along with using objects from the surrounding environment for counting, as well as those specially collected by children (pebbles, buttons, acorns, etc.), models of various objects, which can be used mainly for solving problems, will bring great animation and concretisation.

These models should be beautiful and bright, with colours that are close to natural objects. It is very convenient to store them in envelopes. They are very portable, and their delivery to the Class is not associated with any difficulties.

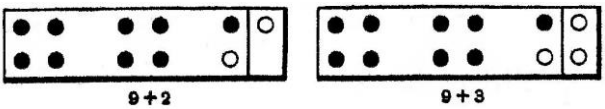
Sticks and bundles. Sticks and bundles are one of the extremely valuable aids in teaching students to count, especially to count to 20 and to 100. Within the first ten, they are used to the same extent as other visual aids: pebbles, cubes, acorns, etc.

When studying numbering in the limit of 20, and then arithmetic operations, especially addition and subtraction, sticks and bundles are an indispensable, basic aid.

By tying 10 sticks into a bundle, the student first gets an idea of a new unit of counting — ten, and by tying together 10 bundles of tens, the student gets an idea of a hundred. The most developed children soon stop needing them, and the less prepared can use them, especially when working independently, until they accumulate sufficient ideas to move on to abstract counting.

The production of these manuals is available to every student.

It is useful to give children measurements: "Make sticks 10 cm long." This will be the first exercise for children in measuring, in using a centimetre ruler.

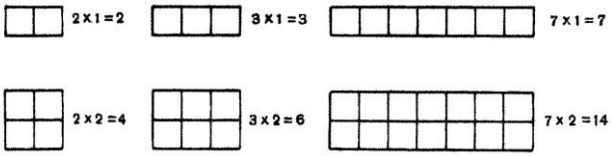


Numerical figures for tabular addition in the limit of 20. One of the valuable aids available to every school are numerical figures for studying addition and subtraction with the transition through ten (9+2 = 11; 9+3 = 12).

Multiplication table in squares. Counting sticks, pebbles, acorns, models of vegetables, domestic animals in pairs, threes, fours, fives, etc., children first depict the results of the counting in squares, and then in numbers.

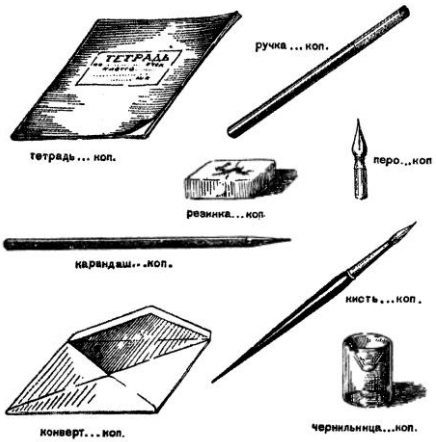
2 X 1 = 2 3 X 1 = 3 7 X 1 = 7
2 X 2 = 4 3 X 2 = 6 7 X 2 = 14.

Drawing squares by students brings clarity and concreteness to the material being studied. Moreover, when children study multiplication by 2, they draw rectangular strips of 2 squares in their notebooks, and the teacher demonstrates this on a poster in front of the whole class.



When learning multiplication by 3, students draw rectangles with 3 squares, etc.

As a result of studying the multiplication table, material will gradually accumulate that relates to all cases of multiplication in the limit of 20.



As a result, all these tables can be combined in one place on a separate sheet of paper or on two adjacent pages of a notebook.

A poster for solving problems. In the first grade, a lot of time is spent on learning the conditions of the problem, memorising it, and writing it down. Using a poster with images of various objects, where their prices are also indicated, makes solving problems much easier (see p. 523).

Arithmetic box. This manual is a must have in every school. It is a cubic box filled with cubes. There are 1000 of them.

Some of the cubes are left on the bars (10 cubes) or whole planks (100 cubes or 10 bars).

The blocks and cubes are used by the teacher mainly for demonstration in front of the whole class (the arithmetic box is used in all primary classes),

Wooden metre. Centimetre tape. Each school should have a wooden meter with divisions into decimetres and centimetres, as well as a centimetre tape. In addition, each student makes a meter from wood, newsprint, cardboard.

Children should be trained to measure the length, width and height of the classroom or room they live in.

Weights. The school should have weights: store-bought weights of 5, 2, 1 and ½ *kg* and homemade ones. For this, children collect small pebbles (or coarse sand), wash them and sew them into pre-prepared bags by weight: 1, ½ and ¼ *kg*. It is necessary to ensure that students more or less correctly distinguish the weight of objects of ½, 1, 2 and 5 *kg*.

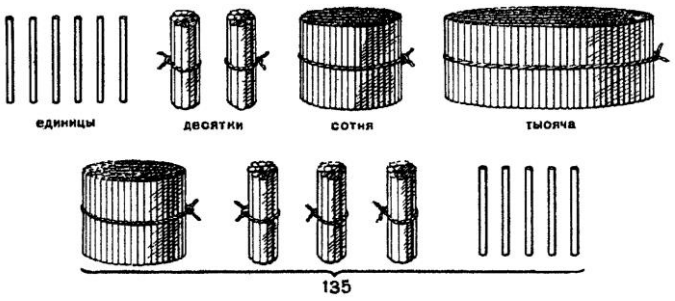
Capacity measures . Models of litre, ½ litre, ¼ litre must be purchased, it is also necessary to have a litre in the form of a cubic decimetre and in the form of a mug.

Model of a clock. The ability to tell time by a clock is of great practical importance. The school should have a wall clock, in addition, a model of the dial, written in both Arabic and Roman numerals. The dial should be equipped with hands. First, the teacher explains the readings of the hour and minute hands, puts the hands in different positions, and the students say the time.

Then individual students do the same thing in turn, and the rest determine the time, or the student sets the hands to one time or another based on the teacher's instructions.

II CLASS

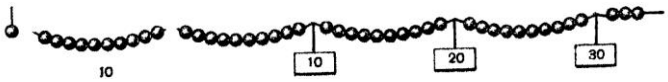
Sticks and bunches up to 1000. Sticks and bunches are very valuable didactic material when studying numbering within 1000.



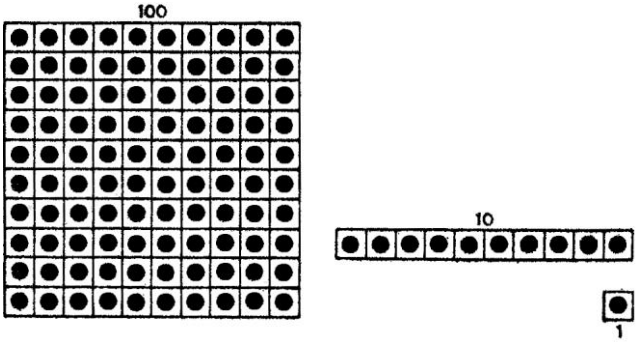
In front of the class, the following are illustrated and compared: one (1 stick), one ten (1 bunch of 10 sticks), one hundred (1 bunch of ten tens), one thousand (1 bunch of ten hundreds).

It is found out how many hundreds there are in a thousand, how many tens there are in a thousand, how many units there are in a thousand. After this, students move on to composing numbers within the limit of one thousand, consisting of several hundreds, tens and units.

Beads. A good addition to sticks when learning numbers up to 1000 are beads. They can be bought in a store, or children can make them themselves from clay or play dough.



Circles drawn in squares to match 1, 10, 100 and 1000. This worksheet is also a good complement to the sticks, bundles and beads when learning numbering to 1000.



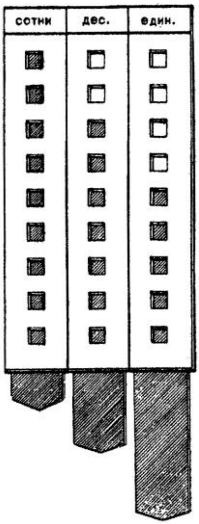
The ability to immediately view on a plane and compare 1, 10, 100 and 1000 will once again bring clarity to the understanding of these quantities.

Hand abacus. The hand abacus is a great success with children. Each student makes one for himself and uses it when learning numbers up to 1000.

The whole class is given the task of drawing a number, for example, 24.

Students move the ribbons and get the desired designation.

Empty digits make it necessary to write zero when writing numbering. An abacus is made from colored cardboard measuring 20 *cm* x 8 *cm* ; three strips of white paper with nine are glued to it from above.
holes, and these strips are glued only at the edges, so that there is space for the free movement of strips of white stiff paper, revealing the required number of units of any category as they move forward.

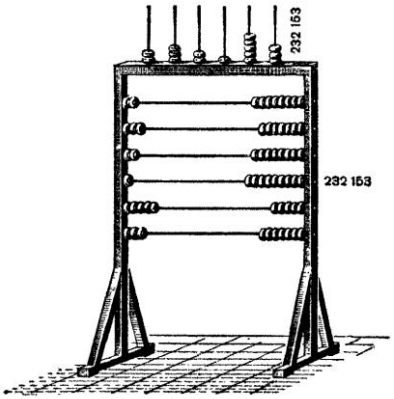


Pythagorean Table. Students use the Pythagorean multiplication table with great interest. It is useful to suggest that students write it down in a notebook in a column, just like a regular table written in a column.

The Pythagorean table can also be used for mental arithmetic.

III CLASS

Class abacus. Class abacus is familiar to children from working with it in the 1st and 2nd grades. But in the first grades, abacus can be successfully replaced by some other aid. The number up to a thousand can be specified by counting a certain number of objects: sticks, beads, cubes, etc. Numbers up to a thousand can be directly seen and felt. When moving on to the next, higher grades, counting objects becomes very difficult. Therefore, in the 3rd and 4th grades, where students move on to studying large numbers, it is necessary to use abacus as a visual aid, on which the local value of the numbers can be shown.



Particularly useful when familiarising yourself with numbering are those abacuses which, in addition to horizontal wires, have a number of vertical ones, fixed to the upper crossbar of the frame.

With the help of these wires, it is possible to give a visual representation of the decimal grouping of numbers in the same order as decimal groups are written, i.e., on the right extreme place are the units, then the tens, etc.

Children should be accustomed to using abacus, therefore, in addition to class abacus, it is necessary to have ordinary trading abacus for practice; it is desirable that each student have an abacus.

When studying numeration, **a numeration table is widely used** . The top row of the table gives the name of the classes, the second row gives the order of the digits, and below it is noted that each class consists of hundreds, tens, and units.

Before moving on to reading and writing multi-digit numbers, students should practice reading numbers using a numbering table.

When studying measures, the corresponding **tables are also used:**

Меры длины	Меры веса	Меры времени
10 *мм* = 1 *см*	1 000 *г* = 1 *кг*	60 сек. = 1 мин.
10 *см* = 1 *дм*	100 *кг* = 1 *ц*	60 мин. = 1 часу
10 *дм* = 1 *м*	1 000 *кг* = 1 *т*	24 часа = 1 сут.
1 000 *м* = 1 *км*	10 *ц* = 1 *т*	Месяц = 30 или 31 дням (кроме февраля)
		Год = 365 или 366 дням

Such tables help to systematize the studied measures and to consolidate them. The teacher, with the participation of the whole class, makes a corresponding table on the board, the children write it

down in their class notebooks. In addition, the teacher hangs a table of measures in the classroom in the form of a beautiful, clear poster.

The poster can be made by the teacher or one of the children.

IV CLASS

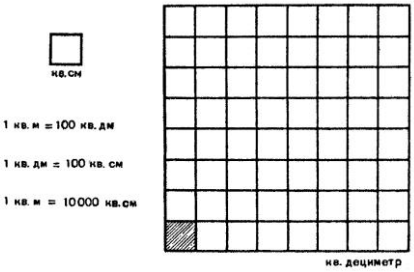
Types of angles. Students in grade 4 should gain an understanding of right, acute and obtuse angles.

During lessons, students use sticks to form different angles by moving them apart and together; draw different angles in their notebooks and on the board; cut out models of angles from paper and cardboard. As a result, the material must be designed.

The corners are drawn in a certain order, and the corresponding inscriptions are made under them (by the teacher on the board, by the students in their notebooks).

It is necessary to train students in drawing angles (on the board and in notebooks) in different positions. As a result, the material can be summarised in the form of a poster.

Square measures. To make the concepts of square measures (square meter, square decimetre and square centimetre) more concrete, it is necessary to depict them on paper or cardboard in natural size and hang them in the classroom.



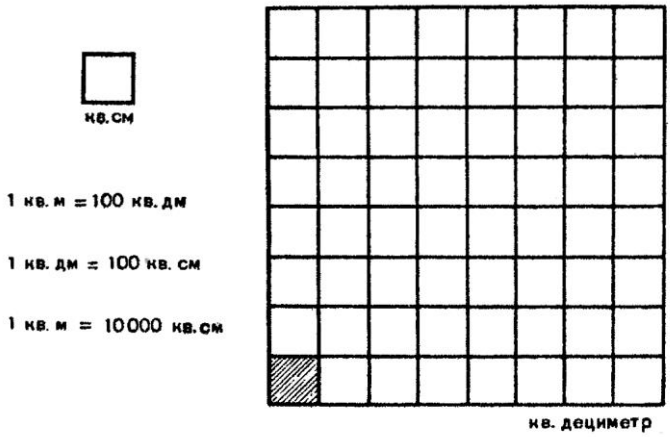
Set of squares and rectangles. In order to provide children with the opportunity to measure and find the areas of rectangular figures, it is necessary to have a set of rectangles and squares of various sizes in the classroom. It is desirable to have from four to eight series, and they should be numbered, and each number should correspond to one specific size. For example:

Rectangle No. 1 has the following dimensions: 6 *cm* and 4 *cm* not.

No. 2 » » 7 *cm* from *cm* etc.

Square No. 1 has a side length of 4 *cm*.

No. 2 » » » 3 *cm* etc.



This makes it easier to check students' work, since the teacher only needs to ask the number of the rectangle or square to know whether the student has found the correct result.

The square decimetre must be divided into square centimetres.

A square meter must be divided into square decimetres, and one of them must be divided into square centimetres.

The children themselves can easily complete this manual. However, they should only be warned that they must be especially careful when taking measurements and correctly marking right angles.

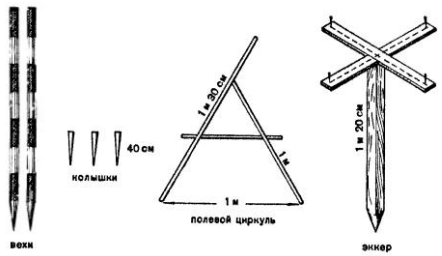
When working, students use a right triangle, a measuring ruler, or a centimetre ruler.

Are and hectare must be measured somewhere on a flat place near the school, and in the classroom they should be drawn on a scale of 1 *cm* = 1 *m* .

The dimensions of both the rectangle and the square must be expressed in whole centimetres.

Each student has a ruler with divisions in centimetres, made of wood or graph paper.

In class, students independently measure the length and width of the figures given to them and calculate the area.



Surveying tools: pegs, stakes, measuring rope, field compass, ecker. At first, students measure on the ground with an ordinary meter or measuring rope. In practice, the measuring rope is often replaced by a field compass of 1 or 8 *m* .

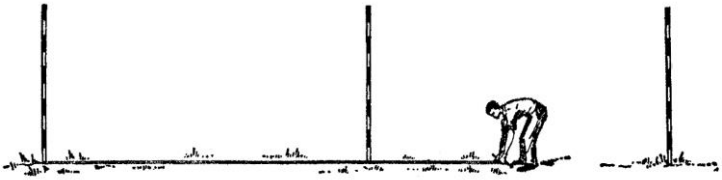
Определение длины своего шага

Ecker is used by students in autumn or spring when measuring on the ground. First, the children mark the Ar (area) on the ground without an ecker by eye, making sure that sometimes a rather big mistake is obtained. After such a primitive measurement of macaw, they try to measure it with the help of an ecker, which makes it possible to correctly build right angles. In addition to measuring macaws, children should be able to measure a hectare, rectangle or square of a given size on the ground.

Samples of cubic measures. Students of the 4th grade study cubic measures and volume measurements. In order to achieve a clear idea of cubic units, children should see them in front of them, hold them in their hands. Samples of cubic measures can be made of wood, cardboard. Difficulties arise when making a cubic meter. It can be knocked together from bars, then covered with cardboard or plywood.

A set of cubes and bars. In addition to samples of wooden cubic meters, it is necessary to make a cubic

decimetre from cardboard and a cubic centimetre from clay or plasticise.



Измерение мерной верёвкой

Measuring with a measuring rope

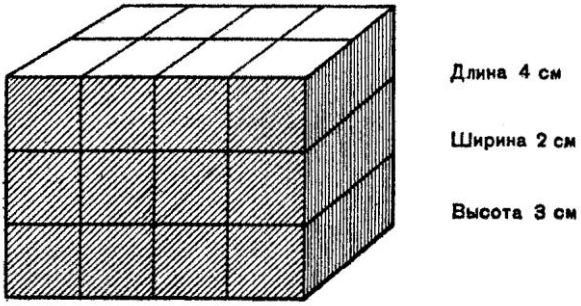
In order to give students the opportunity to practice measuring the surface and volume of bodies, it is necessary to have a set of cubes and rectangular parallelepipeds of various sizes in school. It is best to have several standard sizes with separate numbers. For example, parallelepipeds No. 1 have the following sizes: 8, 6 and 4 *cm* ; No. 2, 5, 3 and 4 *cm* , etc.

It is also necessary to have cubes with edges of 3, 4 or 5 *cm* .

The student receives a parallelepiped (or cube). Using a centimetre ruler or tape, he measures its length, width, and height and finds its surface and volume.

The student only needs to state the number of his parallelepiped so that the teacher can immediately determine whether he has made the calculation correctly. The measurements of the bodies must be expressed in whole centimetres.

A cubic decimetre made of wood and composed of cubic centimetres. This manual must be purchased ready-made (decimetre arithmetic box).



It is useful in studying the relationship between individual cubic units, and, in addition, the cubes can be successfully used to make bars from them, as well as for direct measurement (by stacking) of the volume of various boxes, pencil cases, etc.

All this concretizes the idea of measuring volumes and makes work in this direction more conscious.

Poster "Measuring the volume of a parallelepiped". The poster is made by students as a result of their conscious assimilation of volume measurement. It is necessary to pay attention to the entry when calculating the volume. It is necessary that it corresponds to the reasoning that children carry out when calculating the volume.

Length 4 *cm* ; width 2 *cm* ; height 3 *cm*.

4 cu. *cm* X 2 = 8 cu. *cm*

8 cu. *cm* X 3 = 24 cu. *cm*

4 cu. *cm* X 2 X 3 = 24 cu. *cm* (*see* page 529).

Let's say a student measures the volume of a room. To do this, he measures its length, width and height and, having received, for example, 8, 5 and 4 *m* , he reasons as follows:

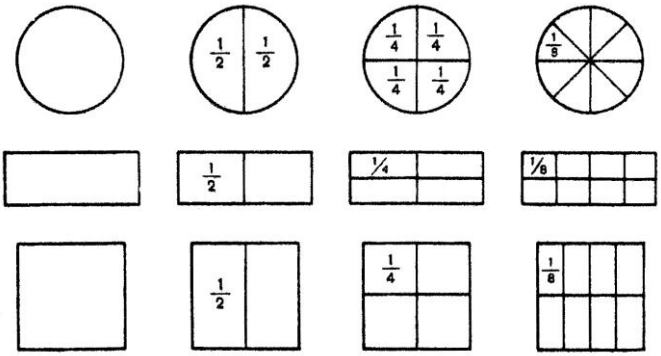
"One row along the wall will fit 8 cubic *meters*. There will be 5 such rows on the floor. One layer or floor will thus fit 8 cubic *meters* X 5 = — 40 cubic *meters*. There will be 4 such layers. Therefore, the total will be 40 cubic *meters* X 4 = 160 cubic *meters* ."

Table of cubic measures.

1 cu. *m* = 1000 cubic meters *dm*
1 cu. *m* = 1000000 cubic meters *cm*
1 cu. *dm* == 1000 cubic *cm* .

A table of cubic measures is compiled after students have derived the relationship between cubic units.

Circle, rectangle, square, divided into 2, 4, 8 parts. To familiarize students with fractions: ½, ¼ and 1/8—you should use a circle, rectangle, square, which students fold in half, then in half again, and in half again.



A square divided into 100 parts. This aid is necessary when studying percentages. Each square is one hundredth of a large square, or 1% of it.

A whole strip will represent 1/10 of a large square, or 10%. The cardboard strips that go with this square make it possible to illustrate fifths and tenths, as well as to specify the given percentage.

This manual is made by children under the guidance of a teacher.