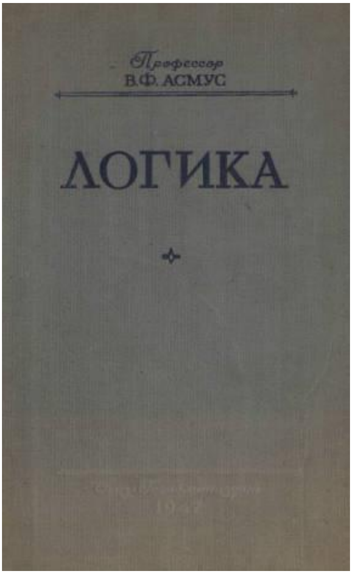
ACADEMY OF SCIENCES OF THE USSR
AND INSTITUTE OF PHILOSOPHY

V. F. ASMUS

LOGIC



OGIZ

State Political Publishing
Moscow 1947

АКАДЕМИЯ НАУК СССР
ИНСТИТУТ ФИЛОСОФИИ

Профессор
В. Ф. АСМУС

ЛОГИКА

Огиз
ГОСУДАРСТВЕННОЕ ИЗДАТЕЛЬСТВО
ПОЛИТИЧЕСКОЙ ЛИТЕРАТУРЫ
1947

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Publishing House: State Publishing House OGIZ (Gospolitizdat)

Moscow, 1947

Annotation:
The proposed book is a systematic presentation of the teachings of logic. It can be used by students of higher educational institutions, graduate students of research institutes and persons embarking on an independent study of logic. Teachers of logic in high school will find in it a detailed coverage of issues included in the program of their subject, but only summarized in the textbooks of logic for high school.

E-Book prepared by
The Socialist Truth in Cyprus-London Bureaux
http://www.st-cyprus.co.uk

&
Direct Democracy (Communist Party)
www.directdemocracy4u.uk

English translation from Russian:: Google Translate.

E-Book: June 2020

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FOREWORD

IN MEMORY of
my wife
Irina Sergeevna
DEDICATING
V. ASMUS

The proposed book presents a systematic presentation of the teachings of logic. It can be used by students of higher educational institutions, graduate students of research institutes and persons embarking on an independent study of logic. Teachers of logic in high school will find in it a detailed coverage of issues included in the program of their subject, but only summarized in the textbooks of logic for high school.

The book deals only with *formal* logic. No questions have been posed regarding the relation of formal logic to dialectics, since consideration of these questions is advisable not in terms of formal logic, but only in terms of dialectic logic.

The teachings of formal logic presented in the book are developed on the basis of the *materialistic* understanding of thinking and scientific knowledge. The laws and forms of thinking, the study of which is the subject of logic, are considered in the book as a reflection of the properties and relations of things in the material world that exist outside of consciousness and regardless of consciousness. The author sought to show that the forms of thinking studied by logic are not fiction of scholastics, but that they represent a generalized abstract expression of the forms and rules of thought applied by all sciences: the sciences of nature and the sciences of society.

Staying *formal* during the 19th and 20th centuries, logic strove to refine and generalize its own teachings. The author used in his work the most valuable results of this refinement and generalization. In particular, in the theory of conclusions, the author relied on in—depth studies of the largest Russian representative of formal logic in the late XIX and early XX centuries — M. I. Karinsky. In the theory of evidence, the author relied on the classification of evidence proposed by Professor S. I. Povarnin, a prominent Russian representative of the *logic of relations*. This classification is more complete than the classification accepted in the usual logic of classes, and is more developed from a logical point of view.

Considering the logic of relations as the development and generalization of the logic of Aristotle's classes, the author expanded the analysis of judgment by introducing into it, in addition to the usual for attributive logic and class logic, the S – P judgment scheme expressing whether an attribute belongs to an object or a class of objects — another class of objects, also a judgment scheme *a* R *b* , expressing all kinds of relations, including other types of relations besides membership relations. In preparation for publication, the manuscript of the proposed work was reviewed by specialists. The author is deeply grateful to all those who conveyed their criticisms, thoughts, and wishes to him.

Professor *V. Asmus*

Moscow, January 5, 1947

CHAPTER I. THE SUBJECT AND TASK OF LOGIC. LOGIC AS THE SCIENCE OF RIGHT THINKING

§ 1. In practical and theoretical activity, a person faces tasks that can be solved only if the thinking involved in this decision is *correct* thinking, that is, capable of leading us to the attainment of truth.

For thinking to be correct, it must satisfy three main requirements: 1) *certainty*, 2) *consistency*, and 3) *evidence*.

Certain thinking is precise thinking, free from any inconsistency. *Consistent* thinking is thinking that is free from internal contradictions that destroy the connection between thoughts where this connection is necessary. *Evidence* thinking is thinking, not only formulating the truth, but together and indicating the grounds on which it must be recognized as truth.

§ 2. Logic teaches how to define concepts, to find out their content, how to divide the scope of a concept, to classify, how to reason, that is, from truths that have already been clarified or recognized, to derive other truths that need to be connected with the former, and etc.

However, although logic formulates a number of laws and rules, the implementation of which is necessary for our thinking to be correct, logic can formulate these rules only because it first establishes *theoretical* the truths on which all these rules are based. Everything that can be learned from logic about the practical rules of thinking stems from the fact that logic finds out about thinking as a *theoretical* science. Not because there is a science of logic, that there are known rules of thinking, but rather: the rules of thinking only matter because, regardless of the existence of the science of logic, there are forms of thinking that are constantly used by us and make up the subject of logic. The meaning of these rules is that wherever thinking is correct, that is, correctly reflecting the order of things and phenomena in thought, certain forms are used in thinking, known relationships and sequences of thoughts are realized, which are the subject of the study of logic.

§ 3. There were logics who thought that logic represented nothing else except that it was a *technical* science of thinking. There were also logicians who thought that logic was not dealing with what was , but with what *should be*, with *duty*. The latter argued that logic is not a science *of being*, but *of the due*, *about the norms of* our thinking. The direction in the development of logic, whose representatives see in it the science of duty, or the norms of thinking, is called *normativism.*

The normative point of view is wrong. And the technical rules and the rules of proper thinking formulated by logic can exist only because logic exists as a *theoretical* science .

As grammar reveals the existing laws of speech, language, and does not create them, so logic reveals in which forms the correct thinking is carried out.

Indeed, the above three requirements or conditions that proper thinking must satisfy are the requirements of *certainty*, *consistency*, and *evidence.*— have power over thinking not by themselves, not as rules of due. These requirements get the meaning of norms or laws of thinking only because independently of these requirements and before they were first formulated by logic, there are three features in thinking itself, which justify three *theoretical* positions regarding logical thinking. These provisions can be formulated as follows:

1. Only *certain* thinking is correct thinking, that is, logical.

2. Only *consistent* thinking is logical thinking.

3. Only *evidence—based* thinking is logical thinking.

These three provisions on logical thinking are really *theoretical.*

They substantiate three rules regarding the proper in thinking.

§ 4. Since only *certain* thinking is logical thinking, it follows that all thinking, in order to be logical, must satisfy certainty conditions.

Since only *sequential* thinking is logical thinking, it follows that all thinking, in order to be logical, must be consistent,

since, finally, only *evidence—based* thinking is logical thinking, it follows that all thinking, to be logical, must to be evidence.

It can be seen from the foregoing that the definition of logic as a science of technology or of the art of correct thinking is true, but not enough. This definition speaks only about the final *practical* task of logic, but does not say anything about logic as a *theoretical* science.

What is the subject of logic as a theoretical science? — To answer this question it is necessary to find out what is a *logical form.*

The Concept of Logical Form

§ 5 . To each utterance and to each row of interconnected utterances, in addition to the special *content, there is* also a certain *form of* the utterance itself. Consider three statements: "Suvorov was brave," "the day was rainy," and "the battle was cruel." In all these statements, we are talking about *various* objects: in the first—about Suvorov, in the second — about the day and in the third—about the battle. In all these three statements, we are talking about the *various* properties of the objects themselves: the courage of Suvorov, the rainy nature of the day and the cruelty of the battle. Considering the idea of the objects of these statements, as well as the idea of their properties as *components* the content of these statements, we can say that in all three statements the components of the content of the statements will be *different.*

But although all these statements spoke of different things, they also have a *common* feature in them all. In each of them, thought reveals *belonging to an* object of a known property. In each of them we are talking about a different subject and a different property. But in every utterance this property is considered as *belonging to the* subject. The property of courage also belonged to Suvorov, just as the property of rainyness belonged to the day and how the property of cruelty belonged to battle.

What is common in all these statements is expressed in them through the word “was.”

The word “was” in this case, obviously, does not express the idea of the component parts of the content of statements. This word shows that in all three utterances there is *one and the same way of communication of* conceivable parts of the content.

The method of communication of the components of conceivable content is called the *logical form*—in contrast to the content itself.

In the first three statements, the logical form was the same, and the components of the content were different.

Let us now take a sentence and begin to successively replace in it each of the components of its content with another. Consider, for example, the sentence: “Glinka wrote music.” Replace the thought of Glinka with the thought of Scriabin. We’ll get a new sentence: “Scriabin wrote music”, already with another component of the content. In this second sentence, we replace the thought of music with the thought of piano sonatas. We get a new sentence: “Scriabin wrote piano sonatas.” Let us now compare all three sentences and see what happened in them as a result of a twofold replacement of the components of the content. These parts have all changed. In the sentences “Glinka wrote music” and “Scriabin wrote piano sonatas”, the components of the content imaginable in them are already quite different.

What remains of these general statements? Remained common *logical form of* utterance, i.e., an conceivable *way of connecting* its components. The same logical form in all three statements is expressed by the word “wrote”, which is repeated in each statement and which shows that the way of connecting the various components of the content has remained the same.

From all these examples we see that the logical form is not an integral part of the conceivable content, but only a way by which the constituent parts of the content are connected in thought among themselves.

The logical form in this sense of the word is actually the subject of the study of logic as a theoretical science. Logic is a theoretical science of the correct forms of thinking.

§ 6. Why is logic a *special* science? Why is such a order of things impossible that the forms of thinking used by each *individual* science would be studied by *this very* science? Is a *special philosophical science*— logic, necessary for this purpose ?

In the concepts of various sciences and in the relations between these concepts, the properties of things themselves and the relations between things existing in reality are reflected. In the concepts and teachings of logic some kind of reality must also be cognized. But what is this reality? Knowing what things and what relationships between things can be logic?

It is quite obvious that logic cannot set itself as its *immediate* tasks the task of knowing the very things that are studied by individual sciences.

The immediate task of logic is the study of forms of thinking that reflects and cognizes reality. The direct subject of study for logic are the forms and laws of correct thinking. For logic, they are the same direct subject of study, which for each science are the subjects it studies.

§ 7. The study of forms of thinking is not only *possible*. Research is *absolutely necessary.* Without this research, our thinking remains unaccountable. Even if it turns out to be correct at the same time, it will lack the distinctness and consciousness that alone can give thought to impeccable accuracy and impeccable consistency and persuasiveness.

No matter how correct our understanding of the components of the content, this understanding alone is not enough to comprehend the utterance. We can understand all the individual words of the sentence, but not understand the meaning of the sentence. This happens, for example, when the proposal is too long or too complicated. In this case, we understand the components of the content, but do not catch the *logical form of the* statement.

That the logical form of thinking is *special* the subject of research is particularly clear when considering the so—called *conclusions*, or *conclusions.*

Compare the following two conclusions:

First conclusion	All ancient poems are written with a hexameter.	Second conclusion	All conical sections intersect the line at no more than two points.
	Homer’s poems are ancient poems.		Ellipses are conical sections.

	Next, Homer's poems are written with a hexameter.		Next, ellipses intersect a line at no more than two points.

In each of these conclusions, the two previous judgments logically substantiate the third proposition as a conclusion from the first two.

The components of the content in both conclusions are completely different. The first conclusion relates to the field of poetics, the second to the field of mathematics. But the *logical* form, i.e., the *way of connecting the* components of the content, is the same in both conclusions. The general form of inference, applied both in the first and in the second case, can be expressed as follows: "If a thing has a certain property and if everything endowed with this property also has some other property, then the thing in question also has this other property."

But precisely because logical forms of thinking are common to thinking in the most diverse fields of knowledge, these forms should not be studied by individual sciences, but should be studied by a special science — *logic*. And since logic studies logical forms of thinking in thinking, this science is called *formal logic*.

There is *only one* science of formal logic — one for all sciences. No matter how different the sciences are from one to the other in their content, the thinking through which these sciences solve each of their special problems is always subject to the rules of logic. The logic of thinking for all sciences is equally required.

§ **8.** The main task of logic as a science is to study the forms of thinking and to elucidate the rules and laws that thinking observes in its application of these forms. Logic studies various forms of concepts, judgments, inferences, and proofs. She clarifies the rules that thinking follows when defining concepts and when classifying, when contrasting judgments, when deciding on their compatibility or incompatibility. Logic explores and classifies various types of inferences, ascertains the structure of the correct inferences, investigates the conditions of conclusions about probability, ascertains the rules of generalization; studies the structure of evidence, classifies various types of evidence, etc. Logic further investigates the premises and structure of the methods of scientific thinking used in science: research methods and systematization methods.

These studies clearly demonstrate what has been said above, namely, that the same logical forms and the same logical actions, or operations, are found in a wide variety of sciences, covering a wide variety of contents.

Idealistic logicians draw the wrong conclusion from this fact. Noting — and quite rightly — that the same logical forms, for example, forms of inference or proof, can cover a wide variety of material belonging to different areas of reality and different areas of knowledge, these logicians conclude from this that the forms of thinking studied by logic are *completely do not depend* on the content of what is thought with the help of these forms.

So there was a direction in the development of logic, which, unlike *formal* logic can be called *formalistic.*

However, *formal* logic and formalism, or *formalism* in logic, are by no means the same thing. Formal logic is the science of the right forms of thinking. While studying formal logic, we at the same time know that the forms of thinking, no matter how common they are for all sciences, no matter how widely they are used to cover the most diverse content, are nevertheless connected with the content and depend on the content. What is reflected in the logical forms of thought is the content of reality itself: its objects, properties and relationships.

The possibility of applying the same logical forms, for example, the same forms of judgment or inference, classification or proof, to various materials of different sciences proves not at all what the formalists of logical science claim: not that the forms of logic do not depend on the content imaginable in them. The ability to apply the same logical forms to different contents proves only that along with the *particular* content *that is* characteristic *only of a given* field of knowledge or a given science, there is also a content common to *a number of* sciences or even to *all* sciences. From this point of view, general logical forms should not be considered as forms that *do not depend* on any content, but as forms *extremely broad* content.

The study of logical forms is as little like formalism as little as formalism is the study of forms, for example, musical or poetic art. He who studies the sonata form in musical art is not yet a formalist of art history. He who explores the form of tragedies or an epic poem, is also not yet a formalist of literary criticism. The formalist will be only the musicologist and only the literary critic who, studying the form — which is a completely honourable and necessary task, studies it in error, as if the form does not depend on the content.

CHAPTER II. THE LOGICAL LAWS OF THINKING. LAWS AS THE LAWS OF DEFINITE , CONSISTENT AND EVIDENCE—BASED THINKING

§ 1. Whatever the tasks of thinking and whatever forms it uses to solve these problems, correct thinking is a definite, consistent, and evidence—based thinking. These three features of correct thinking are not properties inherent in thinking as such. The forms of thinking of a modern cultural person were formed as a result of constant interaction between a person and the material world, on which a person acts with the help of tools and which, on the other hand, continuously influences a person and his thinking. In the forms of thinking, the whole vast experience of the material practice of social man has been put off. These very forms have arisen and developed in their modern form in accordance with the properties of the material world,

Therefore, the logical features of certainty, consistency and evidence are not features that, thinking, has generated from itself and which have no basis in the properties of reality itself. Correct thinking possesses these traits only because they represent or reflect some fundamental properties of reality itself.

§ 2. Everything that exists outside our thoughts and that can be the subject of thinking has the property of *certainty*. Each object — whatever its properties—is, first of all, a *certain* object and, as such, differs from all other objects, without exception, that are conceivable.

Although no object actually exists on its own, in isolation, out of touch with other objects of reality, nevertheless, even belonging to some whole, the object enters into this whole as a *definite* object. So, it *differs* from all other objects, possesses, in addition to properties *common* to it with other objects, also properties that belong *only to it* . Even if the subject exactly as follows, what are the other things of the same species, it differs from them, if only for the number, in order, in a place in space, and so on. D.

Being *certain* in its properties, an object requires that our thinking about it be a specific thinking. This means that our thinking about an object can be correct only if, while thinking about an object, we think *about it*, that is , we distinguish this object with all its properties from all other objects that only may be conceivable by us.

§ 3. Another property is closely related to the certainty property that belongs to each object. Since each object is *precisely this particular* object and in this sense differs from all others, it cannot be that the properties that currently *belong to* it, as distinguishing it from all other objects, *do not* at the same moment belonged to him. If what distinguishes a given object as a certain object from all others at the same time does not belong to it, then the object would not be what it is, would not be a specific object.

But if such is the property of any particular subject, then our thinking about the subject can be correct only if the thinking is *consistent*. This means that, recognizing certain properties that characterize a given object as defined, that is, distinguishing it from all others, thinking cannot at the same time deny that these properties belong to the object.

§ 4. Finally, with the same property of certainty in each subject is connected — another fundamental property. Any property of an object that distinguishes this object as defined from all other objects does not exist in this object on its own, but only because there is something such that this property is determined and without which it could not exist. No object and no property exists without what determines their existence. If an *object* exists , then conditions must exist that make its appearance necessary. If there are known *properties* in an object , then conditions must exist, by virtue of which precisely these, and not other, properties are present in the object.

This dependence of the subject on the conditions without which neither the subject nor its qualities could exist, determines our thinking about the subject. Since neither the object itself, nor its properties can exist without what determines their existence, our thinking about the subject can not think about the subject of any statement that would not be based on anything or would not be sufficiently substantiated. Any correct statement is due to the correctness of those statements on which it is based as its foundation. This feature of thinking, corresponding to the conditioning of each existing fact by other facts, is called *evidence of* thinking.

§ 5. Since the features of certainty, consistency, and evidence are inherently inherent in all right thinking, they have the power of *laws* over thinking . Everywhere where our thinking turns out to be correct, in all its actions and operations it obeys certain laws, the implementation of which tells him the character of thinking of a certain, consistent and evidence—based. There are *four* laws of these *:* 1) the law of *identity,* 2) the law of *contradiction;* 3) the law of the *excluded* third; and 4) the law of *sufficient reason.* Moreover, the law of identity characterizes the *certainty of* thinking, the law of contradiction and the law of the excluded third — its *consistency, the* law of sufficient reason — its *evidence.*

§ 6. The *laws of* thinking should not be mixed with the *norms of* thinking. A rule is a rule or regulation that always assumes the presence of some legislator or a person who dictates this rule. The norm always speaks of what is set for granted, and therefore always presupposes as its condition the prescription of the one who sets this norm.

The laws of thinking are not norms in the sense of this concept indicated here. These laws do not express any precepts. These are genuine laws. They are inherent in *all* actions of right thinking and are present *everywhere,* where thinking is right. These laws have power over thinking, even regardless of whether thinking itself knows anything about them and what is prescribed by them. The force of the laws of thinking, which is obligatory for correct thinking, is caused not by the fact that these laws are the *norms of* thinking, but by the fact that the lines of thinking that affect the operation of these laws express and reflect the properties of reality itself: the certainty of each object, its difference from the others and its conditionality by others subjects.

But precisely because there are *laws that* act in thought even when it does not give itself a clear account either in their character or in their action, these laws can be formulated each not only as a *law,* but also as complying with the law and the stipulated order or *norm.* However, it should be remembered that the source of these regulations, or norms, is *by no means thinking in itself.* The logical laws of thinking are not prescriptions of thinking itself. All prescriptions and requirements that can be deduced from them are themselves “dictated” by the properties of the material world, according to which all forms of thinking have developed.

The laws of thinking are requirements or prescriptions only in the sense that without observing these laws, thinking cannot be right. But in this sense, the requirements expressed by the laws of thinking are absolutely immutable. No thinking should disturb them if it wants to be right thinking. And in *this* precisely defined sense, we have the right to talk about what logical laws require of our thinking.

Law of Identity

§ 7. Wherever our thinking is right, the logical law of thinking, called the *law of identity* , operates . According to this law, the *necessary logical connection between thoughts is possible only if every time a thought about an object appears in a reasoning or conclusion, we will think of this very object in the same content of its attributes.*

So, from two statements — "all ruminants are artiodactyls and all deer are ruminants", the conclusion "all deers are artiodactyls" follows.

But this conclusion is obtained only if, in the course of the whole argument, both for the first time, when we think about "ruminants" ("all *ruminants* are artiodactyls") and the second time ("all deer are *ruminants")*— under the word "ruminant" we will mean exactly the same thing in the same content of its signs. In this argument, we must also think of "artiodactyls" and "deer".

In fact, if, saying or thinking "all ruminants are artiodactyls," we would mean one class of animals under the "ruminants", with one trait, and if we say "all deer are ruminants", this time we would understand the ruminants different class of animals, with other characteristics, then we obviously could not conclude that "all deer are artiodactyls." Only on condition that the "ruminants" included in the number of "artiodactyls" are the same as those "ruminants" to which the "deer" belong, only under this condition we, recognizing both these statements as true, can deduce of them the third — that "deer are artiodactyls."

The law of identity does not mean at all that, thinking about an object, we *always* , *always,* for *any* conditions must think in it the same signs. Since the subject has, generally speaking, an innumerable set of signs, it is quite possible and legitimate that in various cases, depending on which side of the subject in question, we will think of the same subject once in one, in another times — on other grounds. On the other hand, the development and deepening of our knowledge of the subject necessarily leads to the fact that the concept of the subject includes more and more new signs. Finally, due to the constant changes occurring in the subject, the signs conceivable in the concept of the subject are also constantly changing. The law of identity does not prohibit us from thinking in different cases the same subject on its various grounds. The law of identity requires that we think the same subject on the same grounds only if when it is necessary to understand the logical connection between the concept of an object included in the conclusion, with the concepts of other objects also included in this conclusion. In other words, *the law of identity is one of the necessary conditions for the possibility of a correct conclusion.* But in this meaning, the law of identity is the immutable law of *all* thinking. And vice versa: for thinking that does not obey the law of identity, no logically valid conclusion is possible, no transition from substantiating provisions to the provisions that are derived from them. According to the law of identity, an object that we think of should not be replaced by another object, and in this sense it should be thought of as being identical with itself in all those thinking processes in which it is thought. In other words, in all those actions of thinking, where we are talking about a certain subject, this subject should be thought of *as this* an object, no matter how many times it appears in thought, and no matter how the thought of this object is associated with other thoughts about itself or about other objects.

§ 8. The law of identity does not say what exactly the subject of our thought is. This subject can be any: existing or imagined, relatively stable or variable. But whatever it may be, the law of identity requires that: a) while discussing a known subject, we are discussing precisely about him, and not about another subject, only mistakenly accepted as the first, and that, b) including the idea of the subject in the composition conclusion, we thought this subject on the same grounds.

§ 9. The law of identity applies to *every* subject of thought, no matter what we think. Therefore, the law of identity can be expressed in *general* formula — like those formulas that are used in algebra. The formula of the law of identity: And there is A.

This formula means that if we think of a certain object, then we think and should think of this very thing. At the same time, we should think it the way it is: if it is relatively stable, then as relatively stable, if it is changeable, then how changeable, etc. What exactly will be And what we think about is the formula of the law identity does not say anything: Or it can be any object and any property of the object.

§ 10. The law of identity has the widest application in the practice of thinking. Therefore, with all reflection and speech, one should be careful not to violate the law of identity in our reasoning or in our speech.

Often the mistake of logical thinking consists in the fact that the thinker violates the law of identity in his reasoning. So, when discussing a subject or question, the one who thinks in the course of his reasoning, without noticing it, often replaces this subject with another, believing, however, that this is the same subject. As a result, neither the reasoner nor his listeners receive an answer to the question posed.

Law of Contradiction

§ 11. Sometimes contradictions penetrate into our thoughts, which arise due to the inability to abide by the thought in those provisions, those statements that seem to be recognized by the reasoning person himself, but from which he deliberately or involuntarily departs, falling into contradiction with himself by myself.

On the contrary, correct thinking is always *consistent.* This means that, recognizing the known provisions as true and developing conclusions from these provisions, provided that our thinking is correct, we cannot admit in our reasoning or proof any statements that contradict what we have already recognized.

The logical law of thinking, by virtue of which correct thinking does not contain contradictions, is called the law of contradiction. According to this law, *two statements cannot be immediately true, of which one claims something about the subject, and the other denies the same thing about the same subject and at the same time.*

For example, two such statements cannot immediately be true: “Nikolaev can play chess” and “Nikolaev cannot play chess”. These statements contradict each other. Therefore, according to the law of contradiction, two such statements cannot be both true right away.

§ 12. Moreover, the law of contradiction prohibits at the same time only statements that are true in which: 1) we are talking about the same thing; 2) statements relate to the same time; 3) affirmation and denial consider the subject in the same respect.

And really. If the statement refers to one Nikolaev, and the negation refers to the other Nikolaev, then there must not be a contradiction between the statement and the negation: it is possible that the first Nikolaev can play chess, and the second cannot.

There will be no contradiction even if the statement and the negation relate to one and the same subject, but at the same time the assertion refers to one time, and the negation to another. If the statement "Nikolaev cannot play chess" refers to the *past,* and the statement "Nikolaev knows how to play chess" — to the *present,* there will be no contradiction between the two statements, although both relate to the same subject.

Finally, there will be no contradiction in the case when the statement and the negation relate to the same subject at the same time, but the statement considers the subject *in one respect* and the negation *in another.* If, saying "Nikolaev knows how to play chess", only the knowledge of moves is meant by diminution, and in the second case, by the same words they mean the skill of an experienced and skilled player, who knows the theory of openings, is skilled in defence and attack, then between approval and denial it is not necessary there will be a contradiction: it is possible that Nikolaev is able to play chess in the first sense of the word, but is not able to play in the sense that is meant in the second case.

Given the possibility of such cases, logic formulates the law of contradiction in such a way that it is completely clear which particular contradictions are unacceptable in correct thinking. Logic explains that incompatible statements refer *to the same subject, while at the same time they* consider the subject in *the same respect.*

§ 13. Like the law of identity, the law of contradiction is expressed by the general formula. This formula for the law of contradiction will be: "the judgments" A is B "and" A is not B "cannot be true at the same time."

The meaning of this formula is as follows: if we find out that a certain object A, among its properties, has some property

B, then it cannot be argued that the same object A at the same time and in the same respect does not have this property B.

§ 14. Any violation of the law of contradiction leads to the fact that discrepancies arise between our statements, the necessary logical connection is violated.

Moreover, the prohibition of contradictory statements expressed by the law of contradiction applies to everyday thinking and scientific thinking. Logical inconsistency should not be tolerated in any reasoning, speech or scripture. The more important a scientific theory is for life, the more aspects of life and the interests of society it covers, the more important that there are no logical contradictions in this theory.

§ 15. The law of contradiction, in the sense explained above, is valid with respect to all statements opposing each other, regardless of the type of opposite.

The opposition between judgments can be either contradictory or counter. *Conflicting* the opposite will be: a) if one of the opposing statements is general, and the other is particular, and b) if both opposing statements are singular. For example, the statements "all planets have an atmosphere" and "some planets have no atmosphere" are in a relationship of a *contradictory* opposite: they are opposite to each other, that is, one of them claims about one class of objects that about the same class of objects at the same time denies the other, but one of them is general ("all planets have an atmosphere"), while the other is a particular *"some* planets have no atmosphere "). Another example of the contradictory opposite: "this star is Sirius" and "this star is not Sirius". Here both opposing statements are singular, that is, they relate to one single subject.

§ 16. *The* opposite will be the opposite if the opposite statements are both *common*. For example, the statements "all spiders are insects” and “not a single spider is an insect” are in relation to the counter—opposite: both affirmation and denial are *general* statements here .

§ 17. Whatever the opposition between statements is, the law of contradiction retains its force both for the contradictory and for the counter—opposition. According to this law, neither such two statements as “all planets have an atmosphere”, “some planets have an atmosphere”, nor such two statements as “all planets have an atmosphere”, “not a single planet has an atmosphere” can be true at once. , nor, finally, such as “this star is Sirius,” “this star is not Sirius.”

Law of Excluded Third

§ 18. We have established that, according to the law of contradiction, two opposing statements cannot be both true at once. But can not opposing statements be both immediately false? *Three* cases must be distinguished here . 1) If the opposite is counterattack , that is, both statements are general, then they can both be immediately false. Consider two statements: “all planets have an atmosphere” and “not a single planet has an atmosphere”. The opposition between them is a *counteraction,* since affirmation and negation here are *general* statements .

In this example, both statements are false. It is also false that “all planets have an atmosphere”, and it is false that “not a single planet has an atmosphere”. The truth here is the third, namely, that part of the planets (for example, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune) has an atmosphere, while the other part (for example, Mercury) does not have it.

Why, in the case of a *counter* —opposition, can both opposing statements, like in our example, turn out to be both false right away?

This happens because the counter—contrast is the *most extreme* of all possible. If one claims that all the planets have an atmosphere, and the other that no planet has an atmosphere, then one cannot imagine between these two statements the opposite of what they express.

However, two counter—statements may turn out to be both immediately false. They will both be immediately false, if between the extreme cases, which are expressed by both counter—statements, there are cases that form the transition between them, standing in the middle. Between the extreme statements "all planets have an atmosphere" and "not a single planet has an atmosphere", the third statement is possible: "some planets have an atmosphere, and some do not."

Since two counter—statements *can* both turn out to be false right away, it does not follow that they *in all* cases, they will *always* and *certainly* turn out to be false. There are also possible cases where one of the counter—statements is false and the other is true. So, from two counter—statements — "all the planets of the solar system revolve around the sun" and "not a single planet of the solar system revolves around the sun" — the first is true, and the second is false.

Contrary statements are not both immediately false in cases where the opposite expressed by general statements can only be extreme, that is, when between the two extreme cases expressed in both statements there really are no transitional *cases.*

§ 19. 2) If the opposite between two statements is *contradictory*, that is, one of the utterances is general, and the other is particular, then such two utterances cannot turn out to be both immediately false. In this case, the *third* law of logical thinking comes into effect — the *law of the excluded third.*

According to this law, *of two conflicting statements about the relationship of two concepts, one statement* — and *only one — must be true, so no third true statement about the relationship between these concepts is possible.*

So, from conflicting statements about the relationship between the concepts of “dolphins” and “mammals,” namely, “all dolphins are mammals,” “some dolphins are not mammals,” one must be true. It is either true that “all dolphins are mammals,” or it is true that “some (that is, at least some) dolphins are not mammals.”

Since, according to the law of contradiction, two conflicting statements cannot be both true at once, the truth of one of such statements means the falsity of the other and vice versa. But this is not enough. The law of the excluded third does not only say that one of the conflicting statements must necessarily be true. The law of the excluded third says, moreover, that truth lies only within the framework of these two statements. Apart from these two statements, no third is possible about the relationship between the same concepts that would be true. In the case of conflicting judgments, one has to reason according to the scheme: “either — or. The third is not given “(tertium non datur).

The law of the excluded third is so called because this law excludes the truth of any third utterance, except for our two — affirmation and negation, between which we must make a choice.

The law of the excluded third justifies the requirement, which can be expressed as follows: choose one of two statements contradicting each other, since one of them must certainly be true and since there is no third that could turn out to be true instead of the two.

§ 20. The law of the excluded third, as well as the law of contradiction, does not say *which one* of two conflicting statements will be false and which is true. The solution to this last question requires in each case a special study. The law of the excluded third only indicates that the correct answer to the question posed — provided that the question itself is formulated accurately — lies in *one* of two contradictory statements, but does not answer the question itself. This law implies the need to choose one of two contradictory opposites, but the law of the excluded third does not in itself indicate which one. This question in each special case requires special consideration.

§ 21. The law of the excluded third is certainly applicable to *any* two *conflicting* statements. Regarding such statements, it always remains valid that one of them must be true. But this law has no force in relation to the *counter*— opposition. It remains possible here that the truth does not lie in either of two opposing statements, but lies in some third statement.

§ 22 . 3) If the opposing statements both refer to only one single subject, then such an opposite is different from the counter and the contradictory. While in the case of a *counter*— contradiction the possibility is not excluded that both counter—statements will turn out to be false at the same time, in the case of opposing statements *about one single subject* such statements cannot be both false at the same time. "In other words, the law of the excluded third applies to these statements in the same way as it does not contradict statements. So, two statements — "this star is Sirius" and "this star is not Sirius" cannot both be false at the same time: one of them must certainly be true.

So, the law of the excluded third extends to all *conflicting* statements, including opposing statements about *one single* subject. On the contrary, in relation to *counter*— statements, the law has no binding force.

§ 23. Since the law of the excluded third is true for all conflicting statements, it can, like the law of identity and the law of contradiction, be expressed by the general formula. The formula of the law of the excluded third is: *A is either B or not B.*

The meaning of this formula is as follows. Whatever the subject of our thought (A), this subject either has a known property (B) or does not possess it. It is impossible that it was false that Item A has property B, or that Item A does not have this property. Truth is indispensable in one of two contradictory statements. No third statement about the relationship of A to B and to non—B can be true.

Law of sufficient reason

§ 24. The *fourth* logical law of thinking is the law of *sufficient reason.* This law expresses the quality of logical thinking, which is called *evidence* . According to this law, *in order to recognize a statement about an object as true, a sufficient basis must be indicated.* On the contrary, any statement in which the statement is made without specifying a sufficient reason by virtue of which the alleged is approved will be not satisfying the law of sufficient reason.

Evidence will be such reasoning or such thinking, in which not only the truth of a certain position is affirmed, but at the same time the reasons are indicated, by virtue of which we cannot but recognize this situation as true. So, the mathematician does not just claim that the sum of the angles inside a triangle of Euclidean geometry is equal to two right angles, but *proves* this statement, that is, shows that, having adopted the system of definitions and postulates underlying the Euclidean geometry, we cannot but agree with the theorem on the equality of the sum of the angles inside a plane triangle to two straight lines. So, the astronomer does not just invite us to believe that the earth has a shape close to the shape of a ball, but proves this position through a series of observations and arguments: for example, by observing the shape of the earth's shadow approaching the moon's disk during lunar eclipses, or by observing a gradual sinking under the horizon, first the lower, and then the middle and upper parts of the ship retreating into the open sea.

Evidence is a very important condition for proper logical thinking. The vast majority of truths that make up the content of science, the essence of truth, substantiated through evidence. Even such truths that seem obvious, "taken for granted", mathematics always seeks to prove, as far as possible, that is, to bring us to an immutable consciousness of their necessity and truth, to connect these truths with a logical connection with the truths already proved by it or simply accepted as starting points (axioms, postulates). So, for example, a geometer does not just say that every circle is divided by a diameter into two equal parts: the geometer *proves* this is his statement. It would seem, what is there to prove? It is enough to look at the circle drawn on the blackboard with a straight line passing through its centre to verify the obvious truth of this theorem. But the geometer does not trust this evidence, since he knows that evidence sometimes deceives us. If we stand on the railroad track and look into the distance, we will see that as the distance from us to the horizon rails seem to converge at one point. This is obvious, but deceptive. In fact, the rails remain parallel throughout the journey. But if evidence has deceived us in one case, where is the guarantee that it will not deceive us in others? That is why science seeks, without relying on simple evidence, to prove *as* *much as possible,* all its provisions. Science is not a simple sum of true propositions. Science is the sum of truths, sufficiently substantiated, necessarily related.

The rules of this necessary connection are clarified and prescribed by logic — in the sections of this science devoted to derivation and proof in its various forms. But no matter how special the method of substantiating the truth is in each individual case, in any case, the justification must be available so that the situation can be recognized as true. In this case, the basis should be a *sufficient* basis. A justified conclusion is that conclusion which is obtained not from any clauses, but from provisions that can be a valid and *sufficient* basis for this conclusion.

The name of the fourth logical law of thinking under consideration — "the law of *sufficient* reason" — is not free from objections. In philosophical literature it was pointed out that this law should be called simply — the law of foundation. Indeed: the usual name contrasts a sufficient basis with a base *insufficient* . However, an insufficient basis is not, strictly speaking, a basis. Such can only be a sufficient basis. Therefore, the expression " *sufficient* reason" contains *pleonasm* , that is, the unjustified use of an excessive word in the name.

The objection is quite thorough. However, the name "law of *sufficient* grounds "can still be preserved if we take into account that the name emphasizes the *complex* nature of any foundation. Since the foundation is usually difficult, the belonging of a known circumstance to the composition of the necessary conditions of fact does not mean that the foundation is *exhausted by* this circumstance . Only the totality of circumstances or conditions *necessary and sufficient* for the occurrence of a fact or phenomenon constitutes the basis of this fact, this phenomenon. Therefore, the name "law of *sufficient* reason" can be saved as emphasizing the need for *an exhaustive* account of *all the* necessary components of the foundation.

§ 25. The law of sufficient reason expresses the existence for each truth of sufficient reason only in the most general form. Therefore, this law, of course, cannot indicate exactly what the basis should be in each individual case: whether it rests on a direct perception of the fact or on the proof of the situation. This law does not say anything about what this perception and this evidence should be. The law of sufficient reason only expresses that for every true statement there is, and therefore, sufficient reason must be indicated by virtue of which this statement is true. The question of the special nature of the foundation requires *special* consideration in each special case and is related to the *special* content of each branch of knowledge.

§ 26. As well as the logical laws of thinking already considered, the law of sufficient reason can be expressed by the general formula, namely: “if there is B, that is, as its basis — A”.

This formula means that the law of sufficient reason expresses not only the conditionality of our true *thoughts,* but also the conditionality of actual *facts* and *events* . No fact can take place, no event can occur unless they are causally determined by other facts and other events. No thought can be recognized as true if there is no sufficient basis for its truth in other true thoughts. In this case, only a thought that correctly reflects real facts can be true.

§ 27. The significance of the law of sufficient reason becomes immediately apparent in all cases when this law is violated. One of the possible logical errors is an error consisting in the fact that what can not serve as such a basis is taken as the basis for a conclusion or statement. So, simply following in time two events one after the other — no matter how often it repeats — in itself can not be sufficient reason for asserting that the previous event is the cause and the next is the action. Suppose we have seen the sun rise many times after dawn. This observation cannot be sufficient reason to assert that dawn is the *cause of* sunrise, that this connection of events is *necessary*, and that it should *constantly* repeated also in all other cases. To solve the question whether this phenomenon is really the cause of the other, following it, it is necessary to conduct a special study based not only on the observation of a simple repetition of a sequence of two phenomena. Logic sets the rules for such research — in the doctrine of *induction*.

§ 28. The four logical laws of thinking — the law of identity, the law of contradiction, the law of the excluded third, and the law of sufficient reason — are applied in all actions, or operations, of thinking. In all *arguments, proofs* and *conclusions*, wherever *judgments are* opposed, where *concepts* are thought, correct thinking occurs according to the logical laws of thinking.

Moreover, in each special operation of thinking, logical laws are usually applied not only individually, but also jointly. Since the definiteness, difference and conditioning of all objects of thought are not isolated features of these objects, but they presuppose each other, in accordance with this the main features of logical thinking — certainty, consistency and evidence — expressed by the logical laws of identity, contradiction, excluded third and sufficient reason, interconnected and suggest one another. So, in the proof of the theorem, apart from the law of sufficient reason, expressing the condition of evidence in the proper sense of the word, the other logical laws of thinking also appear as the necessary logical conditions for the proof: the law of identity, the law of contradiction and the law of the excluded third. Indeed, without observing the law of identity, it would be impossible to discern any necessary connection between the concepts included in the proof: the same concept, appearing twice or several times in the arguments, would not be identical, i.e. would not be would be the concept of the same subject, conceivable on the same grounds. Further, without complying with logical laws *a contradiction* and an *excluded third* would not have existed an indispensable necessity, recognizing as true the starting points, on which the evidence is based as its basis, to recognize as true those provisions that follow from them: only the law of contradiction explains why it is impossible, while recognizing the true starting point, to be true recognize true contradicting conclusion. And only the law of the excluded third explains why, having come to the conclusion that the known statement is false (as is the case in some evidence), we are thus forced to admit the truth of the statement contradicting it.

CHAPTER III. DOCTRINE OF THE CONCEPT

The Connection of the Concept with Judgment

§ 1. Every thought is always a thought about an object, or, as they say in logic, about an object. The object of our thought can be things, their properties, their actions, relations between them, etc. These things and their properties can really exist and can be non—existent, imaginary, imaginary. But even when the object of our thought is imaginary, such as, for example, the Black Sea in Russia and Lyudmila, it exists as an object of our *thought* . Even when our thoughts about the object are poor, empty, vague, as is the case with people who do not know the subject well, our thoughts still remain thoughts about the object, even if it is not well known to us.

§ 2. But thought not only points to a known object, but, in addition, always reveals to us some part of the content of the object. Consider the sentence: “mortars are guns firing with mounted fire.” This sentence, firstly, expresses the idea of a well—known subject — mortars; secondly, the proposal is not just pointing us to a known subject. It, in addition, reveals for our thought a certain part of the content of this subject: the ability of a mortar, unlike other types of tools, to conduct mounted fire.

At the same time, the idea expressed in the sentence does not reveal to us all the content of the object, but only some part of this content. In addition to the ability to shoot with mounted fire, mortars also have many other properties — calibre, length, construction, features of the control mechanism, etc. These properties are not considered or disclosed in this thought, although all mortars have these properties and although all these properties are distinctive for mortars.

An object is always richer in content than our thought about this object. An object has innumerable properties, a thought — in each individual case — reflects only a part of these properties, considers only those properties that are highlighted by the thought itself and which constitute only a part of the content of the object.

§ 3. So, every thought expressed in the form of a sentence is an idea, firstly, about an object or object, and, secondly, about a known part of the content belonging to the object, which is revealed or separated from the entire content of this object.

But this is not enough. In any thought expressed through a sentence, in addition to the subject matter and in addition to the part of the content allocated from the whole composition of the subject, thirdly, the *relation* between subject and content. Consider once again our proposal: “mortars — guns firing with mounted fire.” In this proposal, in addition to the subject (“mortars”) and in addition to the content (“the ability to shoot with mounted fire”), the connection between mortars and guns firing with mounted fire is also revealed. This connection consists in the fact that “mortars” and “guns shooting with mounted fire” are thought of in this statement as identical objects: “all mortars are guns shooting with mounted fire", and "all guns shooting with mounted fire" are mortars . In turn, this identity is based on the fact that in the composition of the entire content of objects called mortars, there is some part that stands out especially in thought — the ability to shoot with mounted fire. This part of the content of the subject has its basis in the subject itself and must be there.

§ 4. The thought through which: 1) a known *object* is highlighted , 2) a *part of the content of* this object is revealed , and 3) the *relationship between the* object and the allocated part of its content is *affirmed* , which is called a *judgment* in logic . Examples of statements: "the moon shines by reflected light of the sun", "water is not a simple body," "The Battle of Stalingrad — the greatest in the history of an example of the environment and the destruction of the encircled enemy army," etc...

The idea that distinguishes the subject of the judgment is called the logic of *the subject of* the judgment .[1] A thought revealing in a judgment a part of the content belonging to an object is called a *predicate of* judgment.[2] A thought revealing the relationship considered in a judgment between its subject and its predicate is called a *relation* .

§ 5. Judgment is an extremely important form of logical thinking. All truth is logically expressed in the form of judgment. All reasoning consists of judgments. In science, every law is expressed in the form of judgment. Every axiom (for example, "the whole is greater than its part") is a judgment. Every theorem (for example, "the sum of the angles inside a plane triangle is equal to two straight lines") is also a proposition. Everyday speech, story, conversation, debate are also made up of judgments.

§ 6. In each thought it is necessary to distinguish the *logical* composition of thought from its *grammatical* expression.

Grammatically, the thought of an object, its content and the relation between them is expressed in the form of a *sentence* . A sentence is a form of expression of thought in a *language* , or a *verbal* expression of judgment. The idea of ”mortars — guns fired by mounted fire” is a *logical* proposition, but the verbal, grammatical form by which this idea is expressed is a *sentence*.

§ 7. Since speech serves us to express our thoughts and has evolved from the need to express thoughts, generally speaking, the structure of the *sentence* and the structure of *judgment* correspond to each other.

However a grammatical *sentence*— far from the same as logical judgment. A sentence is only a verbal expression of thought. But every thought can be expressed in a word in more than one single way. Generally speaking, there are many ways to grammatically, verbally express one and the same thought. In this case, the logical composition of thought — the subject, the content, the relationship between the subject and the content—can remain the same.

Thus, the statements “Pushkin is my favourite poet” and “I love Pushkin more than all other poets” express the same idea. In these statements, essentially the same logical content. However, the grammatical form by which this thought is expressed appears to be different in these statements. Grammatically there are two different sentences. In the first, the subject is the word “Pushkin”, in the second is the word “I”. In the first, the predicate is complex: “my favourite poet.” In the second — simple: the verb “love.”

§ 8. In some cases, the difference between the structure of a grammatical sentence and the structure of a logical proposition can be very significant. *First* , the subject and predicate *sentences* may not coincide with the logical subject and the predicate of *judgment*. So, in the sentence “the glory of victory over the Turks at Ishmael belongs to Suvorov”, the word “belongs” will be a grammatical predicate, the word “Suvorov” will be a grammatical complement with the verb “belongs”.

On the contrary, in the *logical* proposition expressed by this sentence, the idea of Suvorov can be a logical predicate. If the teacher answers the question “to *whom* belongs the glory of victory over the Turks at Ishmael”, the student answers: “the glory of victory over the Turks at Ishmael belongs to Suvorov”, then it is obvious that the word “Suvorov” will be a logical predicate here. On the contrary, in terms of grammar, this word is just an addition.

Secondly, the difference between a grammatical sentence and a logical proposition is reflected in the fact that the same grammatical sentence can express, depending on which question it answers, not the same thing, but two or even several different judgments. In the same sentence, “The glory of victory over the Turks at Ishmael belongs to Suvorov”, the subject of logical judgment expressed by this proposal may be the idea of not who owns the glory of victory at Ishmael, but the idea of the victory over the Turks at Ishmael, whose glory belongs to Suvorov . But the proposition in which the subject of thought has become different is already another proposition.

Thirdly, the difference between a grammatical sentence and a logical proposition is reflected in the fact that a sentence may not have a subject at all, but a proposition cannot be without a subject. There is no special word in the sentence “evening”, which could be called the grammatical subject of this judgment. But this same sentence “grows darker” is a grammatical expression of a logical judgment, which can no longer be said that there is no subject in it, that is, there is no object conceivable in it. Such an object is here and is conceived. Only this subject here is vaguely distinguished by thought: this is what I have noticed as signs of the coming evening.

§ 9. In a logical proposition, the subject, the predicate, and the relationship between them are thought together as members of a single utterance. In the judgment "quinine bitter," one immediately thinks of the object of thought (quinine), and the part of its content that is highlighted by the thought in this judgment (bitterness of quinine), and the relationship between the subject and the predicate that expresses the property of bitterness to quinine.

However, although in the judgment the subject, the predicate and the relation between them are thought of as constituting a whole or a *single* judgment, each of these members of the statement is expressed by a *special* thought. The fact that we *distinguish between the* subject, the predicate and the relation between them, shows that the wholeness or unity of the judgment is a *special* wholeness and *special* unity. This is such a unity in which we distinguish its *parts* . A single thought of judgment "quinine bitter" is expressed with the help of a *special* thought about the subject of judgment, a *special* thought about the part of the content of the subject that the predicate of judgment reveals, and a *special* thought about the relationship between the subject and the predicate.

Thoughts through which the subject, predicate and relation are expressed are called *concepts* in logic.

Signs of the Subject and Signs of the Concept

§ 10. In each proposition, our thought can highlight the concepts by which the subject, predicate and relation are thought.

But although concepts can be distinguished by our thought from the composition of the judgment as its members, this does not mean that the judgment is composed or composed of concepts, like a pile of potatoes from ready—made potatoes put together or how the wall of a house is made up of separate finished bricks.

In any established judgment, a statement expressing the integral meaning of a judgment is thought before its parts. When I think about the property of quinine and think the judgment “quinine bitter,” I do not “add up” the concept of quinine with the concept of bitterness and with the concept of the relationship between them. I think something integral and, only noticing that in this wholeness there are special “parts” — the subject, predicate and attitude — I express these special parts with the help of special thoughts or concepts — with the help of the thought of “quinine”, the thought of “bitter” and thoughts about “belonging to bitterness to quinine.” In this case, grammatically, the concept of the property of bitterness and the concept of belonging of bitterness to quinine are expressed in the same form — the adjective “bitterness” in the function of the predicate.

Each object has a number of properties common to it with other objects, and a number of properties that distinguishes it from other objects. Thoughts on the properties of objects are called *signs* . So, a pine tree has a number of properties common to pine trees with all other conifers, and at the same time has a number of properties inherent in one pine tree and distinguishes it from spruce, fir, etc. Thoughts about all such properties are common to this an object with other objects, as well as special ones that belong only to this subject and distinguish it from others, even similar to it, are called *signs* .

Moreover, a sign belonging to the subject itself and, therefore, existing in it regardless of our thought about this subject, must be distinguished from the *thought* of this sign. *A conceivable* sign, or *sign of a concept* , is a reflection in the consciousness of the sign of an object.

Salient Features

§ 11. Not all properties of an object have the same value for this object, and not all signs have the same value for our knowledge. Strictly speaking, every item, even the simplest in appearance, has innumerable properties. Therefore, the concept of an object has innumerable signs. To notice all these signs and remember them all would be beyond the power of anyone, even the most learned person. Yes, this is not necessary.

The desire to include *everything* in the concept the signs of the conceivable in the concept of an object or phenomenon are not only completely impracticable, but from a logical point of view it is completely meaningless. For the tasks of practical life and for scientific knowledge, it is enough if, out of the whole huge set of properties of an object, our thought selects only some properties, however, it selects the most important ones and selects in such a way that each of the signs marking these properties, taken separately, will be absolutely necessary, and all the signs taken together will turn out to be completely sufficient in order to distinguish a given subject from all others, to know this subject on some side of its content.

Such a group of features is called a group of *essential* features of the subject, and the thought of the subject, highlighting the essential features in it, is called *the concept* . With the help of essential features, an object can easily be distinguished not only from objects that are clearly dissimilar to it, but also from objects similar, but not exactly matching the one in question.

For example, the exact concept of a square is the concept of a rectangle in which all sides are equal. In this concept, two features are distinguished: 1) rectangularity and 2) the equality of all sides. Each of these two signs, taken separately, distinguishes a square from other quadrangles. The squareness of a square differs from a rhombus, which has the equality of all sides, but there is no characteristic of the square properties — the necessary rectangularity. By the equality of all sides, the square differs from the non—equilateral parallelogram.

The two attributes highlighted in the concept of a square are not only necessary individually. These signs, in addition, are so interconnected and selected from among all other signs of the concept of a square in such a way that the two signs taken are completely sufficient so that, with their help, without resorting to indicating any other signs, we can distinguish the concept of a square from the concept of all other figures. That is why such signs are called *essential* , that is, distinguishing in the concept of an object is not something that is accidental for it, not something that could be in it, but could not be, but what needs to be for compliance concepts to the subject.

§ 12. Each group of essential features that can be distinguished in the thought of an object forms a *special* concept of this subject. This does not mean, however, that for each concept of an object there is only one single group of essential features. Each object is so complex, contains so many all kinds of properties, and these properties are all so interconnected that it is usually possible to indicate, with respect to the concept of the same object, not one unique, but several groups of essential features.

So, the water poured into the jug is one and the same body both for the one who just drinks this water, and for the painter depicting water on a still—life, and for a physicist studying the physical properties of bodies, and for a chemist studying chemical processes and reactions . But if the water poured into the pitcher — the same body, the same subject and for the painter and for Physics, and for the chemist, the *notion of* the physicist will not have the same thing about the subject as the chemist.

For a physicist, water is a liquid that at + 4° Celsius has the highest density, at + 100° Celsius and at normal atmospheric pressure it boils, at reduced pressure (as happens at the top of high mountains) it boils at a lower temperature, at 0 ° freezes, etc., etc.

For a chemist, the same water is H_2O, that is, a substance in whose molecule one atom of oxygen has two oxygen atoms. The physicist and chemist have not only two concepts about water, but in the concepts of these two groups or systems of essential features. At the same time, the concept of a physicist and the concept of a chemist about water are quite sufficient, each separately, in order to distinguish water from all other bodies with the help of the signs contained in them.

§ 13. This does not mean, of course, that the essential features represent something completely *arbitrary* , meaningful only to a person who distinguishes objects from each other, and not to the various objects themselves.

Of course, various types of activities and work, various areas of interest give rise to different concepts about essential features. For the reader browsing the library catalogue, *what matters* is not the format of the books, but their distribution *according to topics and branches of knowledge*.

For a librarian, when arranging books in a book depository, it will be important not only that the book belongs to a particular branch of knowledge or the heading of the content, but also the same format for all books installed on the same shelf, since such a placement system saves a lot of space.

The conscious activity of people is always determined by the *goals* that they set for themselves. Therefore, in everything related to activity, that which is recognized as essential, always to a certain extent relatively, conditionally, depends on the direction of interest, on the point of view on things.

For all this, the definition of the essential features of the concept of an object is by no means something purely conditional, entirely dependent on the point of view. In the example that we examined, the definitions of water given by a physicist and chemist are not conditional at all and express really significant properties of the subject.

The difference between these two groups of essential properties arose not only because the chemist is interested in some properties of water, but in physics — by others. This difference arose because in the substance of water itself — regardless of what the concept of water was made by a physicist and chemist — there are properties that, when separated into two special groups, form for our thought two different systems of signs by which chemical water properties may be different from physical ones.

§ 14. Each group or system of essential features represents *only part of* all, including essential, features of a given subject. In a real subject, all the attributes belonging to him are together, in communication with each other. But the concept selects or selects from the whole huge mass of attributes belonging to the subject only the group that characterizes the subject from the very point of view from which we study or consider it.

Only in this sense can we say that the materiality of these signs is something relative, that is, depending on the point of view on the subject. But at the same time, any group of essential features indicates features that belong *to the subject itself* , gleaned from *its own* content, and in this sense, it is something irrelevant, independent of any point of view. A separate physical and separate chemical concept of water does not exist because the physicist has one, but the chemist has a different point of view on water. On the contrary. Only because these two different points of view could have developed on the same subject — water — that in this subject itself, regardless of how the physicist and chemist look at it, there are both of these groups of interrelated *attributes* that underlie physical and chemical concepts of water.

Content and Scope of the Concept

§ 15. The set of essential features of an object conceivable in a concept is called the *content of the* concept. For example, in the concept of "square" considered by us, the content of this concept will be both of these essential features of a square: rectangularity and equality of all sides. The following features will be the content of the concept of "plane": the surface with which the straight line connecting two of its points coincides. The content of the concept of "impermeability" will be that property of bodies, by virtue of which two bodies cannot occupy the same space at the same time, etc.

To establish the essential features of the concept, it is necessary to compare among themselves a number of objects. A comparison will show what signs are necessary and sufficient to distinguish a given object from all others, to highlight the most important properties in it, to reveal its relations to other objects and their properties, etc.

It follows that in every concept, besides the thought of its *content* , i.e., about its essential features, one should also distinguish the idea of the totality of those objects that are covered by the concept. The set of objects conceivable in this concept is called *volume* of this concept. So, the scope of the concept of "square" is the thought of all quadrangles that have the essential properties of a square. The scope of the concept of "mammals" is the idea of all animals covered by this concept, that is, satisfying the essential features of the concept of "mammal", etc.

§ 16. The content and volume should be distinguished in each concept. What is the relation to the *content of the* concept and its *volume* ? Is the content of a concept determined by its volume, or, conversely, the volume of a concept by its content?

The correct answer to this question depends on whether we consider the *origin of the* concept of an object or the *application of an* existing concept.

From the point of view of the *origin of* our concepts, the content of a concept is usually determined by its volume. A conceivable class of objects exists before the thought of this class arises. Having singled out a well—known circle of objects and noting that in all of these objects there is a similar and distinctive feature, our thought then singles out a group of signs that will make up the content of the concept of the objects examined. From what is the circle of objects highlighted by thought and what are their properties, their relationship to other objects, what will be the content of the concept of these objects, what signs will be thought of in this content.

On the contrary, from the point of view of *applying the* concept that has already arisen, that is, which has developed in its content, the content is higher than the volume and the volume of the concept is determined by its content. As soon as the content of a concept has been clarified, i.e., it has been established which group of essential features forms its content, this determines what is the circle of objects that can be thought of through a given concept or to which a given concept is applicable.

So, for people who do not have an exact idea of what "white nights" are, it would be difficult to say which nights should be called white. On the contrary, there is no difficulty for the astronomer in this matter, since the astronomer knows exactly the *content* concepts of “white nights”. The astronomer calls white nights such nights during which the sun does not fall below the horizon lower than 18° of the arc.

This *content of the* concept of “white nights” quite accurately determines its *volume* . Based on this definition, the astronomer explains that the phenomenon of “white nights” in the European part of the Soviet Union takes place before the parallel of Poltava, since this parallel is the most southern one, in which the sun does not fall below the horizon at the beginning of summer by less than 18°. It follows from here that the phenomenon of “white nights” takes place in June and July on the parallel of Moscow, but does not reach such development here as on the parallel of Leningrad, where in these months the sun sinks below the horizon by an arc much smaller than in Moscow, etc.

Classes of Concepts and Relations Between Concepts

§ 17. Concepts are divided into classes: 1) from the point of view of the *real existence of objects of* concepts, 2) from the point of view of the *number of objects* conceivable through concepts, and 3) from the point of view of the *relationship between concepts in terms of content and volume*.

§ 18. From the point of view of the real existence of objects of concepts, all concepts are divided into: 1) *concrete* and 2) *abstract* or *abstract* .

Concrete are called concepts whose objects really exist as things of the material world. Such, for example, are the concepts: “book”, “tree”, “airplane”.

Abstract or *abstract*, are called concepts in which not a whole thing is conceived, but any one of the attributes of an object, taken separately from the object itself. Such, for example, are the concepts: “whiteness”, “valour”, “rationality”. The objects of these concepts do not exist as independent things: there are white snows, valiant people, rational thoughts and actions, etc., etc., but not “whiteness” as a separate object, not “valour” as a separate object and not “rationality” as a separate subject.

”Distracted” these concepts are called because their objects are formed by thinking through abstraction, or distraction. This is the name of the action of thinking, consisting in the fact that, having noticed a certain property or sign, or attitude in a number of objects, thinking separates (“distracts”) them from the objects in which they only exist, and turns this property, this sign, this relation to special objects—objects of abstract thought, or abstraction.

In each case, abstract concepts reflect only a part of the attributes of an object. In this context, abstract concepts are similar to concrete ones. Every concept of science does not reflect *everything* the content of the subject, phenomenon, process, but only the known side of this content. Such are not only such abstract concepts as “whiteness”, “valour”, but also such concrete concepts as “capital”, “socio—economic formation”.

§ 19. In terms of the number of objects conceivable by means of concepts, all concepts are divided into 1) *general* , 2) *individual* and 3) *collective* . Concepts are called *general* by means of which not a single object is thought, but a whole class of homogeneous objects bearing the same name. For example, the concepts “circle”, “person”, “judgment” will be common.

Single concepts are called through which one single thing is thought, for example, the concepts of “Peter”, “Sirius”, “Kiev”.

Collective concepts are called by means of which a whole group or set of objects is thought, but this group is thought of as a single object. Such, for example, are the concepts: “constellation”, “battalion”, “grove”. So, the constellation is not one star, but a collection of stars. However, this totality is thought of as some unity or whole. Collective concepts combine the properties of general and individual concepts. Like general concepts, they encompass or represent a whole class of objects. Just as by means of single concepts, by means of collective concepts a single object is conceived. However, a single thing conceivable through them exists as a single one only for thought. In fact, its unity is made up of many, and it really exists—as an object—it is many, not unity.

§ 20. Between objects at the same time there is a similarity and difference, that is, in the objects themselves there are both common to all of them, and various signs. If this is the case with the objects themselves, then it should not be otherwise with the *concepts* of these objects. Therefore, one of the important questions of logic is the question of the relationship between concepts in terms of their content and volume.

§ 21. In terms of content, concepts can be either *comparable to* each other or *incomparable*. Comparable are the concepts in the content of which, despite the difference between the well—known and sometimes very many features, there are also some common features that are comparable to them. The objects of such concepts belong to the well—known uniting them, although sometimes extremely broad, area. Thus, the concepts of “man”, “animal”, “plant”, “mineral” are comparable concepts. The contents of all these concepts have common features, and the subjects of all these concepts make up a very vast, common to them all area of *bodies*.

On the contrary, such concepts as, for example, “home” and “valour” are *incomparable* concepts. The subjects of these concepts belong to completely different areas. Therefore, in the content of these concepts there are no common features, except for those that, due to their extreme generality, can be considered to belong to almost all objects without exception. So, both the concept of “home” and the concept of “valour” can both be objects of thought, both are general concepts, etc.

However, if we consider that all concepts, no matter how different their contents and to whatever different areas, their objects belonged, but nevertheless may be objects of our thought, in this sense it can be said that all concepts without exception are comparable to each other and that absolutely incomparable concepts do not exist at all.

§ 22. Comparable concepts may be in content either *compatible with* each other, or *incompatible* or *opposite*. Compatible are two such concepts in the content of which there are no signs that exclude the possibility of full or partial coincidence of the volumes of these concepts. So, the concept of “gun” and the concept of “howitzer” have different contents. But at the same time, in the content of these two concepts there are no such signs that would be incompatible, i.e. would exclude the possibility of coincidence of their volumes. Therefore, no matter how different the objects of these concepts are, the possibility is not excluded that there are such objects that simultaneously belong to the volume of both one and the other concepts. In fact: among the guns are howitzers, and howitzers, in turn, are among the guns. Another example of consistent concepts is the concept of “parasites” and the concept of “plant.” With all the differences in the content of these concepts, they have no signs, which would exclude the possibility of a plant being a parasite. And indeed: some plants (like mistletoe) are parasites, and some parasites are plants. In other words, the volumes of compatible concepts can, at least in a certain part, coincide.

§ 23. For greater clarity, the relations between the volumes of concepts are represented in logic through circles. Each *individual* object belonging to the volume of a given concept is depicted by means of a *point* placed either inside the circle or on its circumference (see Fig. 1).

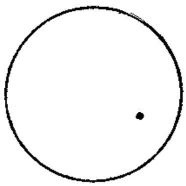


Fig. 1

Since the whole set of objects of a class is included in the volume of a concept, and since a circle (Fig. 1) has any number of points on its surface, a circle drawn to represent the volume of a concept clearly depicts any number of objects of the same class. If the volume of one concept is part of the volume of another concept, in other words, is entirely included in the volume of another concept, then the volume of the first concept is depicted by a circle drawn inside a larger circle and entirely placed on its area.

For example, the relationship between the volumes of the concepts “gun” and “howitzer” can be depicted as shown in Fig. 2.

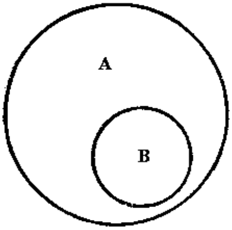


Fig. 2

Here the scope of the concept of “howitzer” is depicted by means of the smaller circle B, and the volume of the concept of “gun” is represented by the larger circle A. At the same time, the smaller circle B is *entirely* placed inside the larger circle A. This figure shows that all howitzers are tools, or, in other words that all the objects included in the volume of concept B belong at the same time and the volume of concept A.

Sometimes the volumes of two concepts, A and B, *partially* coincide. This happens in cases where *part of the* objects included in the volume of the concept A (but *not all the* objects that make up the volume of the concept A) is also included in the volume of the concept B. The relationship between the volumes of such concepts is visually depicted by two intersecting circles (see Fig. 3).

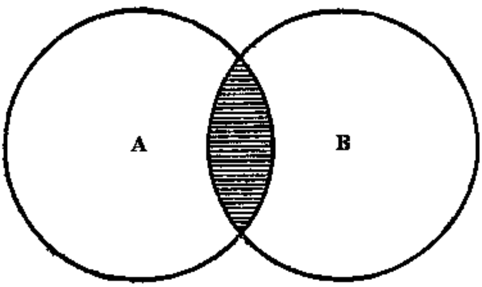


Fig. 3

For example, the relationship already examined by us between the volumes of the concepts of “parasites” and “plants” can be represented as it is shown in Fig. 3: some (but not all) plants are plants, and some (but not all) plants are plants. In this case, the shaded and common to both circles part of the plane of the figure will denote those objects that simultaneously belong to both the volume of the concept A and the volume of the concept B. The unshaded parts of both circles will denote those parts of the volumes of both concepts that cannot match: plants that do not are parasites, and parasites that are not plants.

If no object belonging to the volume of concept A can simultaneously belong to the volume of concept B, then the relationship between the volumes of these two concepts is depicted using two circles placed *one outside the other* so that no point lying on the area of one circle can not be lying on the area of another circle (see Fig. 4).

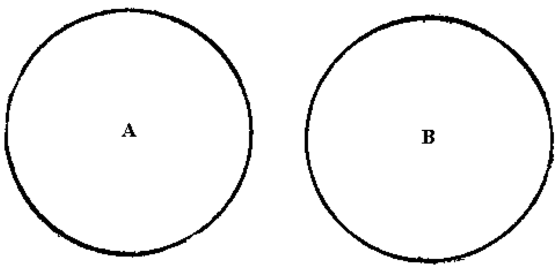


Fig. 4

For example, the relationship between the volumes of the concepts of “acute angle” and “obtuse angle” can be represented as it is shown in Fig. 4: it is immediately evident that not a single sharp angle can be an obtuse angle, and, conversely, not a single obtuse angle can be acute.

§ 24 . In contrast to *compatible* concepts, two such concepts *are* called *incompatible* , the content of which contains signs that exclude the possibility of not only complete, but also partial coincidence of the volumes of both concepts. Such, for example, the concepts of "sick" and "healthy". It is impossible to find such an object that would simultaneously belong to the volume of both of these concepts. In other words, the volumes of such concepts cannot even partially coincide.

Since the volumes of incompatible concepts cannot even partially overlap, the relationship between the volumes of such concepts is depicted as shown in Fig. 4, — in the form of two circles lying one *outside the* other.

§ 25. And the class of compatible concepts and the class of incompatible concepts in turn comprise each further subdivision.

Compatible concepts are either *equivalent*, *subordinate to* each other, or *intersecting*.

Equivalent concepts are those concepts in which the content contains in each of them various signs, but these signs are so interconnected that, due to this connection, the *volumes* such concepts coincide, turn out to be identical. Such, for example, are the concept of a perpendicular restored in the plane of a circle to the end point of its radius, and the concept of an unbounded line having the same direction and passing through the same point of the circle circumference. Both of these concepts have different features in their content, but the same volume, since such a perpendicular and such a straight line coincide. Or, for example, the concept of "founder of the science of logic" and "philosopher — educator of Alexander the Great." And here the signs included in the content of these two concepts are different, but the volumes of both concepts coincide, since the founder of the science of logic and the

philosopher — educator of Alexander the Great was the same person, namely the Greek philosopher Aristotle.
The relationship between the volumes of equivalent concepts is clearly depicted as shown in Fig. 5.

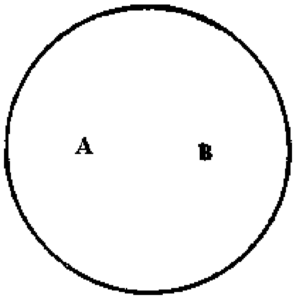


Fig. 5

Here the letters A and B, placed inside the same circle, mean that the concepts A and B have different contents, but the volume is the same.

§ 26. The second type of compatible concepts is *subordinate* concepts. attitude *of submission* concepts—one of the most important in logic. Consider an example of such an attitude. Suppose there are two concepts: the concept of a "triangle" and the concept of a "right triangle". Obviously, both of them are compatible concepts, since there are no signs in the content of both that exclude the coincidence of the volumes of these concepts: some triangles are right—angled triangles. Let us now take a closer look at the relationship between these concepts. Everything that is thought in the content of the concept of a "triangle", obviously, is fully included in the content of the concept of a "rectangular triangle" and is part of this last. In fact: the content of the concept of a "right—angled triangle" includes, firstly, all the features without exception that make up the content of the concept of a “triangle”, and secondly, there are some others besides them, which are peculiar to only one right—angled triangles and which are different from right—angled triangles from all other triangles. This is the case with the content of these two concepts.

Let us now consider the relationship between their volumes. While the content of the concept of a “triangle” is only part of the content of the concept of a “rectangular triangle”, the extent of these concepts is the opposite: the volume of the concept of a “rectangular triangle” is thought of as completely contained in the volume of the concept of a “triangle”, forming only part of this last, since besides right triangles, other triangles also belong to triangles.

Such a compatibility relationship, such as the relationship between the concepts of a “right—angled triangle” and “a triangle,” is called the *submission of* concepts. The relation of submission is the relation of a particular concept to a concept more *general* , and vice versa: the relation of a concept more *general* to a concept more *private* . In this case, a particular notion of “right triangle” refers to *a subordinate* , but a more general—”triangle”— *subordinating*.

The relationship between the volumes of concepts subordinate to one another is depicted by means of two circles, of which one is entirely placed inside the other (see Fig. 2).

Moreover, the larger circle A represents the volume of the *subordinate* concept, and the smaller circle B represents the volume of the concept of the *subordinate*.

§ 27. Some cases of submission of concepts deserve special attention. Such is the case when the subordinate and subordinate concepts are both concepts in common. In this latter case, the subordinate concept is called *genus*, or *a generic* concept, and a subordinate concept—a *species* , or a *specific* concept.

In our example—”triangle”, “right triangle”—concept “triangle”—generic, concept “right triangle”—species [1] .

§ 28. The generic concept, being broader than the generic one *in terms of volume* , contains in its *content a* smaller number of characters compared with the specific concept.

In every concept, if it is a truly scientific concept, are *provided* all special cases that can be deduced from it and from which the full content of the concept is compiled. Every scientific concept is formed according to the rule, knowing that we can consistently cover all the particular cases that its content can represent.

For example, the concept of a “triangle” is the concept of a figure formed by the intersection of three straight lines lying in the same plane. The content of this concept provides as possible all essential features of all particular types of triangles —acute—angled, and rectangular, and obtuse.

But of all these signs characterizing *particular cases*, or *types*, of a triangle and making up the content of the concept of a “triangle”, not one is noted in the *definition* concepts of “triangle”.

This does not happen at all because these signs in no way belong to the content of the generic term “triangle”.

This happens because specifying particular features in the definition is necessary only in special cases when we want to *distinguish* one type of triangle from another, for example, a right—angled triangle or an obtuse—angled triangle.

That is why, in addition to the common signs for all triangles—the figure formed by the intersection of three straight lines lying in the same plane—the definition of the content of the concept of a “right—angled triangle” introduces a *new additional* feature—the presence of one right angle among the interior angles of a triangle.

If, finding out the content of a more general generic concept (“triangle”), we do not note the characteristics that are included in the content of the species concept (“rectangular triangle”), then this is not because species characteristics *cannot be* thought of as belonging to the content of a more general concepts, but because, despite their intended presence in the content, there is no *need to* note all these signs in the definition of a concept.

Indeed, the definition of a triangle is not intended to indicate or list all *possible* particular cases or varieties of triangles, but to distinguish *an y*a triangle — be it acute—angled, rectangular, or obtuse—angled — from any other figure (square, trapezoid, hexagon, etc.).

§ 29. The more general the concept, the smaller the part of the content expressed in the *definition of a* concept, the more signs and relationships of signs are provided for in that part of its content that is not expressed in the definition. The concept of “triangle” provides for the *possibility* to think, in addition to those signs that are thought in the content of the concept of an acute—angled triangle, there are also signs conceivable in the content of the concepts of right—angled and obtuse—angled triangles. Precisely because triangles can be not only acute—angled, but also rectangular and obtuse—angled, all the signs that make up the content of concepts about all these types of triangles can belong to the content of the concept of “triangle”.

But although, thus, in the content of the general concept *all* particular contents are contained, *all* special cases and *all* special features that can be developed from this content or found in it, these particular cases and signs are not indicated in the *definition* more general concepts are not noted *directly* in its content.

They are not noted, not because they are absent in the content of the concept, but because of the possible composition of the content, only those attributes that are necessary and sufficient to distinguish a given object (or class of objects) from all others are introduced *into the definition*. Such— necessary and sufficient — in the case of determining the content of a more general concept will be less special, non—species characteristics.

It is in this sense that they say that in concepts that are related to each other in relation to genus and species, volume and content are *inversely opposite* relation: a larger volume corresponds to a lower content and, conversely, a larger content corresponds to a lower volume.

Essentially, the relation means here the relation of that part of the features that is *directly* indicated or noted in the definition of a concept to the *whole* set of features that are included in the concept's content and are provided for in it, but are not indicated when *determining* its content.

§ 30. The third type of compatibility of concepts—*cross*. This is the name of the relation of concepts in the content of which there are various signs, but which may belong to an object in different respects and therefore do not exclude the possibility of a partial coincidence of the volumes of concepts. These are the concepts of "painter" and "sculptor". The contents of both of these concepts consist of features that do not have the necessary connection with each other. A painter should not necessarily be at the same time a sculptor, and a sculptor a painter. But there may be persons who simultaneously satisfy the characteristics of each of these concepts. Consequently, the volumes of these two concepts in some part of their own may coincide. And indeed: some sculptors, such as Miquel Angelo, were at the same time painters, and some painters, such as Renoir, were sculptors. The ratio between volumes of *intersecting* concepts is depicted by means of mutually intersecting circles (see. Fig. 6).

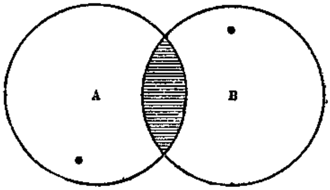


Fig. 6

From this figure it can be seen that the coincidence of volumes of overlapping concepts is possible not for the entire volume of concepts A and B, but only for a certain part of their volumes (not all painters, but only some painters were together sculptors). Points lying *outside the* hatched and common for A and B part of their volumes mean concepts whose signs are so different that they do not allow the coincidence of their volumes.

§ 31. Equivalence, submission and interbreeding are varieties of *compatible* concepts. In turn, incompatible concepts can also be of various types: 1) *contradictory* , 2) *opposite*, and 3) *subordinate*.

Consider first the *conflicting* concepts. So called two such concepts, of which one has a well—known group of features in its content, and the other does not contain anything in its content except the negation of these features alone. These are the concepts of “integer” and “non—integer”. The first of them (the concept of “integer”) has in its content a well—known set of positive signs. On the contrary, the second of them (the concept of “non—integer”) means: any number, except the integer, but which exactly, what are its signs — there is no indication of this in the content of the concept of “non—integer”.

Other examples of conflicting concepts: “neat”—“not—neat”, “valiant”—“not—valiant”, etc.

The relationship between the volumes of two *conflicting* concepts is shown in Fig. 7.

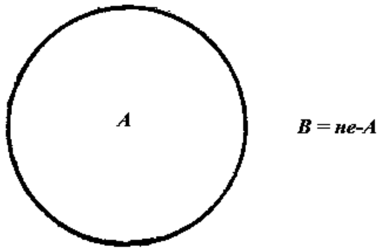


Fig. 7

Here a positive definite concept, for example, the concept of "white" is indicated by the circle A. The contradictory concept B is "non—white", the content of which is the negation of the content of A, indicated by the plane B indefinitely extending around A, not closed by any circle. This way of depicting should show that under non—A, generally speaking, everything can be thought of as anything other than what constitutes the content of concept A.

However, in reality, thinking a contradictory concept, we do not just contrast the denied content A with whatever AND. We contrast "white" not just "all non—white", but contrast it with *some other color*. But this means that even in the case of a contradiction between the two concepts, the contradiction does not lie in the fact that we simply deny the known content, but in the fact that we oppose the negated content to some other, also positive content, relating to the common for A and for a B (non—A) genus. But what exactly will this other content of the same kind be — it remains uncertain.

§ 32. *Opposite* , or *counterattack* , are two such incompatible concepts, of which the content of one not only negates the characteristics of the other, but is also replaced by others — incompatible with it. These are the concepts of “good” and “bad.” The content of “bad” not only contains signs that *deny* the content of the concept of “good”, but, in addition, the denied signs *are replaced by others*— incompatible, but quite positive signs relating to the genus of quality common with the denied concept.

The relationship between the volumes of two opposing concepts A and B, for example, between the volumes of the concepts of “good” and “bad”, is shown in Fig. 8.

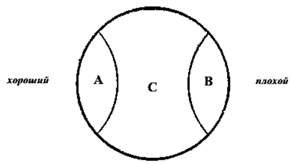


Fig. 8

This figure shows that both opposing concepts belong to the same genus C, in this case, the genus of quality, therefore both A and B are inside the circle they share. In other words, the content of concept B is just as positive as the content of concept A counter to it.

At the same time, this figure shows that between concepts that constitute the extreme opposite (counterattack), A and B there can be concepts that form the transition from A to B. For example, between the extreme opposites of good and bad, there is a “mediocre”, through the many degrees of which one can consistently and continuously move from bad to good and vice versa.

§ 33. The difference between conflicting and opposing concepts in some cases becomes difficult to perceive. In the Russian language, many words that are faced with the negation of “not” can mean not only a simple denial of positive signs, but also some opposite quality, characterized by its special positive signs.

So, the word “not good” can also mean a simple negation of kindness, without replacing the negative concept with a concept of a different quality, and at the same time it can mean the same as the word “evil”, that is, some other quality, not only excluding quality kindness, but at the same time possessing its own special positive signs. Which opposite — contradicting or counteracting — the word with the negation expresses — this can be judged not by this very word, taken separately, but by the whole meaning of speech as a whole or, as they say, “by the context” of speech.

In the Russian language there is a negative particle “without”, which, when put at the beginning of a word, shows that the concept denoted by a word with this particle is not a contradiction, but the opposite.

So, the word “not smart” can also mean a simple denial of the mind (then it will be a concept that contradicts the concept of “smart”) and can be equivalent to the word “stupid” (then it will be a concept opposite, or counter, to the concept of “ clever”). Which of these two meanings the word “not smart” expresses is not visible from the word itself and can only be ascertained from the context.

On the contrary, the word “ *without* smart” means a concept that can immediately be said that it will be the opposite of the concept of “smart,” that is, it will denote, although the opposite of the concept of “smart,” but a very definite content.

§ 34. Incompatible concepts can also be divided according to the degree of generality. Two or more concepts are called *subordinates.* When, being equally common, they are subordinate to the generic concept closest to them in degree of generality. So, the concepts of “gun”, “howitzer”, “mortar” will be subordinated to the common concept of “artillery gun” for them. Moreover, the concept of “artillery gun” is the concept closest in degree of generality in relation to the concepts of “gun”, “howitzer” and “mortar” subordinate to it.

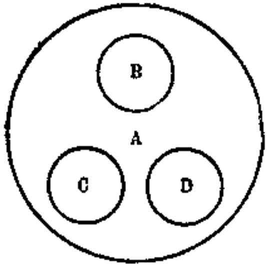


Fig. 9

The relationship between the volumes of *related* concepts is depicted by means of a large circle, inside of which two or more small circles are completely placed without touching each other and without crossing each other. In this case, the large circle represents the volume of the *subordinate* concept, the small circles placed inside the large one represent the volume of concepts *subordinate to the* first. The absence of coincidence or intersection between small circles placed inside a large one shows that the volumes of related concepts are incompatible and that there are distinguishing features in the content of related concepts.

A second example of the relationship between subordinate concepts can be the relationship between the concepts of “acute—angled triangle”, “right—angled triangle” and “obtuse—angled triangle”. In fig. 9 shows this relationship.

Here, the large circle A represents the volume of the subordinate concept of a “triangle.” Small circles B, C, and D depict the relationship between the volumes of the concepts of acute triangle, rectangular triangle, obtuse triangle, which are *subordinate* to the concept of a triangle.

All three of these incompatible concepts are subordinate to the same and common to all of them the concept of “triangle”. Therefore, we can say about all these three concepts that they are *subordinated* to the concept of “triangle.”

§ 35. Incomparable concepts are also called *disparate* . Such, for example, the concepts of “length” and “shine”. The volumes of these concepts cannot be included as the volumes of subordinate concepts in the volume of the concept subordinating them to themselves.

§ 36. In the previous paragraphs, we examined the most important types of concepts and got acquainted with the relations between them in content and volume. All considered types of comparable concepts can be graphically represented by the diagram shown (see Fig. 10).

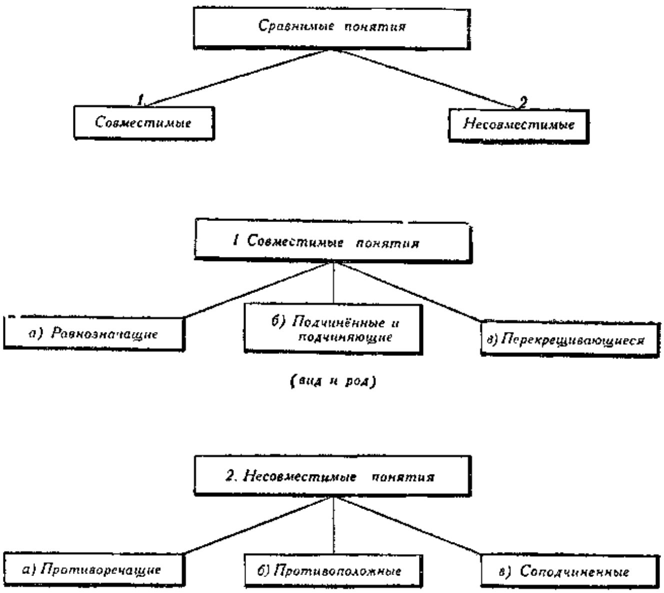


Fig. 10

Tasks

1. Indicate the essential features of the concepts: “circle”, “fraction”, “clock”, “chemical reaction”, “bird”, “romance”, “inspiration”, “courage”, “courage”, “courage”, “courage” “.

2. Define the relationship between the following concepts in content and volume: “scientist”, “professor”, “spider”, “insect”; ”Founder of St. Petersburg”, “winner of the Swedes at Poltava”; ”Lion”, “tiger”, “panther”; ”Spruce”, “pine”, “fir”, “cedar”; ”Circle”, “ellipse”, “parabola”, “hyperbole”; ”Poetry”, “music”, “painting”, “sculpture”; ”Guardsman”, “order bearer”; ”Difficult”, “non—difficult”; ”Difficult”, “easy”; ”Courageous”, “timid”; ”Redness”, “heaviness”; ”Courage”, “caution”; ”Talent”, “hard work”; ”Personal”, “impersonal”; ”Chemistry”, “organic chemistry”; ”Chemistry”, “natural sciences”; ”Fleet”, “squadron”; ”Logarithm”, “good nature.”

3. Using circle diagrams, depict the relationship between the volumes of concepts: “glory”, “dishonour”; ”Poems”, “prose”; ”Poison”, “medicine”; ”Firewood”, “peat”, “coal”; ”Cunning”, “stupidity”; ”Labour”, “idleness”; ”Oil”, “watercolour”, “pastel”, “pencil”; ”Bow instrument”, “violin”; ”Planet”, “luminary”; ”Star”, “planet”; ”Fresh”, “non—fresh”; ”Fresh”, “rotten”; ”Seal”, “mammal”; ”Physicist”, “chemist”, “natural scientist”; ”Stakhanovites”, “caster”; ”Circle”, “straight line touching the circle at a given point.”

CHAPTER IV. LOGICAL ACTIONS ON CONCEPTS

Presentation and Concept

§ 1. In everyday practice of thinking, people use not strictly established concepts, but only *ideas* about the subject. Representation, as well as concept, is a thought that distinguishes well—known attributes of this object in an object. But in the presentation, *firstly, essential* features are not necessarily highlighted . In the idea of the subject, those signs are highlighted that for some reason are striking and which may not be significant.

When the word “ink” appeared in Russian, this word, related to the word “black”, was supposed to provoke the idea: “what they blacken” or “what they paint in black”. Currently, this view does not coincide with the signs that are thought in *the concept of* ”ink”. The concept of “ink” expresses the idea of a fluid through which they write, regardless of its color. This liquid can be not only black, but also violet, blue, green and red. That *idea of the* liquid staining black, which was previously thought, now turned out to be insignificant for the *concept of* ”ink”.

Secondly, in the presentation, the signs are not set equally and depend on the person representing the item, on the psychological circumstances in each individual case. So, if two people imagine, for example, a square, then their representations will, of course, differ from each other. One will imagine a large square drawn on a piece of paper, the other a square drawn on a blackboard, etc., etc. On the contrary, in the concepts established by science, signs are taken only essential and taken in such a way that every time when a given concept is thought, the composition of its essential features is not subjected to random changes, it does not depend on which person and in what psychological state the concept is thought. So, if two people are familiar with geometry, then the *concept* they will have the same thing about the square: both will mean a rectangle under the square, in which all sides are equal.

§ 2. It is these properties of the concept — the hardness and accuracy with which its essential features are established in the content of the concept — that make the concept an important logical form of scientific thinking. In a certain sense, the concept and science are synonyms, that is, words meaning the same thing.

The concept is simultaneously the *first* condition of scientific thinking and its *last* top result. Logical thinking begins with the formation of various concepts about various objects. Through judgments revealing the various relationships between different concepts, knowledge of the essential properties of an object is deepened and enriched more and more. New points of view are being clarified from which the properties of objects, and therefore the signs of their concepts, can be considered. As a result, concepts arise that contain a lot of interconnected, but different groups of essential features that reflect in our thoughts the essential properties of objects. The path leading from the first experiments in the formation of concepts to higher concepts is very complex and long. This path is accomplished not only with the help of judgments, but also with the help of other forms of cognition and thinking, which we are talking about. But as a result, this path leads to the emergence of concepts.

Definition of a Concept

§ 3. Since the most characteristic feature of a concept that distinguishes it from a simple presentation is the accuracy with which the essential features are indicated in the content of the concept, it follows that the most important logical operation, or the first logical action on a concept, is to *establish its content* . This action is called the *definition of a concept* . To define a concept is to indicate which essential features are thought of in its content.

The verbal designation of a concept that is precisely defined and suitable for use in science is called a *term*. The definition does not merely *explain the* meaning of the term. Definition *sets* this value. Such a determination of the value is necessary not only for new concepts introduced for the first time in science and their terms. It is no less necessary to clarify long—used in the language, but inaccurate, confusing terms. Although concepts are usually formed from the materials of everyday notions, they become scientific only after they are transformed from simple and unclear representations into certain concepts.

§ 4. Like any activity of thinking, the definition can be *right* and *wrong*.. A correct definition, that is, a definition indicating the essential features of a concept, is expressed in the form of a sentence in which the subject is the definable concept itself, while the predicate contains a listing of the essential features of the definable. A logical definition is not a simple explanation of the meaning of a word — like those contained in explanatory dictionaries, for example, in Ushakov’s explanatory dictionary. There is no logical definition and a simple substitution of one word for another. The logical definition reveals for thought the signs that are thought by science in the content of the concept.

§ 5. We already know that the scope of a concept is inversely related to its content specified in the definition. Depending on what essential features are thought in the content of the concept, the scope of this concept will be greater or less. But precisely because the scope of each concept is limited by the content of the concept, the definition must be such that it accurately indicates the volume that is thought in this concept.

Therefore, the *first* condition for the correctness of the definition is its *proportionality*. A definition is called proportionate if the volume of the determining concept is exactly equal to the volume of the concept being defined. In the definition of "a square is a rectangle with all sides equal", the volumes of the defined ("square") and defining ("a rectangle with all sides equal") are exactly equal to each other: all squares are such rectangles, and all such rectangles the essence of the squares.

A definition in which the requirement of proportionality is not met will be an incorrect definition. Consider, for example, the definition of "a square is a quadrangle with all sides equal." This definition is incorrect, since it is disproportionate: a rhombus is also an equilateral quadrilateral. The fallacy of this definition is that with its help it is impossible to accurately distinguish a square from a rhomboid. Definition in which the scope of the determinant *more than the* volume defined, is called *too wide*.

A disproportionate definition may be *too narrow*. This is the name of the definition in which the volume of the determinant is smaller compared with the volume of the determined. For example, the definition of "energy is the ability to do useful work" is also a disproportionate, but at the same time too *narrow,* definition. Indeed, the volume of the determinant here turned out to be *smaller in* comparison with the volume of the determinant: the determinant indicates the ability to produce *useful* work, while in reality the concept of energy means the ability to produce *any* work, whether it will be useful or useless.

The second condition for a correct definition is that the definition should not contain a *circle* . A circle refers to such a way of determining when a concept is as if defined by another concept, however, this other concept is such that it itself can only be understood through what is defined. Such, for example, is the definition of "quantity is all that can be increased and decreased." Here, the concept of magnitude is revealed through the concept of increase and decrease, however this concept, in turn, is explained only through the concept of magnitude. And indeed, the increase and decrease is nothing more than a change in magnitude.

A *tautology* forms a particularly clear case of a circle in the definition.. This is the name of the definition, in which the determinant is a simple repetition of what is thought in the definable. In such an explicit form, a tautology, although rare, is encountered. In one bad book on the history of music, the author, wanting to clarify what the essence of Meyerbeer's musical style is, wrote: "Meyerbeer style is a real Meyerbeer style". Definition This is a prime example of a tautology.

In a letter from a 17th century scientist, Noel, to the famous French mathematician and writer Pascal, there is a definition of light that represents an explicit tautology. According to this definition, light is "the light movement of rays consisting of luminous bodies that fill transparent bodies and receive this light only from other luminous bodies."

But even in modern times, the logical mistake of the tautology can be detected in a number of scientific arguments. Only here she usually appears not so explicitly. So, Marx showed that the classics of political economy and their successors in the XIX century. constantly fell into the error of the "circle" or tautology. This mistake consisted in the fact that they determined the value of goods by the value of labour, and the value of labour by the value of goods. However, in doing so, we, as Marx showed, "only push back the difficulty, since we determine one value by another value, which in turn needs to be determined." [1]

The cause of the tautology can be both the negligence and vagueness of logical thinking, and the extreme difficulty that one encounters in determining.

An example of the negligence of logical thinking, leading to a tautology instead of a definition, can be our example with the definition of a "Meyer—burger" style. Such errors are immediately visible and can be easily fixed.

But sometimes it is extremely difficult to notice the tautology in the definition. This happens when the subject, the concept of which is determined, is so simple that in the concept of it is difficult to distinguish the essential features that make up its content.

Some expressions of thought seem at first glance tautologies, but in reality they are not tautologies. There is, for example, the French proverb "in war as in war" (à la guerre comme à la guerre). This expression is not at all a definition, and therefore there is no tautology. This expression means approximately the following: in a war one does not have to show leniency towards the enemy, just as one does not have to wait for mercy from the enemy, that is, war is a war with all its consequences.

Third the condition for the correct definition: the definition should not be only negative. A definition is called negative if it indicates only which features do not belong to a given concept, but does not indicate which features belong to it. So, the definition of "spider is not an insect" is a negative definition. It contains only an indication that the essential features of the concept of "insect" do not coincide with the essential features of the concept of "spider".

In fact: from the point of view of zoology, both spiders and insects belong to arthropods, but at the same time, spiders belong to cheliceous arthropods, and insects belong to tracheal—breathing arthropods. The relation between these two groups is the same as the relation, for example, between acute—angled and right—angled triangles. Both of them are triangles, but it does not follow from this that acute—angled triangles are rectangular.

From the negative definition of the term “spider” one cannot in any way know what the essential signs of a spider will be. Therefore, such a definition is not suitable for expanding the actual knowledge of the subject. A negative definition is limited to an area of obscure features. The question is where, among which particular signs should be sought those that form a *positive* the content of the concept of the subject remains without consideration.

A negative definition is sometimes found in the most serious scientific expositions. For example, in the book of the famous ancient mathematician Euclid “Beginnings” there is such a definition of the concept of a point: “A point is that which has no parts” [1] . This definition is clearly negative. It is caused by the fact that a point is an element of space so simple and homogeneous that any attempt to find any parts in it that could serve as positive signs of a concept for thought fails.

A negative definition, similar to the Euclidean definition of a point, is, for example, Stavin’s definition of a solid: “A solid is one that is neither liquid nor fluid, does not dissolve in water and does not evaporate in air” [2] .

However, some negative expressions, not being definitions, have a very definite meaning.

The possibility of a circle in definition, as well as the possibility of negative determinations in the thinking of even large scientists, proves not only that large scientists can sometimes make logical mistakes. The appearance of tautologies and negative definitions in thinking proves that some concepts are difficult to define. These are the concepts by which the simplest, "taken for granted" objects, the properties of objects, and actions are thought. An attempt to define such concepts leads to the fact that we either don't learn anything new from the definition (as it happens in *tautology*), or we learn what characterizes it not positively, but only negatively (as it happens with a *negative* definition).

Therefore, starting to consider a concept, it is necessary to investigate whether a given concept admits a definition or if an attempt to define it leads only to the fact that we will put a less clear one in place of the clear.

The fourth condition for the correctness of the definition is the *clarity of the* definition, that is, the absence of any ambiguity in it. Many expressions, for example comparisons, being extremely picturesque, figurative and valuable for expressing feelings, for cognition, however, are not at all definitions, since they do not indicate the essential features of the subject.

§ 6. Of all the possible logical errors of the definition, the most important is the error of an overly broad and overly narrow definition. In the first case, the error is that the list of signs is *skipped* any essential characteristic. In the second case, on the contrary, an *excessive* attribute is introduced into the content of the concept being defined , which is essential only for certain objects that are conceivable in the concept. In the first case, the content of the concept being defined becomes one sign less, but its volume is thought to be large. In the second case, on the contrary, in the content of the subject being determined it becomes one more sign, but the scope of the concept is thought to be less.

Definition Through the Nearest Genus and Through Species—Forming Difference

§ 7. Since the definition of a concept consists in establishing its essential features, the rules of definition should obviously include an indication of the methods by which it is possible to find the essential and not other features of the defined concept.

In many cases, listing all of these features is too lengthy. There is a way to define a concept without a detailed listing of all its essential features. This method consists in the fact that, firstly, the closest *genus* to which this concept is defined belongs, and, secondly, a special characteristic (or characters) is indicated by which this concept, as a *species*, differs from all other species the specified kind.

This attribute is called "species difference" or "species—forming difference"; the very indicated method of determination as a whole is called the definition "through the nearest genus and through species difference".

The definition through the closest genus and species—forming difference is applied everywhere where the previous study found that the concept being defined is the concept of an object belonging to one of the species of a certain kind. In other words, the definition of this type applies to concepts that are part of the system of relations of a species to the genus and vice versa. These are many concepts of mathematical, physicochemical, and biological sciences. Thus, the *reflex* is defined in biology as "indispensable natural reaction of the organism to the foreign agent, which is carried out using certain of the nervous system," [1].

Definition is a definition through the nearest genus and species—forming difference. According to this definition, the closest genus for the concept of reflex is the genus of the body's reactions to an external agent. Reflex is one of the types of such reactions. But the definition does not only indicate the closest genus to which the defined concept belongs. The definition also indicates by what features the concept being defined differs as a species from other species of the same genus. Belonging to the kind of reactions of the body to an external agent, the reflex differs from other reactions of the body in that it is 1) an *indispensable* reaction , 2) a *regular* reaction, and 3) a reaction *carried out using a certain part of the nervous system*. Together, these three characteristics make up a species—forming difference, that is, how the reflex, as a *special* kind of reaction of an organism to an external agent, differs from *other* species of the same genus.

Genetic Definition

§ 8. Definition through the closest genus and species—forming difference assumes that the concept being defined is the concept of an object that 1) has already arisen and exists and which is connected by a certain relation of belonging to another class of objects, enclosing it in itself as a genus encloses a species. In this case, the method of occurrence of the subject is not noted in the definition itself.

But the definition can consider the subject and the method of its origin or formation. At the same time, the signs of the content of the concept, which are listed or indicated as already existing in the usual definition, are here considered to be due to the very way the object arose.

So, for example, a circle can be defined as a figure resulting from the rotation of a line segment around one of its ends in a plane.

Definitions of this type are called *genetic* from the word “genesis”, meaning “emergence”.

§ 9. Genetic determinations indicate such a way of origin or formation of an object, which seems as always possible. This is the definition of a circle just given. *Every* circle can be thought of as having arisen according to the method indicated in this definition.

Genetic definitions are based on the fact that, pointing to a possible way of forming or producing an object, these definitions thereby also indicate *the properties of* the object thus created.

Concept Restriction

§ 10. The inclusion of the concept of a new feature usually leads to the fact that the scope of the concept is narrowed, limited. But if a new feature included in the content of a concept is not one of the essential ones, but is derived from the essential ones, then the addition of such a characteristic does not change the scope of the concept. So, for example, if we add the sign of equality of diagonals to the number of essential features of the concept of "square"—to rectangularity and equilateralness, then from this addition the scope of the concept of "square" will not be more or less. In this case, the scope of the concept will not change, since its content has not changed. Indeed, the feature we added is a new feature, but not essential, since it can be deduced as a consequence of the previously established essential features of the concept of "square".

On the contrary, if a new feature added to the content of a concept does not belong to all objects conceivable in the scope of the concept, then the addition of such a feature leads to the fact that the scope of the concept narrows. So, if we add to the number of plant characteristics the sign of reproduction through spores, then we will narrow the scope of the concept of a plant imaginable in this case, limiting it to spore plants and excluding flowering plants from it. The logical operation, which consists in adding to the content of the concept of a new feature, the presence of which in the content of the concept narrows its scope, is called a *restriction* concepts. The limitation is based on the relation between content and volume, already explained above in chapter III (§28 and 29). By virtue of this relationship in terms that are related to each other in terms of genus and species, the addition of new species characteristics always reduces the volume of the genus, i.e., the number of objects in which the properties expressed by species characteristics can actually occur.

§ 11. The logical concepts of genus and species are relative concepts. A concept considered as a species in relation to some generic concept can, in turn, be considered as a genus in relation to another concept. For example, the concept of “officer” is a species in relation to the generic concept of “commander”, but the same concept of “officer” is a genus in relation to the concept of “lieutenant”.

It often happens that the restriction of a concept, i.e., the transition from genus to species, the discretion in this form of a new genus, the transition from it to a new species, etc., can last a very long time, covering a long chain of concepts. At the same time, with each such transition, the volume of each next type will become more and more narrow. Continuing the generalization operation for a sufficiently long time, we can get at the end such a volume that consists of one single subject. So, moving from the generic concept of “Russian” to the generic concept of “Russian scientist”, we get the concept of a smaller volume: there are fewer Russian scientists than Russian people. Further, we can consider the species concept “Russian scientist” in turn as a generic one. Then the concept with respect to it will be, for example, the concept of “a Russian scientist of the 18th century”. The scope of this concept will be even smaller, than the scope of the concept of "Russian scientist." Finally, we can also consider the species concept "Russian scientist of the eighteenth century" as a generic one with respect to the concept of "the greatest Russian scientist of the eighteenth century". In this last concept, the volume will already consist of one single person; this face will, of course, Lomonosov.

A concept whose volume is equal to unity, obviously, can no longer be subject to further restriction. Such a concept is called a concept not about a *species*, but about an *individual* (from the Latin word "individuum" meaning "indivisible").

On the contrary, excluding its species attribute from the concept, we expand the scope of this concept.

GeneraliSation of the Concept

§ 12. The logical operation by which, as a result of the exclusion of a species characteristic, a concept of a broader scope is obtained is called a *generalization of the* concept. This name notes that in the end we get a *more general* concept compared to what was considered before the exclusion of the species trait.

Due to the relativity of the concepts of logical genus and species, a generic concept can in turn be considered as a species in relation to its generic concept. For example, the concept of "pedagogical institute" is generic in relation to the concept of "pedagogical institute of foreign languages." But the same concept of "pedagogical institute" is at the same time a species in relation to the concept of "institute".

In order to turn this specific concept into a generic one, it is necessary to exclude from its content that essential characteristic, which is a species difference. Such an exception is called generalization.

In many cases, the generalization process can cover a very long series of concepts. With each new generalization, the scope of the concept resulting from the generalization will become ever wider. So, the scope of the concept of “pedagogical institute” is wider than the scope of the concept of “pedagogical institute of foreign languages”, the scope of the concept of “institute”, in turn, is broader than the scope of the concept of “pedagogical institute,” the scope of the concept of “higher education institution” is even broader than the scope of the concept of “institute” , the scope of the concept of an educational institution is even broader than the concept of a higher education institution, and, finally, the scope of the concept of an educational institution is wider than the scope of the concept of an educational institution.

When summarizing, the entire volume of each preceding concept is entirely contained within the volume of each subsequent: all pedagogical institutes of foreign languages are among pedagogical institutes, all pedagogical institutes are among institutes, all institutes are among higher educational institutions, all higher educational institutions are among educational institutions and, finally, all educational institutions —in the number of institutions.

Going down by means of restriction from the steps of the *genus* to the steps of the *species* included in this genus , we finally arrive at the individual.

In the content of the concept of an individual, such a wealth of certain signs is thought that in their combination these signs can belong to only one object.

On the contrary, rising through generalization from the steps of species to the steps of childbirth embracing these species, we finally arrive at generic concepts that are so extensive in volume that any conceivable object can be included in their volume — regardless of what certain signs are thought of its contents. Such, for example, is the concept of “object.” Precisely because in the content of this concept no signs can be thought of in a special way, the scope of this concept is so wide that any conceivable object can be summed up under the concept of “object”.

But precisely because of their extreme generality and the indefiniteness of the signs that are conceivable in them (these signs can be any), concepts like “object” can hardly be further generalised.

Separation of Concepts

§ 13. In the content of many concepts, we can find such an essential feature that can be *changed* according to a certain principle or rule. For example, in the content of the concept of “angle”, a sign can be changed that expresses its relation to a right angle. Every given angle has a known magnitude, and therefore in the concept of every angle there is a sign of a known magnitude of this angle. But we can imagine this quantity changing relative to the right angle.

Then in some corners this value will be less than a right angle, in others — equal to a straight line and in third — more than a straight line.

It is quite obvious that for every change in the sign in the content of the concept in all three of these cases there will correspond a certain part of the volume of the concept of “angle”. One part of this volume will be taken by sharp angles, the other by straight lines and the third by blunt ones. And since other cases of changing the angle are not supposed, it is obvious that with such a change in the sign of the angle, we will divide the entire volume of the concept of angle into only three parts.

Moreover, each part of the volume will correspond to one of three possible cases of a change in the value of the angle, and all three parts of the volume in their sum will exhaust the *entire* volume of the concept of “angle”.

The logical method by which we divide all this volume into parts, or into types, is called the *division of the* concept.

The concept, the volume of which is clarified through division, is called “ *divisible* .” Types or species concepts into which the volume of the dividend is divided are called *members of the division* .

§ 14. The volume of one and the same generic concept can be divided into species in more than one single way. What kind of species will be obtained as a result of the division of the concept depends on the basis by which the division itself is performed. So, the scope of the concept of a “triangle” can be divided into types in different ways — depending on whether we will consider the differences between the triangles in terms of their angles or relative sizes of sides.

In the first case, guided by differences in the magnitude of the *angles*, we find that the entire volume of the concept of a “triangle” is divided into species volumes of rectangular, acute—angled and obtuse—angled triangles. In the second case, taking into account the relative size of the *sides* , we find that the same volume of the concept of a “triangle” is divided into species volumes of versatile, isosceles and equilateral triangles.

A sign (or group of signs), by changing which we can divide the volume of the generic concept into species, is called the *basis of division*.

§ 15. The separation of concepts plays an important role in logical thinking. Especially great is its role in science and scientific thinking. Separation—if done correctly—*firstly*, finding out exactly the scope of the concept, reveals the relationship between species belonging to the same genus, and the relationship between subspecies of each species.

Secondly, the separation of the volume of the concept is used, as we will see below, as an integral part of some evidence.

Thirdly, separation is constantly applied — both in practical life and in science—in *classification*. Classification is the distribution of all objects of a known class into categories, in which the transition from one category to another is carried out systematically, according to a certain rule, each object of the class falls into one of one of the categories of the class, and the sum of all objects in all categories is exactly equal the sum of all class items.

To carry out all these tasks, the division must be correct, and for this it is necessary to strictly fulfill the following three necessary conditions.

§ 16. *The first* condition for correct division is that each given division should be made on the same basis. Although the volume of one and the same concept can be divided into types, generally speaking, in different ways, i.e., for different reasons, however, in *each individual*. In the case of division should be made only on one basis. So, the scope of the concept of a “triangle” can be divided into types either by the magnitude of the angles, or by the relative magnitude of the sides. But it is impossible, having started the division of triangles on the basis of the magnitude of the angles and not finished this division, suddenly jump to the division on the basis of the relative size of the sides and continue the division on this — is another — basis. It is also impossible to divide people into thin, thick and stupid or to divide pictures into historical, everyday, landscape and watercolour. In all these examples, the same error: the basis of division is not the same. Since in each of these objects the division is not performed on the same basis, we cannot be sure that we really completely divided the entire volume of the divisible genus into species, nor that.

§ 17. *The second* condition for the correct division consists in the requirement that the sum of the objects in all species obtained by division is exactly equal to the sum of the objects of the divided concept, that is, the sum of the species should *exhaust the* entire volume of the generic concept. If this rule is violated, the division is either too narrow or too wide. So, dividing the scope of the concept of “forest” into types of coniferous and deciduous forests, we obviously get too narrow a division, since besides coniferous and deciduous forests there are also mixed forests, that is, coniferous—deciduous. Here, the sum of volumes of species concepts is obviously less than the volume of the dividend; it does not exhaust the full volume of the dividend and does not contain all of its species.

On the contrary, dividing the scope of the concept of “stars” into the types of setting stars, non—entering stars and planets, we will obviously get too broad a division, since planets are not stars. Here, due to the inclusion of planets in the number of stars, the sum of the volumes of species concepts turned out to be larger in comparison with the volume of the divisible concept.

§ 18. The *third* condition for the correct division consists in the requirement that members of the division are mutually exclusive. This means that, as a result of the division, each object included in the volume of the divisible generic concept should enter the volume of any one of the species concepts, but should not immediately enter two or more types. In other words, separation as a result of division of a concept consists of subordinate concepts, i.e., of species subordinate to the divisible as a genus.

An example of a violation of this rule will be the division of rivers into navigable, non—navigable, rafting and rapids. In this division, some members (rafting rivers and rapids, non—navigable rivers and rapids) do not exclude each other, are not species that exclude each other. This means that, by dividing and moving from one species concept to another, for example, from the concept of non—navigable rivers to the concept of rapids, we introduced some of the objects that were already part of the previous one into this structure.

§ 19. Of all the possible division errors, the most significant is the error consisting in a derogation from the basis accepted for division.

And indeed: the correctness of the division of the volume of the divisible generic concept into subordinate species depends on how consistently and systematically we change the trait that forms the basis of division. Since any part of the volume resulting from the division of a generic concept is determined by a known change in a feature that is part of *the* concept, any mistake in deciding on the principle by which this feature should be changed should lead to an error in the division results. The division in which this error is made is called *inconsistent* or *cross*. The last name shows that in the case of such a division, the same objects are simultaneously included in different types. Who, for example, will divide people into brave, cowardly and cautious, must agree that some brave and some cowardly people may be cautious.

§ 20. Division free from logical errors is far from an easy task. It is easily feasible if the trait by which the genus is distributed among species is so accurate and distinct that all possible changes are easily visible and can be established in an exhaustive way.

In complex objects and phenomena of nature and society, it is often extremely difficult to find and distinguish such a change in signs that would put a group of objects that are a known variety, *outside of* any other group of objects possessing another species of the same species. You can, for example, divide the scope of the concept of "military aircraft" into types depending on the *purpose of the* aircraft. Then the scope of the concept of "military aircraft" will be divided into types: 1) scouts; 2) fighters; 3) bombers; 4) attack aircraft; and 5) transport aircraft.

However, this separation does not take into account the fact that one and the same aircraft can fulfill two purposes simultaneously: for example, it can be used both for attack and for bombing at the same time. But this means that there may be such an attack aircraft, which, entering the category of attack aircraft, is simultaneously included in the category of bombers.

Dichotomy

§ 21. There is a division technique free from errors encountered in other division methods. This technique is called a "dichotomy," that is, division into two.

In the examples we examined earlier, the basis for the division was taken as a possible change in the sign in some certain respect. In a *dichotomy, the* basis for the division is not a change in a sign, but the mere presence or absence of a known sign. In other words, a dichotomy is the division of the volume of a given concept into two *contradictory* to each other, specific concepts, i.e., to two such concepts, of which one represents the negation of the signs of the other. Such, for example, is the division of people into floating and non—floating or plants into spore and non—spore. It often happens that dichotomy obtained by dividing the volume of a concept can be continued. This happens when a negative concept, which is one of the species of a divided genus, in turn, is a complex concept that allows further division into two.

Fig. 11

For example, by dividing the scope of the concept of “scientists” into conflicting species concepts of “mathematician” and “non—mathematics”, we can, in turn, divide the volume of the negative concept of “non—mathematicians” into conflicting species concepts of “natural” and “non—natural” “. In turn, the volume of the negative concept of “non—natural” can be divided into conflicting specific concepts of “historian” and “non—natural”, etc. (see Fig. 11).

Such a division can continue until we reach a specific concept to which the concept of the subject we are studying should be related. Dichotomy is used as an auxiliary tool for orientation, for example, in botany — in the compilation of the so—called plant identifiers. In these handbooks, a long chain of dichotomous divisions ultimately leads to the determination of the species to which the plant in question belongs.

§ 22. The advantage of a dichotomy is that it does not violate the above division rules. In fact, with a dichotomy, species resulting from division turn out to be concepts that *contradict* each other. But the volumes of conflicting concepts cannot be overlapping: an object cannot be found that would simultaneously be included in the volume of the species concept and in the volume of the concept that *contradicts* this species concept. In other words, dichotomy cannot be confused. If the plants are divided into spore and non—spore, it is clear that the plant under investigation should be either among the spore or non—spore. It is also clear that if it is among the disputed, then it cannot at the same time be among the disputed.

With a dichotomy, the sum of the species volumes obtained as a result of the division completely exhausts the volume of the dividend; it cannot be neither more nor less than this volume. Therefore, the separation carried out according to the rules of dichotomy can never be either too wide or too narrow. If the genus of vertebrate animals is divided into winged and winged species, it is clear that, apart from these two species, no third one is possible, which would make up part of the volume of the divided concept.

§ 23. With all these advantages, a dichotomy has its drawbacks. *Firstly*, dividing the volume into conflicting concepts leaves the part of the volume of the given dividend that is expressed by a negative concept too uncertain. If I know only about vertebrates that they are either winged or winged, then the second, negative type of "winged" is a too general, too vague concept. Such a concept, as always happens with conflicting concepts, implies only signs that should be denied in the content of the species concept.

§ 24. *Secondly*, Continuing the dichotomous division, we usually finally arrive at a field with respect to which it is very difficult to decide which type—positive or negative, which contradicts it—will belong to the concept of this subject. So, the difference between animal and plant is striking if we are dealing with higher forms of animal and plant life. No one will be at a loss to say that, for example, a tiger is an animal, and an oak is a plant. But where it is necessary to deal with microorganisms, even specialist scientists often found it difficult to answer the question whether this species should be assigned to animals or plants. In such species, the usual distinctive features of the animal and the characteristics of the plant are often present.

It is therefore not surprising that the role of a dichotomy in the scientific classification of objects and phenomena is very limited; a dichotomy is usually used only as a preliminary auxiliary orientation technique.

Tasks

1. Define the concepts: “circle”; “newspaper”; “unconditioned reflex”; “Island”, “isthmus”, “canal” (in a geographical sense); “Column” (in the military sense); “Mountain”, “hill”; “Bald spot”; “Artist”, “actor”; Iambic; “ Sine of the angle”; “friction”; “barometer”; “revolution”; “cone”; “constitution”; “Berry”, “fruit”; “axis”; “psychology”; “Sign”.

2. Check the correctness of the following definitions and in cases when these definitions turn out to be incorrect, explain which rule of definition is violated in them: “day is the time interval between sunrise and sunset”; “Cylinder — a body formed by the rotation of a rectangle around one of its sides”; “A whale is not a fish”; “Inspiration—the liveliest disposition of the soul to the perception of impressions and to think about them”; “Debut—performance of the artist in front of the public”; “Pie—a boat of the Indians, hollowed out of a tree trunk and driven by an oar”; “Champion is a winner in the competition”; “An exam is a test of a student in any subject”; “Poster—a notice posted in public places about some spectacle”; “Parallel lines— lines that do not intersect at any continuation”; “Liberal is a man of liberal convictions.”

3. Perform the restriction on the following concepts: “order”, “aviation”; “writer”; “the officer”; “geometry”; “engine”; “Flat figure” (in the geometric sense); “Commander”; “body”; “mushroom”; “dance”; “newspaper”; “Russian”; “doctor”; “liquid”; “fat”; “Grain crops”; “picture”.

4. Perform the action of division on the following concepts: “artillery gun”; “Ways of communication”; “planet”; “newspaper”; “Quadrangle”; “Conical section”; “plant”; “clock”; “heating”; “Climate zone”; “fish”; “Railway track”; “Oils”; “engine”; “school”; “vertebrate”; “machine gun”; “General”; “curve”; “loan”.

5. Check the correctness of the following partitions and in cases where the partitions turn out to be incorrect, explain what the error made in them consists of: “cars are passenger, freight, postal, sleeping, reserved seats and black cards”; ”Corners are adjacent, vertical and straight”; ”Volcanoes are active and extinct”; ”Landings are sea and air”; ”Sciences are divided into mathematical, natural, medical and social”; ”Poems are epic, historical, dramatic, lyrical and romantic”; ”Air bombs are divided into high—explosive, incendiary and deep”; ”Communication lines can be land, underground, water, air and interplanetary”; ”Geographical maps are physical, meteorological, economic, political, administrative and maps of communication lines”; ”Teeth are front, upper, lower, incisors, fangs, milk and wisdom teeth “; ”Stars are divided into constants and variables, entering and not entering”; ”There are children’s guns, such as Monte Cristo, hunting, fighting, machine guns, anti—tank and machine guns”; ”Singing is solo, chamber and choral.”

6. Perform a dichotomous division of the following concepts: “officer”; ”musician”; ”book”; ”city”; ”Roads”; ”Substances”.

CHAPTER V. JUDGMENT AND ITS COMPOSITION. TYPES OF JUDGEMENT

The Composition of the Judgment. Subject and Predicate

§ 1. In logical thinking, a concept usually does not occur on its own, but as part of a *judgment* in connection with other concepts that are part of the judgment.

The relation of a concept to a judgment is similar to the relation of a single *word* to a *sentence* . Considering a sentence, we distinguish in it individual words—individual parts of speech. But we usually speak not in separate words, but in whole sentences.

The situation is similar with logical thinking. We think not with separate concepts, but with whole judgments. Only by analysing the composition of the judgment, we begin to highlight the concepts included in this judgment.

We already know (chapter III, § 3—4) that in judgment can be distinguished: 1) the subject, or the thought of a certain subject; 2) a predicate, or the idea of a certain part of the content of the subject, which we consider in this proposition; 3) the thought of the relationship between the subject and the selected part of its content. All these thoughts are parts of judgment and are called concepts.

So, in the judgment "heroism is valour" we can distinguish: 1) the concept of "heroism", 2) the concept of "valour" and 3) the concept of the relationship between them. This attitude consists in the fact that heroism contains all the essential signs of valour, and therefore contains the *basis* for classifying it as valour. In other words, the relation between the subject and the predicate in this example is the relation of belonging; heroism *belongs* to the number of valour.

However, all these concepts that we highlighted in this proposition exist in it not *separately* from each other: they get a logical meaning only in the whole *proposition*.

There are times when the subject of our thought, apparently, is a *separate* a concept taken independently of judgment. But even in these cases, the concept itself is the result of previous judgments. So, I can think of the concept of “heroism” and regardless of judgment. But then this concept itself is the result of judgments and a replacement for previously formed judgments. I can think of this concept separately only because even before that I thought of a number of judgments, for example: “there are acts of higher courage, courage, steadfastness, devotion to duty, manifested in the struggle against difficulties or in the fight against the enemy. ”Such actions are rightly called heroic, and the behaviour of a person who has committed such acts is called heroism, etc.”

That the concept makes sense only as a result of judgments and only in connection with judgments is evident from the following. A concept that we cannot expand into judgment has no logical meaning for us. A student who has never studied astronomy cannot associate any thought that is clear to him with a concept such as “ecliptic.” For him, this is not even a concept, but simply an unfamiliar word. It is unfamiliar because the concept expressed by this word has never been encountered by him in any sentences known to him. Only after the teacher clarifies that the ecliptic is called a large circle in the celestial sphere, along which there is a visible annual movement of the sun between the stars, the word “ecliptic” will become a concept for the student. But it became a concept only because the teacher revealed the meaning of the term “ecliptic” through *judgment*.

Judgment is the main form of logical thinking. As already noted, all scientific truth is expressed in the form of judgment. Not only firmly grounded and verified true thought, but even a simple opinion or conjecture is expressed in the form of a judgment. Even an incorrect, erroneous statement about an object takes the form of a judgment.

§ 2. In the chapter on the concept, we already became acquainted with the members of the judgment — with the "subject" and "predicate". Let us consider in more detail their logical function in the judgment and the possible types of relations between them.

Subject judgments are the thought of some subject. This subject can be either truly existing, or one that is thought to exist. In the judgment "mountains on the moon often resemble circuses", the subject of the judgment will be the concept of "mountains on the moon". Here, this concept denotes an object that actually exists. Such mountains are clearly visible through a telescope. In the judgment "The beautiful Vasilisa has transformed from a frog to a princess", the subject of the judgment will be the idea of the beautiful Vasilisa. This thought is a thought about an object that does not exist, but only imagined, that is, about an object that exists only for thought, but not as an object of the real world.

§ 3. Although the subject of judgment is always the thought of a subject, the subject of judgment and the subject of judgment are not the same.

First *thing.* Thought exists or is thought to exist by itself. An object exists even when no one thinks about it. Mountains on the moon existed before Galileo first brought his telescope to the moon and — the first of the people—saw these mountains. In its existence, the *object of* thought does not depend on whether someone thinks about it or not and whether they think about it correctly or not.

Secondly, the number of properties and relations belonging *to the subject itself* is incomparably richer than that part of the content, or those signs that we think in the *concept* about this subject. In any concept, only a part of the attributes of an object is always thought of. Even from among the essential features of an object, a concept distinguishes only one part or group corresponding to the point of view from which we consider the object in this concept. The concept always reflects only some aspects of the subject. Such a concept, which would reflect in itself in each individual case of its application *completely* all the signs belonging to the subject, does not exist in any thinking. Only in the endless progress of cognition, i.e., provided that the cognitive process is considered as a *whole* , can one say about a concept that it reflects *all* aspects of the subject.

Thirdly, the attributes belonging to the subject matter constitute the *basis* those signs that can be highlighted or marked by the thought in the concept of the subject. Although in the concept of thought only a part of the attributes of an object is distinguished, it does not find this part arbitrarily. She finds her in the subject itself.

The difference between the subject of judgment and the subject of judgment is necessary in order to correctly imagine the relationship between objects of thought and logical forms of thought about objects. The subject of judgment is always the thought of the subject of judgment. In each given judgment, the subject cannot be a thought that exhausts all the features of the concept. The subject of judgment is the thought of only a certain part of the qualities, properties and relations belonging to the subject.

§ 4. Thus, the *subject* judgments do not just indicate the subject. The subject of judgment is a thought—in the content of the subject has not yet been disclosed—about a certain part of the attributes of the subject. So, in the judgment "bamboos are cereals" the subject of judgment is the concept of "bamboo". This concept does not just indicate an object, but embodies the idea of some essential features of this object.

But, this thought would remain limited by what is already thought in the content of the subject of the judgment, if the thought of the subject had not received further definition in the *predicate of* judgment.

The predicate more fully, more precisely, determines that part of the thought about an object that appears in the given judgment as a subject. Namely: to those attributes of an object that are already thought in the content of the *subject of* judgment, the predicate adds *new* signs. In our example, the predicate asserts that the subject of our thought possesses not only those attributes that belong to plants called "bamboos", but that it also possesses those attributes that belong to "cereals".

§ 5. If the judgment included only the subject and the predicate, then expanding the thought of the subject by adding to that part of the content of the subject that is already thought in the subject, another part of the content conceivable in the predicate, would be impossible. For signs that are conceivable in the concept of a predicate to be truly connected with signs that are conceivable in the concept of the subject, it is necessary that in the judgment, in addition to the thought of the subject and the predicate, the thought of the *relation* between them. Therefore, the third, along with the subject and the predicate, the logical member of the judgment is *attitude*. As long as I have a *separate* thought in my mind about bamboos and a *separate* thought about cereals, there is no judgment yet. But as soon as I understood the relation between bamboos and cereals as the *relation of* all bamboos to the class of cereals, a judgment arose. The concept of “bamboos” and the concept of “cereals” are no longer just conceived in it. It also thinks of the *relationship* between these concepts, which gives us the basis for including the entire class of bamboos in the class of cereals.

§ 6. The relationship between the subject and the predicate is the most important logical member of the judgment. Although the initial in any judgment is the thought of an object and although everything that can be thought of about an object has a basis in the object itself or in the relations of this object with other objects, the signs of the object are revealed to thought only through the relationship between the subject and the predicate. Only this attitude turns the idea of the subject and the predicate into a statement called a *proposition*.

§ 7. Since the relation between the subject and the predicate is the most important logical member of the judgment, the classification of *logical* types or types of judgments should be based on what types of logical relations between the subject and the predicate of judgment can be. Whatever the subject of judgment, from a logical point of view, he is always the thought of a part of the attributes of an object. Whatever the predicate of the judgment, from a logical point of view, it is always the thought of some other part of the attributes of the object. The relationship between the subject and the predicate, in contrast, can be different in its *logical* meaning.

Basic Logical Types of Judgments

§ 8. By the logical nature of the relationship existing between the subject and the predicate, judgments are divided into three large groups:

a) *The first* group, or the first type, is formed by judgments in which the relation of the subject and the predicate is conceived as *belonging to a property of an object*. An example of this type of judgment is the quinine bitter judgment. Judgment is, like any proposition, threefold. *The subject* in it is the concept of quinine, the *predicate* is the concept of the property of bitterness and *attitude* is the implied concept of the property of bitterness to quinine.

b) the *second* a group, or the second type, is formed by judgments in which the relation of the subject and the predicate is thought of as belonging, but not as a property of an object, but of an object—a class of objects or a class of objects—another class of objects. An example of judgments of this type is the proposition "quinine is a medicinal substance." The subject in it is the same concept of quinine, the predicate is the concept of medicinal substances and the relation is the concept of quinine belonging to the class of medicinal substances.

c) *Third* a group, or the third type, is formed by judgments in which the relation of the subject and the predicate is no longer thought of as a relation of belonging, but as the ratio of two objects (properties, qualities, etc.) in magnitude, in position in space, in sequence in time or simultaneity, in the intensity of qualities, in the connection of cause and action, in kinship, etc. Examples of this type of judgment are the following judgments: "quinine is better than wormwood", "A is equal to B", "Elbrus is higher than Mont Blanc", "Kazan lies to the east from Moscow"; "Leo Tolstoy was born later than Turgenev"; "Ivan is the brother of Peter," etc.

In judgments of belonging (a sign to an object, an object to a class of objects and one class of objects to another class of objects), the three—term composition of the judgment is not always explicitly expressed in the grammatical form of the sentence. In judgments on relations of the third kind (on relations in magnitude, in space, in time, by reason and action), the three—membered composition of a judgment in the grammatical form of a sentence is usually expressed by the words: “equal”, “more”, “less”, “earlier”, “At the same time”, “later”, “stronger”, “weaker”, etc.

§ 9. Since in *each* there are three members of the judgment: 1) the subject, 2) the predicate and 3) the relationship between the subject and the predicate, then the composition of the judgment can be schematically represented by the general formula. But since the relation between the subject and the predicate can express either the relation of belonging, or the relation of space, time, size, strength, causality, etc., the formula of judgment in these cases will not be the same.

A judgment of the type “quinine is bitter” and a judgment of the type “quinine is worse than wormwood” differ significantly in their logical nature.

Schemes, or formulas, judgments also differ depending on the logical nature of the judgment. The logical structure of membership judgments is expressed by the formula:

S—P

In this formula, S means the subject of judgment, P is its predicate, and the line between S and P is the relation of belonging. In this case, belonging can be either belonging to a property of an object, or belonging to a class of objects, or belonging to one class of objects to another class of objects.

The logical structure of judgments about the relations of space, time, magnitude, causality, etc. is expressed by the formula:

$$a \text{ R } b$$

The letter R—the initial letter of the word "relation"—means the relation here. Letters *a* and *b* mean objects of thought, between which the relation is considered. The logical importance of the distinction between the judgments of belonging and the judgments of the relations of space, time, magnitude, causality, etc., has been highlighted in the latest literature by *English* logicians, starting with Morgan, and *French* logicians, starting with Lachelier.

Judgments expressing relations of belonging of a property to an object and an object to a class of objects are hereinafter referred to as judgments of *belonging*. Judgments expressing the relations of space, time, magnitude, causality, kinship, etc., are hereinafter referred to as judgments about *relations* . The formula for judgments of membership will be S — P. The formula for propositions about relationships will be *a* R *b*.

Judgment as a form of Expression of Truth

§ 10. A statement can have a very different purpose in thinking. A statement can express *feelings* ("I love Borodin's music"), *desire* ("I want to write a letter to my father"), etc.

Unlike statements expressing a feeling or desire, a judgment is a logical form in which *truth* is expressed . Truth is the correspondence between the subject of thought and the thought of this subject. True is a judgment in which our thought reflects reality; *firstly* , it connects that which is connected in reality itself; *secondly* , in a true judgment, our thought connects objects and separates them *in this way* how they are connected and separated in reality itself.

A lie or delusion is a discrepancy between the subject of thought and the thought of this subject. A judgment is called false, in which our thought, *firstly* , connects what is not connected in reality itself, and shares what is connected in reality itself. So, the judgment "Mount Mont Blanc is in Asia" is false, because in this judgment our thought is trying to connect what is actually divided. In fact, Mont Blanc is in Europe, not in Asia; *secondly* , the judgment is false even in those cases when it, trying to connect what is connected in reality itself, connects objects *differently* from how they are connected in reality itself.

As a logical form of thinking, all judgment is the answer to a well—known question posed by our thought. Therefore, the first task when considering a judgment is to correctly understand which question this judgment answers or tries to answer. Having correctly understood the meaning of the question, we thereby obtain a well—known concept of the *subject of* judgment, and the concept of the *subject of* judgment shows in which area we should look for those properties or those relations that belong to the subject and which should be open in it for our thought and cognition.

§ 11. There are judgments that are enough to express so that everyone agrees with them. In such propositions, the relation between the subject and the predicate expressed by the proposition is obvious. This attitude correctly reflects in our thought the connection that exists between objects and their properties in reality itself.

But far from any true proposition, the relation between the subject and the predicate conceivable in this proposition will be obvious. Not in every judgment we immediately see that the relation between the subject and the predicate affirmed in it is such that what is the relation between the objects they represent in reality. When a teacher first informs students that the volume of the planet Jupiter is 1 312 times greater than the volume of the Earth, it is not immediately clear from one statement that this is the case. To convince of this, it is necessary to give proof.

The ability of a judgment to express truth depends on the justification of the judgment or on the means by which it is proved to be true. To prove many judgments (in particular, affiliation judgments), it matters: 1) whether the given judgment will be affirmative or negative, 2) general or particular, 3) expressing the truth under a certain condition or regardless of the condition, 4) expressing the necessary or only possible connection phenomena. In logic these differences between judgments are called differences in *quality*, in *quantity*, in *relation* and in *modality*.

Quality of Judgment

§ 12. The quality of a judgment is its affirmative or negative form. The judgment "all ferns are spore plants" is an *affirmative* judgment . In an affirmative judgment, our thought connects that which is thought to be connected in reality itself. On the contrary, the judgment "cereals are not spore plants" is *negative* judgment. In a negative judgment, our thought disconnects, or shares what is thought to be disconnected, or divided in reality itself. In other words, in an affirmative judgment, the signs conceivable in the concept of a predicate do not stand in relation to the opposite with the signs conceivable in the concept of the subject and indicating the subject of the judgment. The properties of spore plants do not stand in opposition to the properties of plants called ferns.

On the contrary, in a negative proposition, the signs conceivable in the predicate stand in relation to the opposite to the signs conceivable in the concept of the subject and indicating the subject of judgment. Thus, the properties of spore plants are in opposition to the properties of cereals.

§ 13. If we thought with separate judgments, then the question of the quality of judgment in each special case would be solved very simply. Any judgment with a negative utterance would be a negative judgment; any judgment without a negative utterance would be an affirmative proposition.

In reality, however, we do not think in separate judgments, but we connect judgments, compare them with each other, compare, distinguish, etc.

Comparison of judgments reveals that the quality of the judgment, i.e., its affirmation or negativity, is not that belongs to the judgment, *regardless* of its relationship to other judgments. One and the same judgment turns out to be affirmative and negative—depending on what other judgments we will consider it in relation to.

Let us have the judgment "Nikolaev did not defend the graduation project." If we considered this judgment as completely separate, we would say that in terms of quality this judgment is negative.

Let us now consider the same proposition regarding the other two propositions. First, we consider it with respect to the judgment "only persons who have defended a graduation project have the right to the title of engineer". It is quite obvious that in relation to this judgment the meaning of our judgment "Nikolaev did not defend the graduation project" will be negative. And indeed: since Nikolaev did not defend the graduation project, and in the second proposition we are talking only about those who defended the graduation project, it is clear that Nikolaev is *not* among the persons whom the subject of this judgment has in mind.

Now consider the relation of our judgment to another judgment. "Persons who have not defended a graduation project do not have the right to the title of engineer.” It is completely clear that in relation to the subject of this second judgment, the meaning of our judgment “Nikolaev did not defend the graduation project” will be affirmative. And indeed: the second proposition refers to persons who did not defend the thesis project. But Nikolaev, as can be seen from our judgment, belongs to precisely these individuals. Thus, our judgment “Nikolaev did not defend the graduation project” turned out to be both *negative* and *affirmative* . However, in both cases, this meaning will be *relatively* affirmative and *relatively* negative. This means that the quality of judgment is not its unconditional property. The quality of a judgment depends on which judgments and with which concepts in the judgments we compare the meaning of the statement, the quality of which we want to determine.

The complexity of the task increases due to some uncertainty and ambiguity of the language. Often, the same logical thought can be expressed both with the help of negation, and without the help of negation. The judgments “water is a complex body” and “water is not a simple body” express the same idea, but the negative sentence is not included in the first sentence, but in the second.

§ 14. Quality is a very important characteristic of judgment. Affirmation or denial is certainly thought of in all judgment. Any judgment is the answer to the question posed by the thought of the relationship between the subject and the predicate. But this answer will certainly consist either in the fact that thought will connect the concepts whose objects are interconnected, or, on the contrary, will separate these concepts if their objects do not stand in connection with each other.

Judgment Amount

§ 15. By the number of judgments there are *general* , *particular,* and *individual*. The judgment "all birds are warm—blooded animals" is an example of a *general* judgment. In such a judgment, the subject is a whole class of objects in their entirety.

The judgment "some birds fly away to the warm lands for the winter" is an example of a *private* judgment. In such a judgment, the subject is not the entire class of objects (in our example, the class of birds), but only some *part of* this class. Thus, the differences between judgments in terms of quantity are determined by whether the *whole* class or *part* is conceived through the subject of judgment, class. But whether a given judgment will be general or particular, the meaning of a statement always refers *to the entire* part of the volume of the concept of an object that is represented by the subject of the given judgment. In the judgment "some birds fly away to the warm lands for the winter", the meaning of the statement refers to the entire part of the volume of the concept of "bird" that is represented by the subject, that is, to *all* migratory birds, although migratory birds make up only part of the entire class of birds.

§ 16. Only general judgments express the truth of a well—known statement regarding the *whole class,* Items. Thus, Newton's law states that the gravity established by his formula applies to any two parts of matter, no matter in which part of the universe they are. The knowledge that a certain position will be true *for the whole class as a whole* has great cognitive and practical significance. If we know that the cognition of the connection of phenomena is the same within the *whole* areas of these phenomena, then our practical orientation, our ability to predict the course of these phenomena in cases that we have not yet experienced, achieve the greatest confidence. If Leverrier were not sure that the Newtonian law of gravity would remain valid beyond the orbit of Uranus — the last known solar system planet until 1846, then Leverrier would not have taken up his calculations and the planet Neptune would not have been discovered. This confidence was inspired by the *community of* Newtonian law, which was formulated as the law of *universal* gravitation.

A large number of laws of nature are expressed in the form of general judgments. The more general the form of the judgment, the larger the part of the class for which the given judgment will be true, the more accurate our prediction of the expected order of things and events becomes, the more successful and fruitful the practical action based on this prediction becomes.

In accordance with what has been said, the general judgment formula will be: “all S — P” (for membership judgments).

But not in all cases, the subject of general judgment has the word “everything” directly indicating that the subject represents the whole class. Often the word “all” is only implied, but the judgment from this does not cease to be general. So, the proposition “vertical angles are equal to each other” is, of course, a general proposition.

§ 17. Another role in cognition of judgment is *private*. So, the proposition “some plants are parasites”, of course, expands our knowledge. But, expanding our knowledge about the compatibility of characters in the concepts of “plant” and “parasite”, this judgment leaves the question completely unclear for *which part* plant properties of the plant are compatible with the properties of parasites. The meaning of this judgment is as follows: some, exactly which, part of the plants are parasites. This uncertainty means that with respect to any plant that we may encounter in our experience, we cannot have any confidence in advance whether it will be a parasite or not. This question requires special consideration in each case. On the contrary, when the law of nature is expressed by a general proposition, we firmly know that even beyond the limits of the facts we have examined so far, the general relation approved by the law remains valid. For example, a study of the so—called binary stars, i.e., stars projected on the arch of the sky extremely close to each other, showed that some of these stars are *orbital* stars: being physically connected with each other, these stars move in orbits around a common centre of attraction. Extending the universal Newtonian law of gravity to orbiting binary stars made it easy to calculate the masses of these stars using the very techniques that made it possible to determine the comparative masses of the planets of the solar system.

§ 18. The formula for private judgment (for membership judgments) is “some S — P”. In this formula, the word “some” is quite certain — because it means *part of the* class. But this word is not well defined — because it does not show *which* part of the class it represents. This word, firstly, can mean “only some,” that is, “not all,” and, secondly, it can mean “at least some,” that is, “not one single instance of this class” . If the word “some” has the meaning of “at least some”, then this does not exclude the possibility that all S, and not only part of them, will turn out to be R.

§ 19. In addition to general and particular judgments, they also differ in terms of quantity still *single* judgments. The formula of a single judgment (for judgments of membership): “this is S—P.”

Individual judgments, of course, cannot express truths that have the meaning of a general law or characterize the properties of a whole class of phenomena. What is expressed in these judgments is valid only with respect to one single subject. But this does not mean that a single judgment has no value for knowledge. How valuable a single judgment will be depends on the value that the *subject of* such a judgment has for knowledge . The judgment “this bird is a nightingale” has limited cognitive significance, since this nightingale is only an ordinary copy of the class of nightingales.

We have a different example of a single judgment in the judgment: “Alexander Vasilievich Suvorov is a great Russian commander who stormed the Turkish fortress of Izmail.” And this judgment in form is singular. But it no longer refers to the rank—and—file, nor does it matter to which class subject. This judgment refers to a person of extremely great importance in the history of our country. In its predicate judgment, this marks one of Suvorov’s greatest deeds. Such individual judgments play a large role in the composition of knowledge, especially in the historical sciences, as well as in the descriptive sciences: in descriptive astronomy, in geography, etc.

§ 20. Thus, the distinction between general, particular, and individual judgments cannot be understood as if only general judgments are of great value to knowledge, that private judgments are of less value, and single judgments are even less.

Each of these forms of judgment has its own value and its own area where it is mainly applied. There are tasks and questions for the solution of which particular and individual judgments are more suitable than general ones, or for the answer to which only private and only individual judgments are suitable.

If I want to show that the properties of the plant and the properties of the parasite can be compatible, then to solve this problem I do not need to prove that *all* plants are parasites: it is enough to make sure that *some* plants are parasites. An attempt to solve this problem, relying on a *general* judgment, would, on the contrary, lead to failure, since in reality not all plants, but only a part, are parasites.

And in the same way, if I want to write a biography of a major politician, commander, scientist, writer, etc., at every step I will have to make a number of *individual* judgments about him that cannot be replaced by private or general. Nevertheless, such individual judgments are important and completely irreplaceable: only they outline this particular person with all the special features of his character and activity, with all the events and deeds in which he participated.

§ 21. But all three forms of the amount of judgment not only have their own field of application. These forms, in addition, are not unconditionally separated from one another. They are interconnected, each suggests both others.

Thus, a general judgment cannot be thought of independently of the particular and the individual. To verify the truth of the general judgment, for example, that all cereals have inflorescences in the form of spikelets [1] , you must first know that "some cereals bloom in spikelets." I know that rye, wheat, millet, oats, i.e. *some* cereals bloom in spikelets. But I know that in addition to these types of cereals, there are others: corn, and bamboo, and rice. Having ascertained what other grains exist besides those considered, and making sure that all other cereals also spikelets, I have the right to express a general opinion: "All cereals spikelets".

Here we come to a general judgment from a particular one. Our thought makes such a transition at every step. And this is understandable: the general situation is usually not immediately visible. For millennia, people have seen how steam, cooling, turns into water. However, a lot of time passed, a huge work of observation, experience and thought was required, so that from knowledge of this fact people get to the knowledge that any gas can be turned into a liquid body. First found that *some* Gases can be converted to liquids under special conditions. At this stage, the generalization extended only to a part of the gases, while others still could not be transformed. Therefore, the judgment expressing the property of gases to liquefy into a liquid could only be *private* . And only later, when the experimental technique made it possible to achieve very low temperatures, it was found that *any* gas can become a liquid with sufficient cooling for it. At this stage, the generalization became complete, and the judgment " *all* gases are liquefied in a liquid", expressing its result, is general.

This way of turning an individual situation into a particular, and a particular into a general, many judgments pass. At every moment of the development of science in it there are such particular judgments that are in transition to general judgments; today such a judgment is still private, for a complete generalization there is not enough data, but tomorrow this data can be found, and a judgment from the private will become general.

The constantly existing possibility of transferring private judgment to general is reflected in a certain ambiguity of private judgment. We have already seen that a judgment of the type "some S — P" can have different meanings. It can be understood so that only part S — P, and the other part S — non—P. And it can be understood so that *at least* some S — P. In the latter case, there is a possibility that even *all* S will be R. This possibility is constantly available for many provisions of science that are on the way to full generalisation.

§ 22. But regardless of the possibility of the *transition of a* private judgment to a general, any general judgment presupposes particular and individual judgments. And this is true even with respect to the judgments of mathematics.

And indeed, even thinking the general propositions of mathematics, we do not think them unconditionally separate from the judgments of particular and individual. The generality of the theorem means that this theorem, being true with respect to a whole class of mathematical objects—figures, quantities, etc.—will be true for some part of this class and for an individual representative of the class. Since it is true that all equilateral triangles are equiangular, it must be true that some equilateral triangles are equiangular, and that this given equilateral triangle is equiangular.

But a single judgment is not conceived separately from the general. Although the judgment "this bird is a nightingale" is valid only with respect to this and no other bird, general judgments are assumed to be judgments about it. To identify this bird with a nightingale, I must have an accurate understanding of a number of essential properties common to *all* nightingales. A single judgment — on the *subject of a* statement — involves the assimilation of a whole series of general knowledge expressed through general judgments.

§ 23. The question of quality and the amount of judgment is of great importance in logical operations called inferences or conclusions, as well as in evidence. Given the importance of characterizing judgments in quality and quantity for judgments substantiating conclusions about ownership, logic has developed a notation system by which the quality and quantity of any judgment about membership are expressed in one letter. Judgments that are general in quantity and affirmative in quality (for example, "all liquids are elastic") are called affirmative and are denoted by the Latin letter A. Judgments particular in quantity and affirmative in quality (for example, "some metals are alloys") are called partly affirmative and are denoted by the Latin letter I. The letters A and I are the first and second vowels of the Latin verb "affirmo", meaning "I affirm.”

Types of Judgments in Relation

§ 24. We consider the following three judgments: 1) “seals are animal mammals”; 2) “if the lines *AB* and *CD* *are* parallel to each other separately of the third line *EF* , then *AB* and *CD* *are* parallel to each other”; 3) “corners are either obtuse, or straight, or sharp.” Ignoring the differences between these judgments in terms of quality and quantity, we consider what differences exist between them , *depending on the nature of the statement itself*.. In each of these three propositions, the nature of the statement is determined by the content of the subject matter. So, in the judgment “seals are animal mammals”, the basis for this statement is that animals called seals really have all the essential properties of mammals. Similarly, the basis for the judgment “if two lines are parallel to the third, then they are parallel to each other” is also the content of its subject, that is, the property of two lines parallel to the third: since such lines are parallel to each other, then wherever two lines parallel to each separately of the third, they will be parallel to each other. Finally, in the judgment “corners are either obtuse, or straight, or sharp”, the basis for this statement will also be that the class of objects called corners contains only these three types of angles.

But although, thus, in all three propositions the utterance is determined by the content and properties of the subject, the method of this conditionality turns out to be different in each case. In the judgment “seals are animal mammals”, the condition for utterance is those properties of the object that are actually present and found in its content. Therefore, in this judgment, the statement is *categorically* expressed , that is, without limitation by any conditions, other than those found in the very content of the subject.

On the contrary, in the complex proposition “if two lines are parallel to each other separately, then they are parallel to each other”, the statement in the statement is valid only under a certain condition, which is formulated immediately, in the statement itself. This condition is expressed by a judgment beginning with a conditional union “if”. In order to recognize the two data lines *AB* and *CD as* parallel to each other, here it is necessary — as a condition for the truth of the statement — to admit or accept that both of these lines *AB* and *CD are* parallel to the third line *EF* . Such a complex proposition in which the truth of the statement depends on the condition that is formulated in the proposition itself is called *conditional*, or *hypothetical*.

The general scheme of conditional judgments about membership will be the formula: “if A is B, then C is D”.

§ 25. Judgments *affirmative*—as they reveal signs of really belonging to the object, are other substantial benefits to knowledge than judgment *negative* , which only indicates what attributes do not belong to the subject. General judgments are applied in knowledge differently than *private* judgments. *Categorical* and *hypothetical* judgments also have different meanings for knowledge. Since categorical judgments affirm such properties of an object that are thought to be found in *the object itself*, then categorical judgments represent a different meaning for knowledge than *hypothetical* judgments in which the truth of the statement depends on the truth of the condition, not yet found in the subject itself, but only assumed and formulated in the judgment itself.

This does not mean, however, that hypothetical judgments have no value for knowledge. Hypothetical judgments play a large role in all sciences. In no real subject does there exist lines that are only of length. However, the mathematician assumes that the lines that he considers in his reasoning and evidence are precisely these. Assuming such lines, the mathematician further establishes what relations must be between these lines, once the condition is accepted that they are conceivable.

But in other sciences, the technique is widespread and of great importance, consisting in the fact that, assuming that the known conditions are fulfilled or are present, the scientist draws logical conclusions regarding everything that follows with logical necessity from the conditions he proposed. No one, for example, was present during the process of the appearance of stellar nebulae. But the astrophysicist, assuming the known mechanical and dynamic conditions of matter distributed in a known manner in space, then explores what processes would have developed there if such conditions were found to be present. Thus, valuable scientific conjectures arise about the possible course of development of stellar nebulae, the solar system, etc.

In all judgments and studies of this kind, the truth itself is not conditional, as such, but only that assumption, having made which we came to the establishment of the truth. That two lines *AB* and *CD*, parallel to each other separately from the third line *EF*, are parallel to each other, there is nothing conditional in this statement; it *necessarily* follows from the hypothesized condition — from the parallelism of each of the two given lines of the third. The question is only about the extent to which the hypothesized condition is reliable: it is likely or really. In categorical judgments, this question is not posed, since in these judgments the conditions of truth are thought of as found in the subject itself. Therefore, categorical judgments must be distinguishable from hypothetical ones.

§ 26. This does not mean, of course, that the mere form of categorical judgment, as such, already ensures the *truth of the* statement. In a categorical proposition, the condition of its truth is thought of as found in the subject itself. But "thinking" does not mean "really exists" in the subject. The mere *subjective* certainty that a certain condition is rooted in the subject itself is not yet sufficient evidence that the matter is really the way it is thought. The advantage of categorical judgment is indisputable only where the conditions of its truth are indeed found in the subject itself. If the speaker only seems to have found them in the subject, then the mere form of categorical judgment, taken by itself, will not ensure the truth of the statement.

§ 27. We have repeatedly noticed that between the logical meaning of judgment and its *grammatical* form is not always full compliance. This is the case in the case of distinguishing between types of judgments in relation. Since in categorical judgments the truth of a statement is also determined by well—known conditions existing and found in the subject itself, a categorical proposition can be expressed in the form of a complex sentence with a conditional subordinate clause starting with the "if" conjunction. So, the judgment "planets have a visible proper motion between stars" can also be expressed in the following form: "if a star is a planet, then it has a visible proper motion between stars". However, this similarity of the grammatical form of the sentence with the logical form of conditional proposition does not make the proposition taken in our example truly conditional or hypothetical.

§ 28. The third kind of relationship is represented by judgments *separation* (disjunctive). In a separation judgment regarding the subject of judgment, a number of predicates are expressed, the sum of which exhausts all kinds of the genus represented by the subject. Each of them, firstly, excludes all others and, secondly, applies to all others in such a way that a given predicate must be affirmed with respect to the subject if all other predicates are negated with respect to it. So, rectangularity excludes obtuse and acute—angledness. On the other hand, the angle must be recognized as straight if it is established that it is neither blunt nor sharp.

The peculiarity of a separation judgment is that this judgment simultaneously expresses our *knowledge* of the subject and *incomplete.* The insufficient nature of this knowledge. Indeed, the separation judgment leaves the question completely unclear which of all the possible predicates in this case should be affirmed regarding the subject. But at the same time, the separation judgment shows that predicates cannot all at once belong to the subject together: their connection is such that if one of them belongs to the subject, then all the others cannot belong to it.

The general scheme of a separation opinion on ownership will be the formula: "A is either B, or C, or D".

§ 29. The separative nature of the judgment, however, cannot be established on the basis of the mere presence in the sentence of the separation union "or," put between several predicates. In a truly dividing proposition, the predicates that can be attributed to the subject must mutually exclude each other. On the contrary, the grammatical union "or" does not necessarily express the incompatibility of the predicates. In the sentence "rivers are either navigable or non—navigable," the union "or" shares incompatible predicates. Therefore, the judgment expressed by this sentence will be divisive. But in the sentence "good workers are either talented or hardworking" the union "or" does not at all express the incompatibility of the predicates. A good employee can not only be both talented and hardworking, but the best employee will be the one which combines both of these qualities. Therefore, the judgment “good workers are either talented or hardworking” is not, of course, a dividing judgment.

But this is not enough. The union “or”, even when it has a separation value, is not necessary for a separation judgment. It can be replaced by a simple listing of the members of the division or the union “and”. So, the judgment “trees are deciduous and coniferous” is a separative judgment. In this proposition, the role of the separation union “or” is played by the union “and”. In exactly the same way, the proposition “sciences are natural, social” is a separative proposition. In it, the separation union “or” is replaced by a simple enumeration of predicates. As in any separative proposition, and in this proposition, every other predicate excludes all the others, and each predicate must belong to the subject if all other predicates do not belong to it.

Separating judgments, just as categorical and hypothetical, characterize judgment *in relation* . In all three cases of the relationship there is a well—known condition that determines the truth of the statement. It cannot be otherwise. True judgment reflects in thought what is in reality. But everything that exists in reality is always conditioned one way or another, depends on the conditions of the place, time, circumstances, etc. In the case of a *categorical* judgment, this condition is thought of as found in the subject itself and therefore is not formulated in the judgment itself. In the case of a hypothetical judgment, this condition is formulated in the judgments themselves or put forward by the thought itself and is thought as an assumption. In case of *separation* judgments the condition for utterance is an exhaustive separation of the entire volume of the subject and the exclusion by each of the possible predicates of all other predicates.

Separation judgment only determines the range of predicates that may belong to the subject, but does not indicate *which one* should be assigned to it. Therefore, in comparison with categorical and even conditional propositions, in which a definite, although differently justified predicate is conceived, a separative proposition is less definite.

Modality of judgments

§ 30. One of the most important properties of a judgment is its ability to express the need or only the likelihood of the relationship between the subject of the judgment and its predicate being confirmed by the judgment. These differences between judgments are called *modality* differences.

We consider two judgments: "In any flat triangle, the sum of its internal angles must be equal to two straight lines" and "Snow may fall in Moscow in May." These judgments are significantly different from each other and represent a completely different meaning for knowledge. In the first of them, the statement is thought to be *necessary* due to the subject, and the connection of the subject with the predicate is thought of as the *necessary* connection. In the second proposition, the statement is thought of as only probable, that is, as one which, although it does not conflict with the subject and the concept of it (the subject), is not only not necessary, but does not exclude the possibility of denial of the predicate relative to the subject. There may still be snowfall in Moscow in May, there is nothing improbable, incompatible with the weather of the month, which we call May. But in this event, which in itself is quite possible, there is nothing necessary: there may be snowfall in May, but it may not be.

Judgments of the first type, expressing the *need for* approval, due to the subject itself, are called reliable, or *apodictic* . Type 2 judgments expressing *probability* the statement being made and at the same time the absence of an indisputable necessity for him, that is, the possibility of the opposite statement, are called probable or *problematic*.

§ 31. The modality of judgment, that is, its belonging to the number of necessary or probable, expresses not only one *subjective* degree of confidence regarding the reliability of the statement. Necessity and probability, expressed by apodictic and problematic judgments, are based on objective reality, in the subject itself and in relation to other subjects. Necessity and probability do not depend on how the speaker realizes and experiences this reliability and this probability in his thought. On the contrary, the subjective way in which each person who thinks a given judgment sees its necessity or likelihood will be different in each special case.

Logic considers in judgments only those differences that are caused by relationships of necessity, or probability, independent of psychological states of greater or lesser confidence. However, the ability of apodictic judgment to express *the necessary* relationship between the concepts of subject and object cannot be discerned from a mere form of judgment. Thus, the false proposition “spiders *must be* insects” will not be apodictic, despite the presence of the word “necessary”, put before the predicate. The need for a relationship is seen only from the evidence. In this, the *modality of* judgment differs from *quality*, *quantity* and *attitude*, where the belonging of a judgment to an appropriate form is already seen from the form of judgment itself.

§ 32. From this point of view, apodictic judgments, as judgments of *necessity* should be put above judgments only probable or problematic. A statement in which the relation between the subject and the predicate is thought of as a *necessary* relation can become a completely reliable basis for a number of predictions and calculations. If I know that a body immersed in a liquid necessarily loses just as much weight as the liquid displaced by this body when it is immersed, then the need for this position can be used for appropriate calculations and actions decisively wherever the body is immersed in liquid.

On the contrary, in the case of a problematic judgment, the reliability of the prediction based on it or the practical calculation entirely depends on the *degree of probability expressed in the judgment* and in what ways this degree can be calculated or established. If it is known that in May in Moscow the climate possible snowfall, almost considered this circumstance falls, obviously, just depending on how large *the degree* of probability. If it is very large, then the farmer, gardener, gardener, railwayman must very seriously take this opportunity into account, prepare and implement some measures, refrain from others, etc. If it is negligible, then it is practically not taken into account.

In the case of probable judgments regarding each particular case, it remains unclear whether a possible event will occur in this case or not. Therefore, problematic judgments, being a certain form of knowledge, give for each individual case less definite knowledge than apodictic judgments.

But this does not mean that judgments about probability have no practical and cognitive value. No matter how small the accuracy with which a problematic judgment determines the possibility or probability of a known position, a known event, this probability cannot be ignored. In areas where strong earthquakes are possible, one has to reckon with their probability when planning villages, when developing types, materials of architectural structures, etc. An architect introduces details in his technical design that aim to increase the building's ability to withstand earthquakes, does not know reliably when exactly the next destructive earthquake occurs. But if he knows the degree of probability of this event—the average frequency, the maximum intensity of strong earthquakes that are possible in a given area.

§ 33. In mathematics, an extremely important in practical respect section of this science is developed — the calculus of probabilities. The calculus of probabilities plays a huge role in the sciences of nature, in the sciences of society, and in many branches and calculations of practical life. This value of the calculus of probabilities is understandable. It is based on the fact that only a very small part of all our judgments and truths is reliable. On many issues, including those of the greatest theoretical interest and the greatest practical importance, only those answers can be given that reveal the probability, but not the full reliability. But no matter how little value such knowledge seems to be compared with apodictic judgments, revealing the *necessary* the relations of objects and concepts, neither science nor practical activity can neglect problematic judgments and methods of calculating probabilities. Practical life often requires certain actions that must be performed— regardless of whether the conditions of these actions are reliable or only likely. Who would want to be satisfied only with completely reliable knowledge, he would be doomed to complete ignorance and practical inaction when solving a huge number of questions and practical tasks.

As the English philosopher Locke correctly noted, a person who would not want to eat until he received evidence that food nourishes him, and who would not want to move his finger until he unmistakably knows that the business ahead of him will surely be successful, — such a person would only have to sit quietly and die.

§ 34. Everything that exists for a well—known reason, and in this sense exists by necessity. However, the nature of the existence of everything that has a reason for its existence is twofold. Everything exists either in such a way that it could not in any way turn out to be non—existent, or it exists in such a way that it could under other conditions turn out to be non—

existent.

Consider from this point of view two judgments: 1) “Leo Tolstoy was born on August 28, 1828” and 2) “a square constructed on the hypotenuse of a right triangle equals the sum of the squares built on its legs”. And in the first and second propositions, the necessary relationship between the subject of the proposition and its predicate is conceived. Leo Tolstoy could not be born on a different day than the one on which he was born. Equally necessary is the relationship between subject and predicate in the second proposition. However, the nature of this need for both judgments is not the same. The necessity conceivable in the first judgment (“Leo Tolstoy was born on August 28, 1828”) is the necessity of one fact only. Therefore, there is no contradiction between what is thought in this judgment and the assumption that under different conditions Leo Tolstoy might not have been born on August 28, 1828.

On the contrary, in the second judgment—“a square built on the hypotenuse of a right triangle is equal to the sum of squares built on its legs”—the need for a conceivable relationship is such that the possibility of any other relationship is excluded.

Based on this difference in the nature of the need for judgments of the first kind (“Leo Tolstoy was born on August 28, 1828”), they are called *assertive*, i.e., simply stating a known fact (without denying the possibility of a contradicting fact). In contrast to assertive, apodictic judgments affirm such a relationship between the subject and the predicate that, as a fact, excludes the possibility of a fact contradicting it. This is our judgment “a square built on the hypotenuse of a right triangle is equal to the sum of the squares built on its legs.”

In assertive judgments, or judgments about available facts, the relation conceivable in them between the subject and the predicate is verified through simple perception or stating of the fact. Hence the name of such judgments is “assertive”—from the Latin verb “asserto” (“I assure”).

In apodictic judgments, or judgments in which the relation between the subject and the predicate conceivable in them excludes the possibility of a contradictory relation, the affirmed relation is verified by means of *evidence* clarifying the indispensable necessity of the conceivable relation. Hence the name of such judgments—“apodictic”—from the Greek word “apodeikis” (“proof”)

Tasks

1. Indicate the logical subject and logical predicate in the following propositions: “in July the days are getting shorter”; ”The enemy cannot stop the advance of the Red Army”; “Moscow is the capital of the Soviet Union”; “Long colonels, and serve recently”; “To expel the enemy from Russian land was the main task of Kutuzov in the war of 1812”; “Horsetail is a spore plant”; “Not a hut is red in corners, but red in cakes”; “Far from the sandpiper to Petrov’s day”; “Some birds build nests in the ground”; “There is no beast stronger than a cat”; “The outer corner of the triangle is equal to the sum of two inner ones that are not adjacent to it”; “The material for sculpting is wood, stone, metal, gypsum.”

2. In each pair of the following judgments, determine the quality of the second order of judgment—first without reference, and then with respect to the first judgment preceding it: “Persons suffering from color blindness cannot be drivers, railway guards, signalmen, etc.” and “Petrov does not suffer from color blindness”; ”In order to study at a music school, it is not necessary to have absolute hearing” and “Sergeyev does not have absolute hearing”; ”One who is not proficient in mathematics cannot be an astronomer” and “Krasnov received an excellent mathematical education.”

3. Determine the number of judgments: “man is a public animal”; ”One half of the globe is called the eastern hemisphere, the other — the western”; “Some trees (eucalyptus trees) do not give shade”; “there is safety in numbers”; “Conscience is a clawed beast scraping the heart”; “This person is always the cause of my terrible frustration”; “Astronomical instruments —precision instruments”; “Some stars periodically change in brightness”; “True heroes do not boast of their exploits”; “Blessed is he who visited this world in his fateful moments.”

4. Come up with two type I judgments and three type E judgments.

5. Determine the type of relationship in each of the following judgments: “as long as there is no agreement in the comrades, their work will not work out in the way”; “Bodies are simple and complex”; “If the enemy does not surrender, they destroy him”; “If the angles in the triangle are all equal to each other, then all its sides are equal to each other”; “Few birds and few insects live in dense spruce forests”; “Come—see”; “Give your heart free will lead into captivity”; “The source of movement may be either living force, or steam, or electricity, or the energy of falling water, or the energy of internal combustion”; “Some planets have satellites”; “Many great generals began their service with a private”; “Hunting is seasonal and throughout the year”; “If a person’s temperature rises above 37 degrees, this means that this person is sick”; “Centres are regional, regional, republican, union, all—Union.”

6. Determine the modality of judgment: “severe chills can be one of the signs of malaria”; “The area of the triangle is equal to half the product of the base to the height”; “The cause of the fire is the ignition of densely stacked raw hay”; “the sky is clear”; “Suvorov made a heroic campaign through the Alps”; “Frosts are possible in May”; “The capitalist structure of society necessarily creates contradictions in public life”; “The hero is not afraid of death”; “Perhaps he is right”; “Potatoes can produce a crop of twenty—one”; “Nothing can be fixed and unchanging in the world.”

CHAPTER *VI.* SUBJECT AND PREDICATE OF JUDGEMENT. DISTRIBUTION OF TERMS

The Relationship Between the Subject and the Predicate of Judgment

§ 1. To the signs that are thought in the concept of the subject, the judgment, firstly, adds new signs that are conceivable in the concept of a predicate. Secondly, the judgment reveals the relationship between signs conceivable in the concept of the subject, and signs conceivable in the concept of a predicate.

So, in the judgment "an anti—aircraft gun can fire direct fire at a ground target", the predicate does not simply repeat the signs that were thought in the subject. To the signs of the concept of the subject (anti—aircraft gun—a gun designed to shoot at an air target—at enemy aircraft), our judgment, firstly, added a new sign, conceivable in the concept of a predicate—the ability of an anti—aircraft gun to fire direct fire and at a ground target. Secondly, our judgment has established a relationship between signs that are conceivable in the subject and signs that are conceivable in the predicate. In this example, the ratio is the ratio is *a sign of belonging subject*: within the meaning of the judgment, AAAs *belongs* function or purpose to shoot point—blank range and ground target.

In judgments of this type, the content conceivable in the concept of a predicate reflects *part of the* signs conceivable in the concept of an object. An item called an anti—aircraft gun has a number of different attributes. Already the concept of the subject—"anti—aircraft gun"—reflects part of these signs—

the purpose of the anti—aircraft gun to strike at an air target. The concept of a predicate *expands* these signs, attaches to them a new part of the signs, except the one that was thought in the concept of the subject. The relation conceivable between the subject and the predicate connects the attributes conceivable in the predicate with the subject of judgment.

§ 2. The relation between the subject and the predicate is a necessary member of the judgment. And indeed: the knowledge of an object cannot be limited to the knowledge of the signs that we find in this subject without regard to other objects. Not a single object exists completely separate from the surrounding world. Each object is part of a wider whole than himself. Each object is in a known relationship to other objects: it is either larger or smaller, closer or farther, heavier or lighter, harder or softer, etc.

Therefore, the knowledge of the subject is not only the discretion of those signs that can be directly detected in the subject. Cognition seeks to consider also in what relation the object and its properties are to other objects and their properties. So, in the judgment “Moscow is more than Kiev”, one thinks of such a relationship between both of these cities in magnitude that could not be found directly in the subject, without its relation to other subjects of thought. Only by *comparing the* magnitude of Moscow and Kiev—two *different* cities—the ratio of the magnitude of Moscow to the magnitude of Kiev, which is conceived in the predicate of judgment, can be found. This attitude (the larger size of Moscow compared to Kiev) was not just extracted from the content of the concept of the subject of thought (about Moscow). This ratio was established after the size of Moscow was compared with the size of Kiev.

In all such judgments, the relation conceivable in the judgment is no longer just the thought of the attribute belonging to the subject. In judgments of this type, the *ratio of* two objects in magnitude is established.

But whether in a judgment it is thought that the attribute belongs to an object or the *relation of an* object to another object, for example, a relation in magnitude, in either case the judgment reveals the content *subject*. The relationship of the subject to another subject also characterizes the subject as well as signs that directly belong to the subject. In the broad sense of the word, the concept of "relation" is also a sign. The difference between the *attribute of ownership* and the property of the *relationship* consists only in the following. Thinking a sign, we focus our thought on the subject itself. Thinking attitude, we direct our thought to the connection of an object with other objects.

However, this difference is not unconditional. On the one hand, the relation is always the relation of the *subject* , and therefore there is also its attribute. On the other hand, a feature belonging to an item is revealed only after the relationship of the item to other items is clarified.

§ 3. From the foregoing it follows that the knowledge of an object depends on the knowledge of the relations in which the object itself is with other objects. And since knowledge is logically expressed in the form of a *judgment*, the judgment should reveal the relationship between the subject of thought and other objects. So it is in reality. Relationships are reflected in the judgment as the relationship between the concept of the subject and the concept of the predicate. Whatever the meaning of the judgment about the subject — whether it indicates that the attribute belongs to the subject itself, or whether the subject is related to other subjects — in either case, the judgment expresses the relationship between the concept of the subject and the concept of the predicate.

§ 4. Every relation between concepts in a judgment is first of all a relation *between content* of both concepts. In the judgment expressing that the attribute belongs to the subject ("quinine bitter"), the relationship between the essential features of the concept of "quinine" and the essential features of the concept of "bitter" is considered, that is, between the *content of the* subject and the *content of the* predicate. This judgment makes it clear that among the attributes of the subject there is a characteristic constituting the content of the predicate.

In a judgment of the type "quinine—a medicinal substance", which expresses the belonging of an *object* ("quinine") to a *class of objects* ("medicinal substances"), it is not *directly the* relation between the content of the concept of the subject and the content of the concept of the predicate, but the relationship between the *volumes* subject and predicate; the entire volume of the subject is thought of as being part of the volume of the predicate.

In a judgment like "quinine is better than wormwood", which expresses the *intensity ratio* (between the bitter taste of quinine and the bitter taste of wormwood), this relationship is thought of as the relationship between the *characteristics* that determine the taste of these two substances, i.e. as the relationship between the *content of the* concepts of the subject and the predicate.

§ 5. The relationship between the *volumes of the* subject and the predicate depends on the relationship between the content of the subject and the content of the predicate. If bitterness is a sign of quinine, then this means that quinine is one of the bitter substances, that is, the scope of the concept of "quinine" is part of a wider scope of the concept of "bitter substances".

§ 6. The relation between the volumes of the subject and the predicate is far from in any judgment constitutes the immediate subject of our thought. Consider, for example, the judgments: "quinine is better than wormwood", "Kazan lies east of Moscow", "the Battle of Austerlitz was earlier than Borodinsky," etc. The subject of all these judgments is not the relation of the property *belonging to an* object or object to the class of objects, but the relation between objects by *size* , by *places* in space, by *sequence* in time, etc. Of course, and in these judgments, the relations between the content of the subject and the content of the predicate substantiate certain relations between their volumes. If quinine is sweeter than wormwood, then this means that according to the intensity of bitterness, the volume of the concept "bitter like quinine", according to this judgment, is *outside the* scope of the concept "bitter like wormwood".

In judging the relationship between objects in terms of size, strength, space, time, comparative value, etc., the *difference* between the compared objects is conceived. Therefore, the relationship between the volumes of concepts will be the ratio of *turning off the* volume of one concept from the volume of another concept.

§ 7. In judgments about the belonging of a sign to an object, as well as in judgments about the relations of magnitude, space, time, etc., an analysis of the relationship between a subject and a predicate by volume is usually *not* performed. True, even in these judgments, the relation between the content of the concepts of the subject and the predicate can be deduced from the volumes between these concepts. If I know that quinine is bitter, *that is, that the property of bitterness belongs to quinine*, then I can say on this basis that the scope of the concept of "quinine" is included, as part, in the wider scope of the concept of "bitter". But it is quite obvious that the reduction of the judgment "quinine bitter" to the relation between the volumes of the concepts "quinine" and "bitter" does not answer that question, the answer to which is the judgment "quinine bitter". In this judgment, the question is not about which class of objects the quinine belongs to, but about the *property* that belongs to the quinine.

§ 8. On the contrary, the analysis of the relations between the volumes of concepts is successfully applied in the analysis of judgments about the relations of *belonging of an* object to a class of objects.

If the content of concepts that accurately outlines their scope has already been established, we have the right to continue to focus our attention on the relationship between volumes. This right is based on the fact that in judgments about the belonging of an object to a class of objects, any consideration of relations between *volumes of* concepts is based on a consideration of relations between *contents* which outlines the most volumes. In the judgments expressing the belonging of an object to the class of objects, the relationship between the volumes of the subject and the predicate is the very question to which these judgments answer. And indeed: in practical life and in science, one has to find out at every step whether a given species is part of a known genus or not. So, a botanist, studying a new plant species, must decide whether this species belongs to flowering plants or to spore plants.

Depending on the solution of these issues, the concept of the subject is included in or disconnected from the known class.

§ 9. Thus, according to the meaning, which for understanding the judgment is related between the *volumes of the* subject and the predicate, the judgments are divided into two groups. The first group includes, firstly, judgments about relationships and, secondly, judgments about the belonging of a sign to an object. In all the judgments of these two kinds, the consideration of the relations between the subject and the predicate usually does not go further than the consideration of the relations between the *content of* these concepts. In these propositions, although the relations between the *volumes of the* subject and the predicate can be deduced, they will not correspond to the question to which these propositions are the answer.

The second group consists of judgments about the relationship of *belonging of an* object to a class of objects or a class of objects to another class of objects. In these judgments, consideration of the relationship between the volumes of the subject and the predicate is not only *possible* (as it is possible in the judgments of the first group), but it is also *advisable* , since it corresponds to the question the answer to which is the judgment.

§ 10. Since in all true judgments about the belonging of an object to a class of objects, the relation between the *volumes of the* subject and the predicate exactly corresponds to the relation between the *content* of these concepts, one can consider the relationship between the concepts of the subject and the predicate not in content, but in volume. By doing so, we will not make a mistake if the judgment in question is true.

In judging whether a subject belongs to a class of objects, consideration of the relationship between the volumes of the subject and the predicate greatly simplifies the analysis of the judgment, since these relationships are extremely simple and can easily be represented by means of visual diagrams. Therefore, judgments about the belonging of an object to a class of objects are usually distinguished by logic from all judgments into a special group. In the judgments of this group, logic considers the relationship between the volumes of the subject and the predicate in all types of judgments that differ from each other in quantity and quality.

The Relationship Between the Volumes of the Subject and the Predicate in Judgments about the Belonging of an Object to a Class of Objects

§ 11. In affirmative judgments about the subject belonging to the class of objects (A), the volume of the subject is fully included in the volume of the predicate. So, in the judgment "all bamboos are cereals", the volume of the subject (the concept of "bamboo") is completely included in the volume of the predicate (the concept of "cereal").

But from the fact that the volume of the subject is completely included in the volume of the predicate, it is not yet clear *which* part of the volume of the predicate will be the volume of the subject. *Two* cases are possible here . First, the subject's volume may turn out to be just a *part* predicate volume. So, in the judgment "all bamboos are cereals", the volume of the subject enters into the volume of the predicate in this way. All bamboos are cereals, but all grains are not exhausted by bamboos. In addition to cereals — bamboo, there are other types of cereals: rice, corn, rye, wheat, oats, millet, etc.

In the case when the volume of the subject is entirely included in the volume of the predicate, but is only part of the volume of the predicate, the relationship between the concepts of the subject and the predicate can be represented by the following diagram (see Fig. 12).

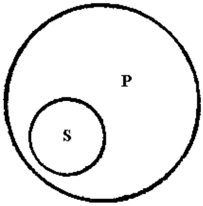


Fig. 12

Here, the large circle P means the volume of the predicate, the smaller circle S means the volume of the subject. It can be seen from the diagram that the entire volume S is entirely included in the volume P, but it is only part of the volume P, so, in addition to S, the volume of P can include, as its parts, volumes of other concepts. Secondly, the subject's volume may not be part of the volume P, but it may turn out to completely coincide with the volume P. Thus, in the judgment "all squares are equilateral rectangles", the subject's volume not only completely enters the predicate's volume, but also completely exhausts the predicate's volume: not only all squares are equilateral rectangles, but there are no other equilateral rectangles except the squares.

In the case when the volumes S and P completely coincide, the relationship between the concepts of the subject and the predicate can be represented by the following scheme (see Fig. 13).

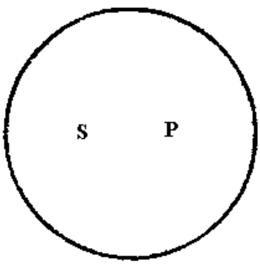


Fig. 13

Here, the volume S and the volume P are represented by the same circle SP, i.e., the concepts of subject and predicate turn out to be equivalent. It is not difficult to understand that in this latter case, judgment is nothing but a *definition of a* concept. To say that all squares are equilateral rectangles, this means to define the concept of “square”. And since in the correct definition the volume of the determined is exactly equal to the volume of the determining, it is not surprising that the volumes S and P turned out to be the same.

§ 12. In particular affirmative judgments about the belonging of an object to a class of objects, the volume of the subject is not completely included in the volume of the predicate, but only in some part of it. So, in the judgment “some mathematicians were astronomers”, the volume of the subject (the concept of “mathematics”) is included in the volume of the predicate (the concept of “astronomers”) only in some part: not all mathematicians, but only some mathematicians were astronomers.

Partial belonging of the subject’s volume to the predicate volume is of two types.

The first form is formed by judgments in which the concepts of subject and predicate are concepts that *intersect* . This is the judgment “some mathematicians were astronomers.” For judgments of this kind, the diagram representing the relationship between the volumes of the subject and the predicate is the same as the diagram for intersecting concepts (see Fig. 14).

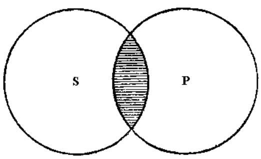


Fig. 14

It can be seen from the diagram that some part of the volume S is included in the volume P. The common part of their surface in both circles, hatched in the figure, represents that part of the subject's volume that will be common to the volume of the predicate.

The second type of propositions expressing the partial belonging of the subject's volume to the volume of the predicate is formed by judgments in which the concept of the predicate is *subordinate to the* concept of the subject. So, in the judgment "some weapons are missile", the entire volume of the predicate (the concept of "missile weapon") is only part of the volume of the subject (the concept of "weapon"). For judgments of this type, the relation between the volumes of the subject and the predicate can be represented in Fig. 15.

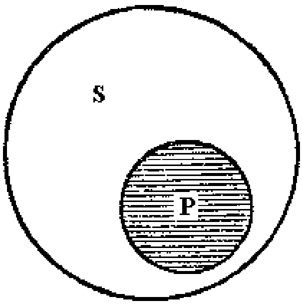


Fig. 15

It can be seen from this diagram that the volume of the predicate (circle P) is entirely included in the volume of the subject (all missile weapons are the tools), but the volume of the subject (circle S) is only partly equal to the volume of the predicate (only part of the tools is rocket tools). The circle P, hatched in the figure, representing the entire volume of the predicate, is that part of the volume of the subject that coincides with the predicate.

§ 13. In general negative judgments about the belonging of an object to the class of objects (E), the volume of the subject in no part coincides with the volume of the predicate.

So, in the judgment "no hero can be a coward", the volumes of the subject and the predicate are thought of one outside the other: there can be no cowards among the heroes, nor heroes among the cowards. This relationship between the volumes of concepts is presented in Fig. 16.

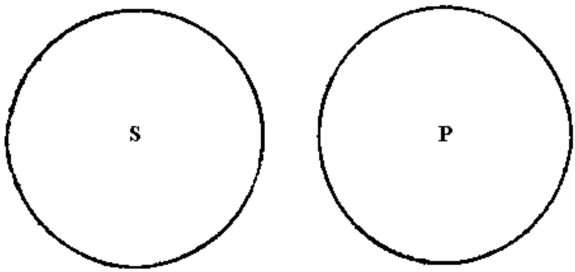


Fig. 16

From this diagram it can be seen that in the volume of the subject (circle S) there is not a single part that would appear to belong simultaneously to the volume of the predicate (circle P). And vice versa: in the volume of the predicate there is not a single part that simultaneously belongs to the volume of the subject.

§ 14. In particular negative judgments about the belonging of an object to a class of objects (O), not the entire volume of the subject is excluded from the volume of the predicate, as is the case in general negative judgments, but only part of the volume of the subject. So, in the judgment "some aquatic animals are not vertebrates", not all aquatic animals, but only a part of them, are excluded from the volume of vertebrates. Another part of the volume of "aquatic animals" turns out to be common with the volume of vertebrates. This ratio of partial exclusion of the subject's volume from the predicate's volume is shown in Fig. 17.

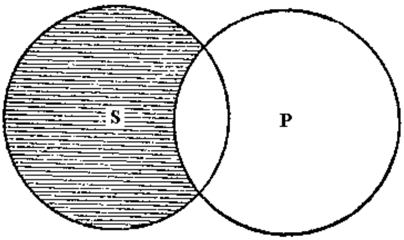


Fig. 17

It can be seen from the diagram that in judgments of this type, not the entire volume of the subject, but only a certain part of this volume, is excluded from the volume of the predicate. In the figure, this part, which is outside the circle P, is hatched. The same diagram shows that another part of the subject's volume (the unshaded part of the circle) is included in the predicate volume (some aquatic animals are vertebrates).

In this form of particular negative judgments, the relations between the volumes of the subject and the predicate will be the relations of intersecting concepts.

We met with such relationships when considering private affirmative judgments. But while in the partly affirmative judgments with intersecting concepts, the subject of the statement was the part of the subject's volume, which *coincides* with the volume of the predicate (cf. Fig. 14), in particular negative judgments of the same kind, the subject of the statement is, on the contrary, the part of the subject's volume that is *not included* in the predicate's volume.

Another type of partial negative propositions is formed by judgments in which the relations between the concepts of the subject and the predicate are *subordination* relations . So, in the judgment "some languages do not have forms of declension and conjugation", the volume of the predicate (languages that do not have forms of declension and conjugation) is excluded from the volume of a part of the subject (from the number of languages having forms of declension and conjugation). But at the same time, the concept of a predicate is subordinated here to the concept of the subject, since languages that do not have forms of declension and conjugation are nevertheless the essence of languages, that is, they are completely included in the volume of the subjugating conception of "languages".

This ratio of the volumes of the subject and the predicate can be represented in Fig. 18.

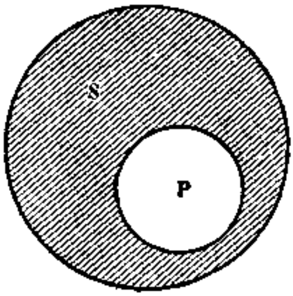


Fig. 18

In this diagram, the part of the subject's volume that coincides with the volume of the predicate (languages that have forms of declination and conjugation) is represented by the circle P. The part of the subject's volume that is not included in the predicate volume (languages that have no forms of declination and conjugation) is represented by that part of the circle S, which turned out to be not covered by the circle R. This part is shaded.

Comparing pic. 18 c. 15, we see that in partial negative judgments, the concepts of which are subordinate to each other, the volume of the predicate is also subordinate to the volume of the subject, as is the case in partial affirmative judgments with subordinate concepts. But, while in the partly affirmative judgments of this type, the subject of thought is that part of the subject's volume, which *coincides* with the volume of the predicate, in particular negative judgments of this type, the subject of thought is, on the contrary, that part of the volume of the subject that is *outside the* volume of the predicate.

The Distribution of Subject and Predicate in Judgment

§ 15. We have examined (§ 11—14) the relationship between the volumes of concepts in judgments about whether an object belongs to a class of objects. In this case, we took the ratio of the volume of the subject to the volume of the predicate, depending on all possible cases of quality and amount of judgment.

But the question of the volume of concepts included in the judgment can be posed in another way. One may not ask about the relation in which the volume of the subject stands to the volume of the predicate. It is possible to raise separately both the subject and the predicate of judgment, namely: whether the subject or predicate is thought in this proposition in its entirety or only in a certain part of its scope.

The study of this issue is called the study of the *distribution of* terms (i.e., the concepts of subject and predicate) in a proposition. *A distributed* term is called if it is thought in a judgment *in* its entirety. In other words, the term is distributed if what is expressed in the judgment refers *to the whole class* items. On the contrary, a term is considered not distributed if, in this judgment, it is thought only in some part of its volume. So, in the judgment "all bamboos are cereals," the concept (or term) of "bamboos" (the subject of judgment) is distributed, since the meaning of the statement refers to the *entire* volume of the concept of "bamboos," and not to any part of it. On the contrary, the concept of “cereals” (a predicate of the same proposition) is not distributed. In fact, although the judgment says about bamboos that they are all among the grains, it is by no means said about the cereals that they are all exhausted with bamboos: besides bamboos there are other cereals—wheat, rye, corn, oats, etc. In other words, including the entire volume of bamboos in the volume of cereals, we do not think in this case the entire volume of cereals, but only that part of this volume that bamboos occupy.

§ 16. It can be seen from the above example that in the same proposition one term may be distributed, the other—unallocated.

The analysis of the distribution of terms included in the judgment is important not only for a better understanding of the meaning of the judgments themselves. This analysis is necessary to establish the rules of possible *transformations of the form of* judgments, and also—in particular—to establish the rules of *conclusions* that can be obtained from judgments.

Considering, for example, the proposition “all bamboos are cereals”, we may ask ourselves: is it possible to transform the form of this proposition in such a way that what is expressed in this proposition about its subject is also expressed about its predicate. Such a transformation is obviously possible. Without changing the meaning of the judgment, instead of saying “all bamboos are grains,” we can say “some grains are bamboos.”

Looking closely at this transformation, we immediately note that, in the same sense, the form of judgment turned out to be different. Our judgment (“all bamboos are cereals”) was *general* . When we transformed this proposition into a predicate proposition (“some cereals are bamboos”), we received not a general, but only a *particular* judgment.

Why is this so? Why can’t the proposition “all bamboos are cereals” be converted into the proposition “all grains are bamboos”? To answer this question, it is necessary to pay attention to the distribution of terms in our judgment. In fact: we wanted to get a statement regarding the *predicate of* our judgment, that is, regarding the concept of “cereals”. But this concept in our judgment is not thought in its entirety, therefore, it is not distributed. It is clear, therefore, that in the transformed form of judgment, where the concept of “cereals” becomes the *subject of* judgment, this concept cannot be thought of in its entirety.

§ 17. But the way the concepts are distributed in judgments is important not only in the transformation of the form of *judgment*. The way the concepts are distributed in the judgment is also important in all cases when we draw *conclusions* , that is, from these judgments we get, by sense, not the same, but *new* judgments.

Consider, for example, the following two arguments:

1. If I know that “all cereals bloom in spikelets” and that “all bamboos are cereals”, then I must deduce from this that “all bamboos bloom in spikelets”. This conclusion will be correct, and it will be new in comparison with the judgments from which it was obtained.

2. But if I know that “all bamboos are cereals” and that “wheat is cereals”, then I cannot say anything new about the relation of wheat to bamboos. In other words, a conclusion from these two propositions is impossible.

The question arises: why in the first example from two true judgments the new third is correctly obtained, and in the second example from two also true judgments no new third can be obtained, and if we tried to do this, would we get a logical error?

Looking closely at both of these examples, we can notice that here it is all about the distribution of terms. In fact: in both the first and second examples, we are only trying to establish a relationship between the two concepts, because we know what relation each of them has to some third concept. In the first example, we are trying to establish the ratio of bamboos to plants blooming with spikelets. In the second example, we are trying to establish the ratio of wheat to bamboos. In the first case, we are trying to establish the attitude of “bamboos” to “plants blooming in the ears of wheat”, on the grounds that we already know the attitude to “bamboos” and the attitude to “plants that blossom in spikelets,” of a third concept — the concept of “cereals”. In the second case, we are trying to establish the relation of “wheat” to “bamboos” also on the ground that we already know the relation of “bamboos” and, in addition, the relation of “wheat” to some third term—to the concept of “cereals”. And in the first and second case, the relationship of the two concepts with each other is clarified through the relationship of each of them individually to the same third term.

§ 18. Let us examine how in each of these two cases the third term is distributed in judgments, through which we try to clarify the relationship between the subject and the predicate of inference. Let us first consider the proposition: “all bamboos are cereals.” From the judgment of this it is clear that the entire volume of bamboos is fully included in the volume of cereals. However, we do not know how much of the cereal volume is bamboos, since the concept of cereals in this proposition is not distributed. However, the second judgment— “all cereals bloom in spikelets” — frees us from the need to know which part of the cereal volume is bamboos. From the second judgment, it turns out that all cereals bloom in spikelets. *In this judgment, the concept of cereals is already distributed, it is thought in its entirety*. Knowing that *all* bamboos are among the grains and that *all* cereals bloom in spikelets, we obviously got the right to deduce from here that all bamboos bloom in spikelets. And indeed: although we do not know exactly how much of the cereal volume is occupied by bamboos, but since the property of blooming with spikelets extends to the *whole* volume of cereals without exception, it is obvious that this property will also apply to the whole part of the cereal volume occupied by bamboos. And since from the judgment “all bamboos are cereals”, we know that the volume of bamboos is fully included in the volume of cereals, in other words, that in the volume of bamboos there can be no part that does not enter into the volume of cereals, it follows that *all* bamboos should bloom in spikelets.

So, the third concept, through which we tried to connect the subject and the predicate in the conclusion, was not distributed in only one of the propositions substantiating the conclusion—in the proposition “all bamboos are cereals”. On the contrary, in another of these judgments (“all cereals bloom in spikelets”), it turned out to be distributed, and this circumstance, in conjunction with what the first judgment found out, which included the *entire* volume of bamboos in the volume of cereals, made the very conclusion possible.

§ 19. Now consider how the third concept is distributed in the second example. And here this third concept will be the concept of “cereals”: we are trying to establish a relationship between “bamboos”, on the one hand, and “wheat”, on the other hand, only because we hope that these concepts will be interconnected through the concept of “cereals” “, The volume of which fully includes both bamboos and wheat.

However, looking at both judgments, which would have to clarify the attitude of bamboos to wheat, we see that these judgments do not clarify any such relationship, and therefore do not substantiate any conclusion. At the same time, we see that the reason for the inability to establish any connection between the concepts of “bamboo” and “wheat” is that *the third concept (the concept of “cereals”) is not distributed in either the first or second judgment*.

§ 20. Now it is clear why in the second case our judgments cannot substantiate any conclusion. In the first of them (“all bamboos are cereals”) we think of some kind — it is not known which one — part of the cereals. But even in the second proposition (“wheat is cereal”) we think of some — it is not known which, exactly, is part of the cereals. In the first proposition, the part of cereals that we think of, but not exactly determined, is occupied by bamboos, in the second, by wheat. But since we do not know in what relation these two parts of cereals are among themselves, we cannot know anything about how the ratio of bamboos to wheat will be. It is possible that these parts will coincide, and that they will turn out to overlap, and, finally, that they will completely lie one outside the other.

Thus, it was precisely the fact that the third term was unallocated in both judgments that made any conclusion from these judgments impossible. It turns out that the ability to obtain a conclusion from two propositions depends on the distribution of terms of these judgments.

In view of this, logic studies all possible cases of the distribution of the subject and the predicate in judgments, depending on the differences between judgments in terms of quality and quantity.

The Distribution of Subject and Predicate in Judgments about the Belonging of an Object to a Class of Objects

§ 21. *In general affirmative judgments (A) about the belonging of an object to a class of objects, the subject is distributed, the predicate is not distributed*. We were convinced of this above, considering the proposition "all bamboos are cereals". In this judgment, the subject ("all bamboos") is distributed, because what is expressed in the judgment is expressed in relation to the *entire* volume of the subject: not a part of bamboos, but *all* bamboos belong to cereals.

On the contrary, the predicate of this proposition ("cereals") is not distributed, because what is expressed in the proposition is not expressed about the whole volume of cereals, but only about that part of cereals that make up bamboos. At the same time, it remains unclear whether all the cereals are exhausted by bamboos or whether besides bamboos other types of plants are included in the number of cereals (see Fig. 19).

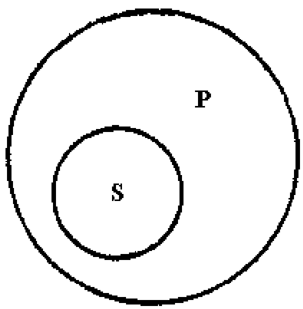


Fig. 19

§ 22. *In particular affirmative judgments (I) on the subject belonging to the class of objects, the subject is not always distributed; the predicate is not distributed in judgments, where the subject and the predicate are concepts intersecting, and distributed in judgments where the predicate is subordinate to the subject.*

Let us first consider a partly affirmative proposition of the first type, in which the subject and the predicate are concepts intersecting, for example, the proposition “some guardsmen are order bearers.” In this judgment, the subject (“some guardsmen”) is not distributed, since the statement does not refer to the entire volume of the concept of “guardsmen”, but only to that part of the volume that is included in the volume of the concept of “order bearers”.

But the predicate of this proposition (“order—bearers”) is also not distributed. Although the guardsmen who are awarded orders are all order—bearers, however, from among the order—bearers, only order—bearing guards think this. This judgment leaves it unclear whether, apart from the order—bearers—guards, there are other order—bearers or all the order—bearers are order—guards.

Let us further consider a private affirmative judgment of the second type, in which the predicate is subordinate to the subject, for example, “some tools are missile weapons”. In this judgment, the subject (“some tools”) is not distributed, since the statement does not apply to the entire volume of the concept of “tools”, but only to part of this volume. On the contrary, the predicate in this proposition is distributed. Indeed, in this proposition, it is not a part of missile weapons that is thought, but all missile weapons: those “some weapons” that are included in the scope of the concept of “missile weapons” *exhaust* its entire scope.

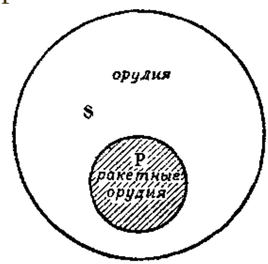


Fig. 20

This relationship between the volumes of the subject and the predicate is clearly shown in Fig. 20. The shaded part of the circle S in the figure indicates those "some weapons" that are "missile weapons" (circle P). The figure shows that the shaded part of the circle S completely exhausts the *entire* volume of P.

§ 23. *In the generally negative judgments (E) about the belonging of an object to the class of objects, the subject and the predicate of the judgment are both distributed* . Consider, for example, the judgment: "not a single hero showed himself to be a coward."

In this judgment, both his subject ("not a single hero") and his predicate ("did not show himself a coward") are distributed. The subject is distributed, because what is expressed in this judgment is expressed regarding the *entire* volume of the subject: *all* heroes, and not about a part of the heroes, claim that they have not shown themselves to be cowards.

But the predicate of this proposition is also distributed, since the statement refers to the *entire* volume of the predicate, not about a *part of the* volume of cowards, but about the *whole* volume of cowards it is stated that not a single hero was in the volume (see Fig. 21).

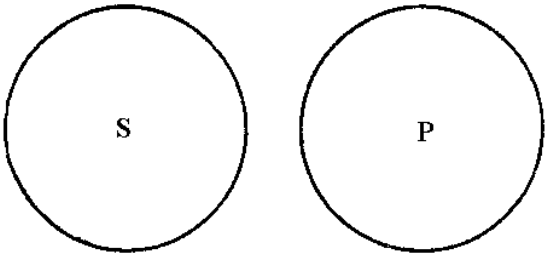


Fig. 21

§ 24. *In particular negative judgments (O) about the* *belonging of an object to a class of objects, the subject is not distributed* , but the predicate is distributed.

Let us first consider the first variety of partial negative judgments about the subject belonging to the class of objects. In the judgments of this variety, the concepts of subject and predicate are *intersecting* . So, in the judgment "some aquatic animals are not vertebrates" the concepts of "aquatic animals" and "vertebrates" are intersecting, as shown in Fig. 22.

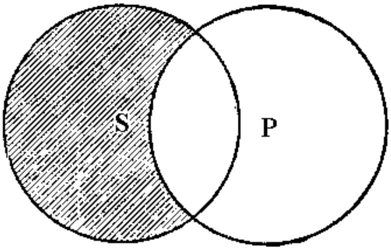


Fig. 22

The hatched part of the circle S denotes that part of the volume of the subject, which, according to the meaning of this proposition, is excluded from the volume P.

In judgments of this kind, the subject is not distributed, since it is thought only in part of its volume. On the contrary, the predicate in them is distributed, since the part of the subject's volume (shown in the figure by the shaded part of the circle S) that is conceivable in these judgments is placed outside the *entire* predicate volume (in the figure outside the entire circle P), and not just its part.

Then we consider the second variety of partial negative judgments about the subject belonging to the class of objects, i.e., those in which the concept of predicate is *subordinate* the concept of the subject. Such, for example, is the judgment: "some arthropods are not insects." In this judgment, the subject ("some arthropods") is not distributed, since the statement does not apply to the entire subject: not all, but only some arthropods are said to not belong to insects.

On the contrary, the predicate of this proposition ("not insects") is distributed. Indeed: although only part of the arthropods is said to be insects, this part of arthropods is no longer excluded from the *part of the* volume of insects, but from the *entire* volume of insects (see Fig. 23).

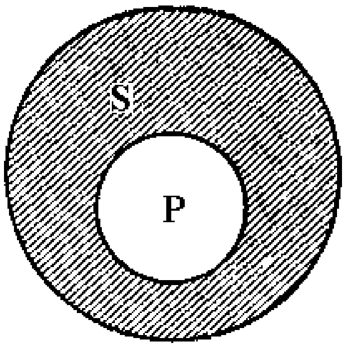


Fig. 23

§ 25. We examined the distribution of the subject and the predicate in judgments about the subject belonging to the class of objects of all kinds of quality and quantity. Let us now summarize this consideration separately for the subject and separately for the predicate. Summarizing all cases of the *subject's* distribution, we easily obtain the following rules:

1) *The subject is distributed in general judgments (general affirmative and general negative) and not distributed in private judgments (partial affirmative and partial negative).*

2)*The predicate is distributed in all negative judgments (general negative and particular negative) and in those particularly affirmative, in which the predicate is subordinate to the subject. The predicate is not distributed in general* *affirmative and in particular affirmative propositions in which the subject and the predicate are concepts that intersect.*

Tasks

1. Determine which of the following judgments express relations of belonging and which are relations of *magnitude* , *force* , *cause and action* , *space* , *time* , and also the relationship of *comparative evaluation*: “The Kursk—Oryol battle took place after the battle of Stalingrad”; “Some engines are internal combustion engines”; “All pines have needles instead of leaves”; “The Himalayas are higher than the Alps”; “Pulkovo is located near Leningrad”; “Pulkovo—one of the most famous observatories in the world”; “The morning is wiser than the evening”; “Flowers wilted from frost”; “Rimsky—Korsakov—one of the composers of the Balakirev circle”; “Leo Tolstoy and Chernyshevsky are one—year—olds”; “Chess is an ancient game”; “Saturn is farther from the Sun than Jupiter”; “Saturn is the outer planet”; “Saturn is surrounded by a ring”; “There is no beast stronger than a cat”; “The human skeleton consists of two hundred and fifty—six bones”; “The serial number of uranium in the periodic system of Mendeleev is 92”; “Uranium is heavier than mercury”; “The human heart consists of two atria and two ventricles”; “Spectacle snakes are very poisonous”;

2. Using visual diagrams, depict the relationship between the subject and the predicate in the following judgments about belonging: “some scientists were composers”; “Some mushrooms are lamellar”; “Some birds do not swim”; “Some composers did not have absolute hearing”; “Some conical sections are closed curves”; “Some trees do not give a shadow”; “All equilateral triangles are equiangular”; “All rhombuses are parallelograms with equal sides”; ”Not a single enemy left the battlefield alive”; “All the works of ancient Greek writers that have come down to us have come down to us in later lists”; “Some volcanoes have long ceased to function”; ”Not a single so—called shooting star is really a star”; “All birds of prey soaring at high altitude have keen eyesight.”

3. Define the distribution of the concepts of subject and predicate in the following propositions: “some organisms reproduce by division”; “Many plants do not have chlorophyll”; “Some sturgeons do not enter the sea at all for breeding”; “All birds have excellent eyesight”; “There are no fortresses that the Bolsheviks would not take”; “Not all volcanoes are active volcanoes”; “Nitric acid is easily washed out of the soil”; “The reason for the lodging of bread cannot be a lack of silica”; “Mushrooms cannot grow on soil that does not contain prepared organic matter"; "All the outer planets known to us have a low density"; "History is a social science"; "A commodity is a thing that satisfies some need and is able to exchange for another thing"; "The republic is one of the forms of government."

CHAPTER VII. ESTABLISHING THE EXACT LOGICAL MEANING OF JUDGEMENTS. TRANSFORMING JUDGEMENTS

Establishing the Exact Logical Meaning of Judgments

§ 1. In all logical operations on judgments, the first task is to establish the exact logical meaning of the judgments over which we want to make logical actions. In order to compare two or more propositions, to establish whether there is a logical connection between them and which one, it is necessary first to establish exactly the logical meaning of each proposition. Regarding each of them, it is necessary to know exactly what its subject is and what its predicate is, what is the relation that is thought between them in the judgment. It must be precisely determined whether this is a judgment of belonging or a judgment of attitude.

If the judgment is a judgment on belonging, then it is necessary to establish whether it is a question of belonging of a property to an object ("quinine is bitter", "sugar is white", "guardsman is brave") or about belonging of an object to a known class of objects ("quinine is a medicinal substance", "Sugar is a product of the food industry", "a guardsman is a fighter of a unit that has especially distinguished itself in battles for its homeland").

If the judgment is a judgment on the relation, then it is necessary to establish what this relation is: whether it will be a relation in *space* ("Kazan east of Moscow"), or *in time* ("Pushkin was born before Lermontov"), or a relation *in* *magnitude* ("Kiev is more than Poltava"), Or *by a causality* relation (" the stack lit up from a lightning strike "), or by an attitude *by comparative dignity* ("Pushkin as a poet is superior to Derzhavin"), or by relationship *of kinship* ("Peter is Anna's brother"), etc.

§ 2. Far from always the exact logical meaning of the judgment is immediately clear and transparent to thought. The fact is that in the practice of everyday thinking, we do not always feel the need to accurately clarify the relations between concepts. Often we are satisfied with only approximate accuracy. This accuracy is sufficient for the initial approach, but insufficient where a more thorough definition of the relationship between the concepts is required.

Since the expression of thought in language serves primarily the tasks of everyday practice, where often only approximate accuracy is sufficient, it follows that the logical structure of judgment does not always exactly coincide with the grammatical structure of the sentence.

We are already familiar with examples of such a mismatch. We already know that, for example, the logical subject of judgment is far from always expressed by means of a grammatical subject sentence. In ordinary life, the word, speech, for us is not only a way of expressing *thoughts* , but also a way of expressing *feelings* and *desires*. We use grammatical forms of speech in order to express not only the known logical content, but also our emotional attitude to this content. Through the words of speech, we express not only *concepts*, but also *images*, not only *knowledge*, but also the *impression* that this knowledge makes on us.

This explains the possibility of a not quite exact correspondence between the logical construction of a proposition and the grammatical construction of a sentence. Until we need a particularly precise clarification of the relations between our concepts, we use the methods of expression of judgments in speech, which are satisfied with ordinary practice.

But as soon as our task becomes to clarify the exact *logical* meaning of the judgment, i.e., to establish the exact relationship between its subject and predicate, we often can no longer be content with those forms of grammatical expression that introduce ambiguity or ambiguity into the expression of thought.

Meanwhile, such ambiguity and ambiguity are often found in speech. We already know that, for example, belonging of a concept to the number of contradictory or counter—ones can often not be established due to the ambiguity of a word with a negative particle “not”. What does the word “unkind” mean, for example: is it only the lack of kindness or the presence of the opposite quality? Is everything that does not belong to the volume of “good”, or only that from the volume of “unkind”, which is called “evil”?

We also know that, for example, to decide whether a given judgment will be negative, we cannot always by the mere presence of negation before the predicate of the sentence. The quality of a judgment, i.e., whether it is affirmative or negative, is determined, as we have seen, not only by the grammatical form of the sentence, but also by the *ratio of the meaning of the* judgment to the meaning of other judgments with which our judgment is connected.

The foregoing remains valid in deciding the question of *relativity* judgments. A judgment expressing the truth under a certain condition, which is put forward by our thought, will be hypothetical even when this condition is not noted by means of the conditional union “if” (“if you go quietly, you will continue”, “if you claw it, you’ll be lost to the whole bird”). And vice versa: the presence in the complex sentence of a subordinate clause with a conditional union “if” does not prove yet that the judgment will be hypothetical — if only the truth expressed in this judgment is derived from the content of the subject itself and does not depend on how we think of the subject (“If you draw a diameter through the circle, the circle will be divided into two equal parts”).

But the question of whether this judgment will be separative or not cannot be resolved on the basis of only the grammatical form of the sentence. The union “or” in some cases expresses a separation relation, in others it does not. The judgment “Ivanov lost a chess game either out of inability or carelessness” is not a dividing one, despite the existence of an “or” union. Inability and inattention do not exclude each other. And vice versa, the judgment “stars can be set and not set” is a separation — despite the absence of a separation union “or”.

§ 3. Precisely because the grammatical structure of a sentence does not always correspond to the logical structure of a proposition, the first task that arises with any logical actions on a proposition is to establish the exact logical meaning of the proposition.

To solve this problem, sometimes it is enough to delve into the meaning of the proposal without any transformation of the form of judgment. Thus, the judgment “all ferns are spore plants” is expressed through a sentence, the form of which does not require transformation, since it clearly reveals the logical meaning of the judgment. In this proposition, the logical subject coincides with the grammatical subject, the logical predicate with the grammatical predicate. It can be immediately seen from the form of this judgment that, for example, by quantity it will be *general* ; we are talking about all the ferns, and not about any part of them. It is just as easily solved in this case the question of whether this judgment belongs to affirmative in quality, categorical in relation to, and assertive—in the way of expressing reliability.

However, in many cases it is possible to correctly judge the logical meaning of a judgment only *after some transformation of the* form of judgment. In all these cases, the logical meaning of the judgment, i.e., the logical relation expressed in it between the subject and the predicate, is established only after it is possible to eliminate all the ambiguities and ambiguities caused by the grammatical form that is not transparent enough from the point of view of *logic* .

Therefore, logic includes in its doctrine of judgment the indication of those ways of transforming the form of judgment, as a result of which the establishment of the exact logical meaning of judgment is achieved.

§ 4. The first of the necessary actions for this is such a transformation of the sentence form in which in a judgment one could clearly distinguish: subject, predicate and logical relation between them. For example, the judgment "few songbirds live in spruce forests" in this grammatical form is not entirely convenient for logical analysis. In this sentence, the grammatical subject is "little", the grammatical predicate is "living". On the contrary, the logical subject, or subject, here is the concept of "spruce forests", the logical predicate, or predicate, is the concept of the small number of songbirds inhabiting spruce forests.

Transforming this sentence into the sentence "all spruce forests belong to forests with a small population of songbirds," we give it a form that, without changing the logical meaning of the sentence, makes this meaning clearer, more accurately indicates the subject and predicate of judgment, more accurately expresses logical relationships between them. After this transformation, we immediately see that this proposition is a typical proposition about the class ("all spruce forests") belonging to another class of objects (“all forests with a small population of songbirds”).

§ 5. The transformation of the form of judgment should only better reveal the logical relation of concepts expressed in the judgment, but should not change the content of the statement itself. Otherwise, we get no longer a *form* conversion judgments, and the replacement of one judgment by *another* , expressing a *different* content. It would be a mistake, for example, if, wanting to transform the form of the judgment “Misha is not reading the newspaper,” we would turn this judgment into this: “Misha is not reading the newspaper.” It is obvious that the sentences “do not read the newspaper” and “reads not the newspaper” do not express the same logical content: the sentence “do not read the newspaper” does not contain any indication that Misha is reading anything or reading nothing . On the contrary, the sentence “Misha is not reading the newspaper” means that Misha is reading something, but what he is reading is not a newspaper.

§ 6. The transformation of the form of judgment, which does not change the logical meaning of judgment, should not only correspond to the logical type of judgment. This transformation should, in addition, make clear the *quality of the* judgment, its *quantity*, its belonging to a certain rubric of *attitude* and *modality*.

Not every grammatical form of judgment accurately expresses its *quantity* . So, for example, belonging to a number of judgments of *the general* does not always celebrated staging of the word “all” or “none”, “no” to the subject of the judgment. However, even without these words, judgment can be general in a logical sense. The judgment “ferns is a spore”, of course, is a *common one* judgment, since it is not about any part of the logical class of ferns, but *about everything* without exception in this class. Similarly, the judgment “spiders are not insects” is general, since the whole class of insects in this proposition excludes the *whole* a class of spiders, not any part of this class.

§ 7. When transforming the form of judgment, the number of the subject must be marked with special words: “all”, “everyone”, “not one”, “nobody”, etc. Thus, the judgment “ferns—spore” is converted into the judgment “all ferns”— controversial, the proposition “spiders are not insects”— in the proposition “no spider is an insect”, etc.

The word “everything”, set before the subject of judgment, usually indicates that this judgment is general, for example: “all airplanes are heavier than air”. But in some cases, the word “everything” has a collective meaning, that is, although it means a group of objects, however, such a group, which is considered in this proposition *as a whole* . In such a judgment, the meaning of the statement does not apply to each member of the group individually, but to the whole group as a whole. So, in the judgment “ *all* books cost forty—five rubles”, the meaning of the statement does not, of course, apply to each book individually, but to all books together, that is, to a group that is thought of *as a whole* in this judgment. The point of this judgment is not that each book taken separately costs forty—five rubles, but that forty—five rubles are all books put together. A judgment in which the word “everything” confronted by a subject means that a certain group of objects is thought of as a whole, will not be a general judgment, but a singular one.

Since the word *”everything”* in front of the subject of the judgment does not always show that the judgment is general, it is necessary to replace the word *”all”* with the word *”everyone”* to correctly determine the amount of judgment . If it turns out that the meaning of the statement applies to each subject taken separately, then the judgment will be general. Consider, for example, the judgment “all planes are heavier than air.” Replace the word *“all”* the word— “everyone”, then the judgment will take the form: “every plane is heavier than air”. Since this replacement did not change the meaning of the judgment and did not violate its truth, then, obviously, the judgment will be truly general. But let’s take the judgment “all shells weighed ten tons.” Replacing the word “all” with the word “everyone,” we get the judgment: “every shell weighed ten tons.” It is immediately obvious that from this replacement not only the meaning of the judgment changed, but the judgment itself turned from true to absurd: not every shell weighs ten tons, but only all shells combined. From this we conclude that our judgment is not general, but singular. Indeed, the meaning of the statement refers to that single whole, which in this case means the word “everything”.

§ 8. Affiliation of judgment to *private* usually denoted by the statement of the word “some”, “others”, “not all”, “many”, “part”, “majority”, “minority” in front of the subject of judgment. For example, “ *some* writers are playwrights”, “ *others* , you’ve been killed,” (Griboedov), “*not all* students learn French,” “*many* fighters went ford, *some* went swimming”, “ *most of the* lakes in the deserts are brackish”, “ *the minority of the* participants in the performance were not occupied in the first act, “etc.

The most clear sign of the judgment being *private* is the word “some” before the subject of judgment. Therefore, any other form of private judgment can be reduced to a form in which the particular nature of the judgment is marked by the word “some,” which is confronted with the subject of the judgment. So, the judgment “most fighters lit a cigarette” can be expressed in the form of the judgment “some fighters lit a cigarette”. Since “some” can mean “majority”, contradictions regarding the meaning of the first judgment will not work. At the same time, the word “some” indicates that we are not talking about the whole class, but only about some part of it, that is, that our judgment will be *private*.

However, the word “some” is not free from the known ambiguity. This word can be understood, firstly, in the sense of “not the entire class of these objects, but only part of it.” For example, the proposition “some fighters smoke” can be understood in such a way that not all fighters smoke, but some part of the fighters are scribbled. And you can understand the word “some” in the sense of “at least some.” With this understanding, our judgment—“some fighters smoke”—will mean: “at least some (or maybe even all) fighters smoke.”

If the quantity indicator in the subject of judgment is not sufficiently defined, so that the whole class and only part of the class can be thought of in the subject, then the quantity of judgment should be considered *indefinite*.

Some private judgments in their logical meaning constitute a special group within the entire field of private judgments. Such, for example, are the judgments in which it is stated that a known property or relation belongs to the entire class of objects, except for a certain number of instances of this class. For example: “all planets, except Mercury, Venus and Pluto, have satellites.” Judgments of this type, of course, will be private, since the subject of these judgments is not a whole class. But since these judgments indicate exactly how many particular instances of a class the property or relation conceivable in the judgment does not belong to, these judgments, being private, nevertheless differ from other particular judgments in which it remains unclear which part of the class their subject represents.

Such judgments, containing an exact definition of a particular quantity, constitute a special group of private judgments and are called *exclusive* judgments.

Each *exclusive* private judgment can be logically expressed in the form of *two* judgments. One of them indicates that a known property or relation does not belong to a known number of instances of the class, the other indicates that this property or relation belongs to all other instances of the same class. Consider, for example, the judgment “all the vowels of the Russian language are preserved, except for the nasal,” yat “(ѣ),” ery “(b) and” ery “(b)”. This judgment is an exclusive private judgment. The logical meaning of this proposition can be accurately expressed through two propositions: 1) “nasal vowels,” yat “(ѣ),” ep “(b) and” yer “(b) were not preserved in Russian” and 2) “all the others vowels of the Russian language are preserved.

The second special group within the entire field of private judgments is comprised of judgments that indicate that a known property or relation belongs to only one single instance of the class and does not belong to the rest of the instances of the same class. For example: “only the Bulgarian language, one of all Slavic languages, has not preserved the forms of declension”; ”February alone, one of all months has twenty eight or twenty nine days.”

Judgments of this type are called *distinguishing* judgments, since in them a known instance *stands out* from a whole class, which, therefore, is not conceived in its entirety. Each distinguishing proposition can be logically expressed in the form of two propositions. The first of them notes that a certain property or relation does not belong to one particular object, the second that it belongs to all other objects of the same class. So, our judgment “only Bulgarian one of all Slavic languages lost the forms of declension” can be expressed in the form of the following two judgments: 1) “Bulgarian did not preserve the forms of declension” and 2) “all other Slavic languages, except for Bulgarian, retained the forms declensions.”

§ 9. Belonging to the number of judgments is also not always clear from the grammatical form of judgment. In the *logical* single judgment will be any judgment, the subject of which is conceived as a single object. For example: “Peter I founded Petersburg”; ”All brochures cost ten rubles.” But a subject conceivable as a single entity in some judgments can represent a truly separate person or an individual object of the class (“Peter I founded Petersburg”), in other judgments, the subject appears to be united only in a collective sense, that is, only in thought, as conceivable by us the whole (“all brochures cost ten rubles”).

Therefore, the grammatical form of a sentence alone does not give an unmistakable indication of whether or not this judgment will be singular. Often a sentence in which a subject is expressed by a quantitative numeral or plural nouns, logically turns out to be *singular* in quantity judgment. This happens in cases when a group of objects defined by a quantitative numeral is thought of as unity, or when the plural of a noun also means some collective whole or unity. For example, the judgment “three regiments constitute a division” is an *individual* judgment. The point of this judgment, of course, is not that each regiment, taken separately, forms a division. The meaning of this proposition is that only all three regiments form a division together, as a single whole. And the judgment “the Greeks defeated the Persians at the Marathon” will be just the same. The point of this judgment, of course, is not that each Greek individually defeated the Persians, but that the Greek army as a single whole defeated the Persian army as well as a single whole.

In case of doubt whether the given judgment will be singular, it is necessary to check the logical meaning of the judgment by applying the quantitative designation “each” to its subject. If at the same time the logical meaning of the judgment changes and the judgment turns into nonsense, this means that the judgment is singular.

§ 10. We have examined some methods of transforming the form of judgment. These methods, without changing the logical meaning of the judgment, make the logical composition of the judgment, its logical construction, logical meaning and the logical relationship between the subject and the predicate more clear.

In addition to the considered methods for clarifying the logical form of judgment, there are a number of other ways of transforming the form of judgment that are useful in logical operations on judgments. These forms are 1) *appeal*, 2)*transformation* and 3) *transformation* by *contrasting the predicate* .

Appeal

§ 11. *Inversion* is a transformation in which the predicate of judgment becomes the subject, the subject becomes a predicate, but the logical content of the judgment remains the same. For example, the proposition "all Heroes of the Soviet Union are order bearers" is called into judgment: "some order bearers are Heroes of the Soviet Union".

It is easy to verify that this transformation only changed the form of judgment, without changing the logical relationship between the subject and the predicate. True, at first glance it might seem that after the appeal we received a judgment with a different content. Firstly, the predicate and the subject changed places, and secondly, the *amount of* judgment changed : before the appeal, the judgment was general, after the appeal became *private*.

However, closer looking into the content of the reversed judgment, we see that the content is the same. And indeed:

although the amount of judgment after the appeal has changed and the judgment has changed from general to particular, this change in quantity does not mean either a change in the number *of concepts of the* subject and predicate *themselves*, or a change in the logical relationship between them. In fact: the meaning of the judgment in its original form can be expressed as follows: “all the Heroes of the Soviet Union are part of all the order bearers.” As in any affirmative judgment on belonging (cf. Chapter VI, § 21), in our judgment the concept of the subject (“all Heroes of the Soviet Union”) is distributed, but the concept of predicate (“order—bearers”) is *not distributed*: Heroes of the Soviet Union do not exhaust the entire class of order bearers, which, in addition to Heroes of the Soviet Union, also includes other awarded orders.

The same opinion is expressed by the judgment resulting from the appeal: “some order—bearers are Heroes of the Soviet Union”. The concept of “Heroes of the Soviet Union” in *both* propositions is thought in its entirety. The concept of “order bearers” in *both* judgments is thought only in some part of its volume. The difference between a reversed judgment and a judgment before conversion, therefore, is not that the number of concepts whose relationship is considered in the judgment has changed.

The difference between a reversed judgment and a judgment before conversion cannot also consist in changing the *logical relation* between the concepts of these two propositions. Both judgments are judgments about the subject belonging to the class. Both argue that the entire scope of the concept “Heroes of the Soviet Union” is fully included—as part of—the broader concept of “order bearers”.

What has changed as a result of the appeal?—Not the content of the judgment, but only its logical form: the predicate became the subject, and the subject became the predicate. If at the same time the amount of the converted judgment has changed from the general to the particular, then this is again a simple result of the rearrangement of the predicate and the subject: changing the judgment into a particular is only an understanding of what was already thought in the unconverted judgment, namely, that the concept of “order bearers” is considered not in its entirety (see Fig. 24).

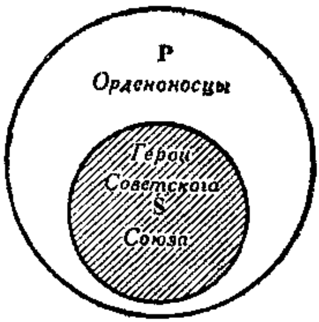


Fig. 24

It can be seen from this figure that the entire logical class “Heroes of the Soviet Union” (S) is only part of the logical class “order bearers” (P). It is this relationship between the concepts of S and P that is conceived in the original (unconverted) form of judgment.

But from the same figure it can be seen that the same relationship between S and P can be thought of in a different way, namely, it is clear that not the entire volume of the concept of “order bearers” (circle P), but *only part of* this volume (the area shaded in the figure is limited circle S) is the volume of the class “Heroes of the Soviet Union” (circle S). It is this relationship of identity between the *part* the volume of class P, hatched in the figure, and the entire volume of class S, refers to the reversed judgment: “some order bearers are Heroes of the Soviet Union” (or in general terms: “some P belong to S”).

§ 12. Being a transformation of only one form of judgment, conversion is not, however, an empty and useless transformation. *Conversion makes for us more distinct the number of subject and predicate, as well as the relationship between their volumes* in the judgment. Prior to conversion, the amount of the concept included in the original proposition as its *predicate* , although it was thought only in part of its volume, however, this partiality of the volume remained unstressed, as if hiding behind the form of an affirmative judgment.

If the relationship between the volumes of the subject and the predicate, as well as the amount of the predicate in the general affirmative propositions, were completely clear to thought, then no one would ever make any logical mistake in making general affirmative propositions. In fact, such mistakes are made very often. Many, surprisingly as it may seem, turn a general affirmative judgment not into a private affirmative one, but into a general one. For example, a judgment like "all artists are impressionable people" many turn into a judgment "all impressionable people are artists".

Such treatment is, of course, erroneous. He who draws judgment in this way obviously does not give himself a clear account in the logical relation between the concepts of "all artists" and "impressionable people." He thinks that the scope of the concept of "artists" completely exhausts the scope of the concept of "impressionable people". In fact, the meaning of the judgment is different. The judgment expresses that the artists, all taken together, make up only a part—unknown what—of impressionable people. Since besides all artists other types of impressionable people can exist (and really exist), the correct appeal of our judgment will be only the judgment "some impressionable people are artists".

An error like the one given would obviously be impossible if the relationship between the volumes of concepts in a general affirmative judgment was completely clear to thought. This attitude can elude attention due to the generally affirmative form of the judgment drawn. The generality of judgment, i.e., the distribution of the *subject*, can be mistakenly transferred by our thought to the *predicate*.

But it is the prevalence of this error in circulation that proves the use of the rule of circulation. The appeal clarifies the relationship between the subject and the predicate, which was not completely clear in the judgment before the appeal.

§ 13. On what is the logical operation of inversion based? What gives us the right to swap the predicate and subject of judgment?

The appeal is based on *the identity of the content of* those concepts that exchange places in the reversed judgment. In our example, “some order bearers” is precisely that part of the volume of the concept of “order bearers”, which coincides with the volume of the concept of “all Heroes of the Soviet Union”. This part (P) does not belong to all the order bearers, but only those of them, in the concept of which the essential features are identical with the essential features of the concept “Heroes of the Soviet Union”. Only on this identity of essential features, i.e., the identity of the *content of* both concepts, is the equality of *volumes* the concepts of “all Heroes of the Soviet Union” and the concepts of “some part of the order bearers”. Only order—bearers who coincide in their characteristics with the Heroes of the Soviet Union are thought in our judgment, which includes all the Heroes of the Soviet Union in a certain part of the class of order—bearers. It is this coincidence of *content*, and only one, that substantiates the equality of volumes: the *entire* volume of Heroes of the Soviet Union and *that part of the* volume of order bearers that make up all Heroes of the Soviet Union.

In turn, it is the equality of these volumes that makes it possible to rearrange the concepts of subject and predicate in a reversed proposition.

§ 14. Since in judgments about the belonging of an object to a class of objects, the relation between their volumes is precisely determined by the relation between the content of the concepts of the subject and the predicate, all logical conditions and rules of treatment can easily be deduced if we consider in the judgments about the belonging of an object to the class of objects the relations between the concepts of the subject and the predicate in all cases of the quality and quantity of these judgments (A, I, E, and O).

§ 15. *If, in a general affirmative proposition, the subject is subordinate to the predicate, then such an affirmative proposition gives not a general affirmative, but only a private affirmative judgment*. So, the proposition "all birds are vertebrates" (A) turns into the proposition "some vertebrates are birds" (I). This rule is derived from the general rules for the distribution of terms in judgment. And indeed: what is expressed in such an affirmative proposition does not mean the entire volume of the predicate, but only that part of it that is identical with the volume of the subject. It is clear, therefore, that during conversion, when the predicate of judgment becomes a subject, i.e., a concept about the subject of a statement, this subject cannot have a volume larger than that which is conceived in the reversed proposition. But to this we must add that this ratio of volumes itself is in turn derived from the identity *between the content of the* subject and the *content* the predicate of the reversed proposition: "some vertebrates", which the predicate of the reversible proposition means, are precisely the "birds" and no other species of the same logical class of vertebrates. Just because the essential features of these "some vertebrates" are the same as the essential features of "birds", the volumes of these concepts are also equal, and we have the right to reverse the judgment, that is, instead of the judgment "all birds are vertebrates," we get the equivalent the meaning of the judgment: "some vertebrates are birds."

§ 16. *If in a general affirmative proposition the subject and the predicate are equivalent, then such a proposition also gives a general affirmative proposition*. For example, the judgment "all dimes are coins of a ten—cents dignity" correctly turns into an affirmative proposition: "all coins of ten cents are dimes of a dignity". Indeed, in this proposition, the subject and the predicate are *equivalent* concepts . But this means that their volumes coincide. Therefore, everything that is affirmed about the entire volume of the subject remains valid with respect to the *entire* volume of the predicate. In other words, the appeal, or rearrangement of the predicate in the place of the subject, is made here without changing the amount of judgment.

This is the case with definitions. Indeed, the proposition "all squares are equilateral rectangles" correctly turns—without changing the quantity—into the proposition: "all equilateral rectangles are squares". Such an appeal is possible, because, being a definition, a reversed judgment, like any correct definition, is proportional: the volume of the determinant in it is exactly equal to the volume of the determined. In such a judgment, thinking the *entire* volume of the subject (" *all* squares"), we thereby think the *entire* volume of the predicate (not a part, but " *all* equilateral rectangles").

Any definition expressed by a general judgment can be reversed. In this case, the judgment remains general.

§ 17. In some cases, it may seem that the rule of appeal of an affirmative judgment is violated, i.e., it can be obtained from an affirmative judgment expressing the subject's subordination to the predicate by means of appeal not only a partly affirmative, but also an affirmative judgment.

The case of the apparent violation of the rules of treatment is represented by the so—called “inverse theorems”. From geometry, for example, it is known not only that equal angles lie in every triangle against equal sides, but also that equal sides lie in every triangle against equal angles. In this case, the converse theorem is as true as the direct one.

But the whole point is that the inverse theorem is *not* obtained *at all through inversion*. If we knew only that equal angles lie in every triangle against equal sides, then, transforming the form of this judgment, we could get from it according to the rules of treatment only a partial affirmative proposition: “in some triangles against equal angles there are equal sides.” And if we actually know that not only some, but all triangles have equal sides against equal angles, then the truth of this statement is established in geometry not by inversion, but through special *proof* .

§ 18. A *private affirmative judgment gives a private affirmative judgment upon appeal, provided that the subject and the predicate are concepts that intersect. If the predicate is subordinate to the subject, then a partly affirmative proposition turns into a general affirmative* is.

Let us first consider the appeal of a partially affirmative proposition, in which the subject and the predicate are concepts that intersect. Such, for example, is the judgment: “some scientists are publicists” (I). This judgment gives a private affirmative judgment when appeal: “some publicists are scientists” (I). This rule is derived from the distribution of concepts in a reversed proposition. In this judgment, neither the subject nor the predicate are distributed. The fact that some scholars form part of the publicist class, of course, does not mean that these certain scholars exhaust the entire scope of the predicate, that is, the entire publicist class: the publicist need not be a scientist. Since in a reversed proposition the predicate is not thought of in its entirety, in a reversed proposition where this predicate becomes a subject, speech also cannot deal with the entire volume of this subject. But that also means

The rule that is easily derived from the distribution conditions of the subject and the predicate is in turn based on the relationship between the *content of the* concept of the subject and the *content of the* concept of the predicate. And indeed: conversion is possible here only because “some publicists” whose concept is thought in the predicate of the reversed judgment are precisely those “some scientists” whose concept are thought in the subject of the reversed judgment. The volumes of these two concepts are equal only because their contents are identical: the signs of that part of the publicists that are thought in the predicate of the reversed judgment are the same as the signs of that part of the scientists that are thought in the subject of the reversed judgment (see Fig. 25) .

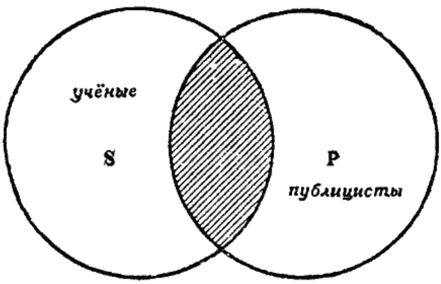


Fig. 25

It can be seen from the figure that the shaded part of the volume of the concept of “scientists” represented by circle S coincides with a part of the volume of the concept of “publicists” represented by circle R. This means that part of the

scientists are publicists. It is this relation of identity between the part of the volume of the concept of “scientists” and the part of the volume of the concept of “publicists” that is thought in the original—unconverted—form of judgment.
But from the same figure it can be seen that the other way round: part of the volume of the concept of “publicists” coincides with a part of the volume of the concept of “scientists”. This means that some of the publicists are scientists. It is this relation of identity between the part of the volume of the concept of “publicists” and the part of the volume of the concept of “scientists” that is conceived in the reverse judgment: “some publicists are scientists”.

Let us now consider the appeal of a partly affirmative proposition in which the predicate is subordinate to the subject. Such, for example, is the judgment: “some writers are playwrights” (I). This judgment gives a general affirmative opinion when addressing: “all playwrights are writers” (A). This appeal is deduced from the distribution of terms in the reversed judgment. In this judgment, the concept of the subject (“some writers”) is not distributed, but the concept of predicate (“playwrights”) is distributed (see Fig. 26).

Fig. 26

As can be seen from the figure, not all writers (S) belong to the volume of playwrights (P), but only part of this volume. This is the part of volume S that coincides with volume P and which is hatched in the figure. But with this part of volume S belonging to volume P, volume P is *completely exhausted: the* entire volume of playwrights is included in the volume of writers. Therefore, the reverse judgment does not think of “some playwrights,” but “all playwrights.”

It is easy to verify that the relationship between the volumes of the subject and the predicate, conceivable in judgments of this type, is based, as always, on the relationship between the *content of the* subject and the *content* predicate. Since all the essential features that make up the content of the concept of “writers” are included as part of the content of the concept of “playwrights,” the *entire* volume of the concept of “playwrights” is part of the volume of the concept of “writers”.

§ 19. A *generally negative judgment also gives a generally negative judgment upon appeal.* Thus, the proposition “no planet is a star” (E) becomes the proposition “no star is a planet”. This rule follows from the distribution of concepts in negative judgments. In such a judgment, both subject and predicate are distributed. First, the utterance refers to the entire volume of the *subject* ; it is impossible to say about any part of the volume of planets that it is a part of the volume of stars. Secondly, the statement applies to the whole volume *predicate* . The judgment “no planet is a star” means that the entire logical class of stars does not enclose in any part of its volume of luminaries called planets.

From this, the rule of reversing negative judgments is easily derived: since the predicate of the reversed proposition is thought in its entirety, then when reversed, where this predicate becomes the *subject* , it will be thought in its entirety, that is, the reversed proposition will be *general* . But it will also be *negative*.

And indeed: the reversed judgment confirms that the entire volume of the planets is *entirely outside the* entire volume of stars. But this means, and vice versa: the entire volume of stars is *entirely outside the* entire volume of planets.

Derived from the conditions of the distribution of concepts, the rule for reversing negative judgments in turn is derived from the relation of the subject's content to the content of the predicate in negative judgments. In these judgments, the entire volume of the predicate is outside the entire volume of the subject only because the subject and the predicate are *incompatible* concepts. So, for example, the scope of the concept of "stars" is outside the scope of the concept of "planet". Due to this incompatibility of these concepts, neither planets can be included in the number of stars, nor, conversely, stars — in the number of planets (see Fig. 27).

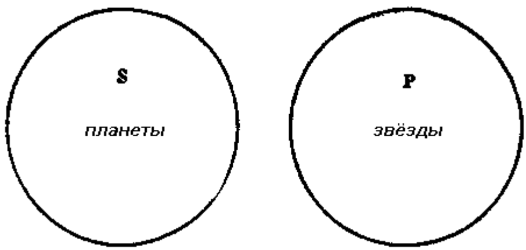


Fig. 27

It can be seen from the figure that not a single part of the volume of the concept of the "planet" represented by the circle S coincides with any part of the volume of the concept of the "star" represented by the circle P. It is this relationship between the concepts of S and P that is thought in the original—unconverted—form of judgment: "Not one S is P".

But from the same figure it can be seen that the other way round: not a single part of the volume of the concept of a "star" matches any part of the volume of the concept of a "planet". It is this relationship between concepts that is conceived in a reversed proposition: “no star is a planet”, or, in a general form: “not a single P is S”.

§ 20. A *particular negative judgment is not usually addressed in practice.* Consider, for example, the judgment “some composers are not pianists” (O). Let’s try to reverse it. When converted, its predicate (“pianists”) should become the subject of reversed judgment, and its subject (“some composers”) — the predicate. But the predicate of a partial negative judgment, as we know, is always distributed, that is, it is thought in its entirety. In our case, the drawn judgment expresses that out of some part of the volume of the concept of “composers” there is not a part, but the *whole* volume of the concept of “pianists”. Therefore, when converting, we should get a general judgment, that is, a judgment about *all* pianists. This judgment must be negative in quality. Since the reversed judgment is negative, that is, it places some part of the class of composers *outside* the whole class of pianists, then when speaking about pianists—if we do not want to change the meaning of the judgment—we will obviously have to put the whole class of pianists *outside* the same part of the class of composers that was thought in the appeal being drawn.

At the same time, however, the subject of the appeal (“some composers”) leaves it completely unclear which part of the class of composers is thought in this case. Therefore, in conversion, when this subject becomes a predicate, we obviously will have to think all pianists outside some completely indefinite part of the composers. This means that we can only say about the whole class of pianists that some composers are not included in it. In other words, the reversed judgment should take the following form: “not a single pianist is among some composers”.

Since the predicate of a judgment so reversed is too vague, in practice, a partial negative proposition is not addressed. Possible formally, and in this case, the appeal loses its meaning here. Indeed, the purpose of the appeal is not just a rearrangement of the subject and the predicate in the judgment, but only such a rearrangement of them, which, without changing the content of the judgment, contributes to a more clear understanding of the relationship between the concepts of judgment.

It is this goal that is not achieved in the case of reversing a partial negative judgment. Such a judgment *before* conversion turns out to be more understandable and definite than *after* treatment. To say “some composers are not pianists” means to say something more definite than if to say “not a single pianist belongs to some composers”. Here the question immediately arises: to which “some composers” does not one pianist belong? And only returning to the original—unconverted—judgment, we see that these are those composers who are not pianists (see Fig. 28).

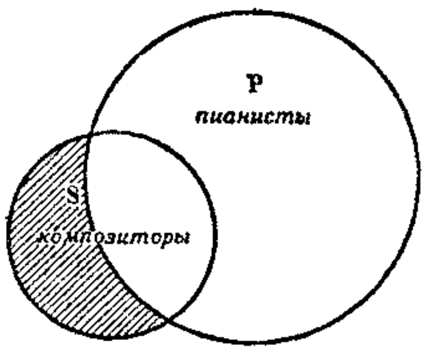


Fig. 28

This figure shows that part of the volume of the concept of “composers” represented by circle S is *outside the* volume of the concept of “pianists” represented by circle R. It is this meaning that expresses the original — unconverted — form of judgment "some composers are not pianists" or—in general form "Some S are not P."

But from the same figure it can be seen that the same relationship between the concepts of S and P can be thought of in a different way. The figure shows that the entire volume of the class "pianists" (P) is outside that part of the volume of the class "composers" (S), which is shaded in the figure and which represents some of the composers who are not pianists. It is in this way that the relation between the concepts in the reversed judgment "no pianist belongs to some composers" or in the general form "no P belongs to some S" is conceived.

Conversion is one of the most common types of transformation of the form of judgment. The purpose of the appeal is that the relationship between the two concepts, which is conceived in a reversed proposition from the point of view of its subject, becomes an object of thought also from the point of view of its predicate. In this case, the concepts included in the judgment, and the logical relationship between them remain the same. Changing the form of judgment, the appeal does not change its content.

Turning

§ 21. The second type of transformation of the form of judgments, which does not change the content of judgments, is *transformation*.

Transformation is different from conversion in that in the transformed proposition the subject of the utterance is not a predicate, but the *subject of the* initial proposition. At the same time, in contrast to conversion, during the transformation, the attitude of the subject of the initial judgment is considered not only to the predicate, but to a concept that *contradicts the* predicate. In other words, if the scheme of judgment is S —

P, then the transformation will be a transformation of the form of judgment, as a result of which the relation of the concept of S is clarified not to the concept of P, but to the concept of *not*—P.

Consider, for example, the judgment "all ferns are spore plants." Let us ask ourselves: What statement can be obtained from it about the attitude of his subject (the concept of "all the ferns") to the notion of " *ne*—sporovye plants", i.e., the notion, contrary to the predicate of the original proposition..? This will be the saying: "not a single fern is an unspoiled plant."

Let us now consider the transformation of a *generally negative* judgment. Let the initial form of judgment be the judgment: "not a single fern is a flowering plant". When transformed, the judgment will obviously take the form: "all ferns are *not* flowering plants."

So, *all the general judgments* (affirmative and negative)*when transformed, they change quality, but retain quantity: general affirmative ones become general negative and vice versa*.

Consider a *partly affirmative* proposition, for example: "some people are agile in movements." Turning this judgment, we get: "some people are not awkward in their movements."

Finally, *we* consider a *partial negative* judgment, for example: "some engines are not steam engines." Turning this judgment, we get: "some engines are non—steam."

So, *all private judgments* (partial affirmative and partial negative) *during transformation, like general judgments, change the quality, but retain the quantity:* private affirmative become private negative and vice versa.

§ 22. Transformation is only a transformation of *form* judgments. Through transformation, the same relation between concepts is thought that was thought in the original form of judgment. But no matter how significant this transformation is from a logical point of view, it nevertheless reveals, from some new perspective, the relation between the subject and the predicate that is conceivable in the initial proposition. And indeed: in the original form of judgment, an object is thought of as having a known property. In the transformed form, it is revealed that the same object cannot possess a property or relation incompatible with a property or relation expressed by a predicate, i.e., a property or relation, the concept of which contradicts the concept of a predicate. The same specific property or relation of an object that was thought in one way in the original form of judgment.

Opposition to the Predicate

§ 23. The third type of transformation of the form of judgment is the *opposition to the predicate* . It is based on the fact that each concept can be thought of not only in its own positive content, but also in relation to a concept that contradicts it. So, the concept of “hero” we can think of not only as a group of positive significant features that make up the content of this concept. We can think of the concept of “hero” also in relation to the concept of “non—hero” that contradicts it.

In contrast to the appeal, the *opposition to the predicate* is not a statement about the predicate of judgment, but about a *concept that contradicts the concept of a predicate*. For example, we have the judgment “all mushrooms are spore plants.” In it, the predicate has the concept of “spore plants”. The concept that contradicts the predicate is obviously the concept of “non—spore plants”. We ask ourselves: how will the form of our judgment change if the subject of the statement is not a subject in it, as it was before the transformation, and not a predicate, as it was when reversed, but a concept that *contradicts the predicate* ? ”Obviously, our judgment will take the form:” not a single non—spore plant is a mushroom. “ And indeed: if all the “mushrooms” are fully included in the volume of “spore”, then it is obvious that within the volume of “non—spore” can not be a single “mushroom”; all of them are distributed without remainder within the volume of “spore”.

So, a general affirmative proposition is transformed by contrasting the predicate with a general negative proposition.

Contrasting with the predicate, as well as conversion, does not change the meaning of the judgment and is a transformation of its form alone. In thinking the proposition “no non—spore plant is a mushroom”, I think in essence the same relationship between the concepts of “fungi” and “spore”, which I thought in the statement “all mushrooms are spore plants”. Only the form of expression has changed. However, I think of the same logical relation, firstly, not from the point of view of the *subject of the* initial judgment, and secondly, not from the point of view of the *positive* content of the predicate, as it happens with conversion, but from the point of view of its *negative* content, i.e., from the point of view of a concept that *contradicts the* concept of a predicate (see Fig. 29).

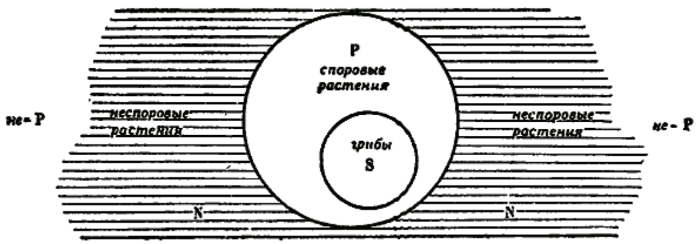


Fig. 29

In the figure, circle S means the entire volume of the logical class “mushrooms”, circle P — the entire volume of the logical class “spore plants”. The logical class “non—spore plants” (non—P) is represented by a plane extending indefinitely in all directions outside the circle P. The figure shows that the entire volume S is part of the total volume P. This relationship between the concepts of S and P expresses the initial form of the judgment “all mushrooms — spore plants “(in the general form:” all S — P “).

But from the same figure it is seen; that the same relationship between S and P can be thought of in other ways, namely, as the relationship between a concept that contradicts the concept of “spore plants”, that is, the concept of “non—spore plants” (non—P), and the concept of “mushrooms” (S) The figure shows that no part of the concept of “mushrooms” (S) can be found in any part of the concept of “non—spore plants” (non—P). It is this meaning that expresses the judgment obtained after the transformation by contrasting the predicate: “no non—spore plant is a mushroom” (or in the general form: “no non—P is S”).

§ 24. A *private affirmative judgment is usually not transformed in practice by contrasting a predicate.* Consider, for example, the partial affirmative proposition “some plants are spore plants”. What statement can be obtained from it regarding “non—spore plants”, that is, regarding a concept that contradicts the predicate of our judgment? — Formally, transformation by opposing the predicate is possible in this case too. In a transformed form, our judgment will take the following form: “not a single non—spore plant belongs to some plants.” The possibility of such a conversion is illustrated in Fig. 30.

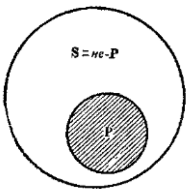


Fig. 30

This figure graphically represents the relationship between the subject and the predicate in the judgment “some plants are spore plants”. Those “some plants” which are “spore plants” are represented here by a part of circle S, which coincides with circle P. This part is shaded. It can be seen from the same figure that the “non—spore plants” are represented on it by that part of the circle S that lies *outside the* circle P and which remained unshaded. It’s clear that inside *this* part of the volume S, nowhere can be found none of those “some plants” (S) that coincide with P, that is, they are “spore plants”. But it is precisely this attitude that the transformed form of judgment expresses: “not a single non—spore plant belongs to some plants” (or in the general form: “not a single non—P belongs to some S”).

However, being possible in relation to partial affirmative judgments, the transformation by contrasting the predicate in this case has no practical meaning. After all, the meaning of any transformation of the form of judgment is that, as a result of the transformation, the relation between concepts in the judgment becomes more definite for thought. But it is quite obvious that in the case of a partially affirmative judgment this certainty fails. The proposition “some plants are spore plants” is much more definite than the proposition “no non—spore plant belongs to some plants”. Regarding the last judgment, the question immediately arises: to *which* “Some plants” do not belong to any of the “non—spore plants”? And only adding “to the number of those precisely which are controversial”, we make the meaning of the transformed judgment definite. But at the same time, we make it extremely empty. Indeed: in an clarified form, our opinion expresses only the idea that non—spore plants are not spore plants. For the sake of such a result, it was not worth making the conversion.

§ 25. A *general negative judgment is transformed by contrasting a predicate with a private affirmative judgment.* Consider, for example, the proposition “no spider is an insect.” What statement can be derived from it regarding “non—insects”? Obviously, this statement would be: “some non—insects are spiders.” Indeed, the transformed proposition establishes that within the logical class of “insects” there can be no part of the logical class of “spiders”. But this means that some of the animals that are not insects belong to spiders (see Fig. 31).

Fig. 31

In this figure, the circle F represents the entire volume of the logical class of arthropods, which includes the volumes of the logical class of “spiders” (S) and the logical class of “insects” (P) as subordinate to it.

The figure shows that not a single spider is an insect. It is this meaning that expresses judgment before conversion. In the same figure, “non—insects” are represented by the entire part of the circle F that is outside the circle P. It can be seen from the figure that some non—P will be S, that is, some of these non—insects will be spiders. This is precisely what the form of judgment expresses, which is obtained as a result of transformation by contrasting the predicate.

§ 26. **A** *private negative judgment is transformed by contrasting a predicate with a private affirmative judgment.* Consider, for example, the partial negative proposition that "some aircraft do not belong to the number of aircraft." We ask ourselves: what statement can be obtained from it regarding a concept that contradicts a predicate? Since the predicate of judgment is the concept of "airplanes," the concept that contradicts it will obviously be the concept of "non—airplanes." What can be said about this concept? It is obvious that "some non—aircraft belong to the number of aircraft" (see Fig. 32).

Fig. 32

The figure shows that the volume of aircraft (circle P) is part of the volume of aircraft (circle S). The same figure shows that some aircraft are not aircraft. It is this meaning that expresses the form of judgment before conversion. The part of the volume of aircraft, which does not belong to the volume of aircraft, is represented in the figure by the hatched part of circle S, i.e., the part of circle S lying outside circle P.

This same figure shows that the volume of non—aircraft, as the volume of any contradicting concept, is depicted by the whole plane indefinitely extending in all directions outside the circle P.

It can be seen from the figure that this plane includes the shaded part of the plane of the circle S lying outside the circle R. But this is precisely what expresses the form of judgment resulting from the transformation by contrasting the predicate: “some non—aircraft belong to the number of aircraft”. The general form of such judgments is: “some non—P belong to S”.

§ 27. It is easy to verify that each of the obtained rules for transforming judgments by contrasting a predicate corresponds to a certain rule of appeal. When appeal, for example, affirmative judgment obtained private affirmative judgment. Transformation by contrasting the predicate, obviously, corresponds to the transformation of a negative judgment into a partial affirmative one.

And in the same way, the rule by which a partial negative proposition is not usually addressed is obviously consistent with the rule that when transformed by contrasting a predicate, a *partial affirmative* proposition is usually not transformed.

There is nothing surprising in the fact that there is a correspondence between the rules of treatment and the rules of transformation by contrasting the predicate. And indeed: when transforming by contrasting a predicate, a statement is always obtained regarding a concept that contradicts the predicate. From this it is clear that each specific case of conversion must correspond to a certain specific case of transformation by contrasting the predicate.

Contrasting with a predicate is a combination of transformation with circulation. To make a contrast, first a transformation is made, and then the converted judgment is converted.

§ 28. Considering the transformation, we clearly see that in operations on a judgment our thought, as well as in operations on a concept and conclusion, is based on the laws of identity, contradiction, excluded third and sufficient reason.

According to the law of identity, the concept of an object is thought of in its original form of judgment as having a certain sign or relation. According to the law of contradiction, in the transformed form of judgment, it is thought that this particular sign or relation, which belongs to the concept of the subject, is incompatible with conflicting signs or relations. According to the law of the excluded third, in a transformed form of judgment it is thought that between the concept of a particular sign or relation of an object and a concept that contradicts it, there is no third concept of a sign or relation that could be attributed to the concept of an object. Finally, according to the law of sufficient reason, a sufficient reason is needed to transform a form of judgment.

Tasks

1. Give the following judgments a form convenient for logical analysis: "there is no comrade for taste and color"; "Philosophize—the mind will spin" (*Griboedov*); "When there is no agreement among the comrades, their work will not go in the way" (*Krylov*); "Genius and villainy are two incompatible things" (*Pushkin*); "You go to the left—you lose the horse, you go to the right—you yourself will disappear."

2. Determine which of the following judgments are general and which are singular: "all bodies exist in space"; "All credit tickets amount to a thousand rubles"; "All credit cards are printed on watermarked paper"; "Workers took their places at the machine tools"; "Workers are the advanced class of bourgeois society"; "knowledge is power"; "Turks are the people of poets"; "Life is a form of existence of protein bodies"; "The circle is divided by diameter into two equal parts"; "Planets move around the sun in ellipses"; "The total mass of all the planets is one seven hundredth of the mass of the sun."

3. Determine which of the following statements are *exclusive* and which are *distinguishing.*: "Only eucalyptus forests do not give shade"; "All genres are good, but boring"; "The planet Saturn is the only one of all the planets surrounded by a system of rings"; "All but one aircraft returned to their bases"; "One plane did not return to base"; "The entire detachment, with the exception of three scouts, was there"; "Only Kutuzov clearly understood that the battle of Borodino, despite the subsequent retreat of the Russian army, was a victory for Russian weapons."

4. Pay the following judgments: "some of the bugs are aquatic animals"; "Bullfinches are not migratory birds"; "Some chess players were mathematicians"; "Insects are arthropods"; "Every student must take an exam"; "Some trees do not drop leaves for the winter"; "Russians are Slavs"; "Not one of the great men could be indifferent to the fate of the fatherland"; "Some vowels in the English language are long vowels"; "In some languages there is no distinction between long and short vowels."

5. Convert the following judgments by contrasting the predicate: "all birds have excellent eyesight"; "Some birds live on the water"; "Most students received good and excellent grades"; "Not one of the planets shines with its own light"; "Not a single genre of literature has remained untested by Voltaire"; "Some mushrooms appear in the spring"; "Many storytellers of folk poetry do not know literacy."

6. Turn the following judgments: "some animals are difficult to tame"; "All people are capable of logical thinking"; "Some great poets were also great prose writers"; "Modern astronomers should have a good knowledge of not only mathematics, but also physics."

CHAPTER VIII. JUDGEMENT COMPARISON

Types of Judgments to Match

§ 1. When we think of a proposition, we can either concentrate on this particular proposition alone, or in addition to this proposition, we can also think of its relation to other propositions. For example, thinking the proposition "some students of our course study English," I can limit myself to considering this proposition alone. However, I will not wonder what the truth of this proposition is to the truth of other judgments.

But thinking this judgment, I can also ask the question, what is the relationship between it and another judgment, for example, a proposition: "not a single student of our course studies English." As soon as I compare these two judgments, I immediately see that they, firstly, cannot be both true right away. It cannot be immediately true that no student is learning English, and that some students are learning English. Secondly, a comparison of both of these judgments shows that they cannot be both false. It cannot be false at the same time that some students are studying English, or that no students are studying English. One of these two propositions must necessarily be true.

Thus, in thinking a proposition, we think either only that which is expressed by this proposition alone, or we also think of the relation of what is affirmed in it to what is affirmed in another proposition.

§ 2. Such a comparison of the two judgments is possible, however, not in all cases. Suppose I compare judgments:

“some students of our course study English” and “not a single student of our course studies Spanish”.

It is quite obvious that from a comparison of these two propositions one cannot see either that they cannot be both true at once, nor that they cannot be both false at once. It is possible, firstly, that both of them are true and, secondly, that both of them are false. From the truth of one of them does not follow the falsity of the other and vice versa.

It is easy to understand why this is so. In the case under consideration, there is no logical connection between the compared judgments, from which it would be clear in what relation the truth or falsity of one judgment is true or false of the other. But there is no connection because the predicate in each of the judgments being compared is different: in one predicate is the concept of “students learning English,” in the other is “students learning Spanish.”

But this connection would not exist if there were different subjects in the judgments under the same predicate. So, from a comparison of the judgments “all sailors learn English” and “some students do not learn English” it is not clear in what respect the truth or falsity of one judgment stands for the truth or falsity of another. Such judgments may be true in one case and false in another. There is no necessary logical connection between them. It does not exist because, under the same predicate (the concept of persons studying English), the subjects of both judgments will be different (“sailors” in one, “students” in the other).

Thus, from a comparison of two propositions, for which subjects are either different or have different predicates, one cannot see in what relation the truth (or falsity) of one stands for the truth (or falsity) of the other.

On the contrary, if the subjects and predicates of both juxtaposed judgments are the same and the judgments differ from each other not in terms of the subject and predicate, but only in quality and quantity, juxtaposing two such judgments makes it possible to immediately see in what relation the truth (falsity) of one of them stands for the truth (or falsity) of the other. So, in the first example ("some students of our course study English," "no student of our course studies English"), the subject and predicate of the first proposition are the same as the subject and predicate of the second proposition. They differ from each other only in their quality and quantity: the first judgment is partially affirmative, the second is generally negative. A contrast of these judgments allows us to see that they cannot be both true and cannot be both false.

Of these two propositions, in which the subjects and predicates are the same, but the quality and quantity are different, they are said to be the same in *material* but different in *form*.

§ 3. Since judgments that are identical in material but different in form can be contrasted, and since from this contrast one can see in what relation the truth of one is to the truth of the other, then logic systematically considers all possible cases of such a contrast. The meaning of this opposition is that with its help we can immediately determine whether opposed judgments, which have the same material but different shapes, are both true right away, and if one of them is true (or false), will it be true (or false) other.

Different types of opposed judgments are based on the fact that different in form, but identical in material judgments can, firstly, be in relation to the *opposite* to each other, secondly, can be in relation *to submission to* each other. Consider, for example, the judgments: "all gases can be liquefied in a liquid" and "some gases cannot be liquefied in a liquid". These judgments are the same in material and different in form: the first is affirmative, the second is private negative. These judgments are in relation to the opposite of each other. They cannot be both true at the same time.

Let us now consider two other judgments: “all gases can be liquefied in a liquid” and “some gases can be liquefied in a liquid”. Between these judgments there is no longer a relation of opposites, but of *subordination* : if it is true that *all* gases can be liquefied in a liquid, then the *Some* gases may be liquefied in liquids.

And indeed: saying that some gases can be liquefied in a liquid, we do not want to say that not all gases, but only a part of the gases, can be liquefied into a liquid. We express only the idea that some part of the gases, which is unknown, belongs to the number of gases liquefied in a liquid. It is quite obvious that in this sense of the judgment “some gases can be liquefied in a liquid” it will stand in relation to submission, and not the opposite of the judgment “all gases can be liquefied in a liquid”. What is true of an entire class will be even more true of its part.

Opposing Judgments by Opposition

§ 4. In contrasting opposing judgments, the following *three* cases are possible:

1. One of the opposing judgments is general, and the other is particular. This kind of opposite is called, as we already know, the *contradictory* opposite. For example: “all gases can be liquefied in a liquid”, “some gases cannot be liquefied in a liquid”. Or judgments: “no gas can be liquefied in a liquid”, “some gases can be liquefied in a liquid”.

2. Both opposing judgments are general, but one of them is affirmative and the other is negative. This kind of opposition is called, as we already know, *counterattack (or nasty)*the opposite. For example: “all gases can be liquefied in a liquid”, “not a single gas can be liquefied in a liquid”.

3. Both opposing judgments are private: one is private affirmative, and the other is private negative. This kind of judgment is called opposites *podkontrarnoy* or *subkontrarnoy (podprotivnoy)* opposite. For example: “some gases can be liquefied in a liquid”, “some gases cannot be liquefied in a liquid”.

The most important kinds of judgments are opposites contrast *contradictory* and opposed to *contraries*.

Conflicting judgments

§ 5. The relationship of the *opposing* opposite is determined by the following rules:

a) *Two conflicting judgments cannot be true at the same time.* This rule is based on the law of contradiction. If we immediately recognized the truths of judgments A and O (“all gases can be liquefied in a liquid”, “some gases cannot be liquefied in a liquid”) or E and I (“no gas can be liquefied in a liquid”, “ some gases can be liquefied in a liquid “), this would mean that we recognized the incompatible statements as true, that is, violated the law of contradiction.

b) *Two conflicting judgments not only cannot be true together, but, in addition, cannot be both false together.* This rule is based on the law of the excluded third. If we recognized that A and O are both false (for example, we would have recognized that it is false both that “all gases can be compressible in a liquid” and that “some gases cannot be liquefied in a liquid”), then this would mean that besides statements contradicting each other, some other third statement is possible regarding the contradictory opposite that is conceivable in these propositions. But this would mean that the law of the excluded third is violated.

Since conflicting judgments can neither be true nor false together, it follows that one of them is true and the other is false. For example, if it is true that “all gases can be liquefied in a liquid,” then it is false that “some gases cannot be liquefied in a liquid,” etc.

Counter judgments

§ 6. *Counter—judgments cannot both be true together*. This rule, common to both types of opposing judgments, is based on the law of contradiction.

Unlike conflicting judgments, *counter—judgments can both be false*. Since the opposite expressed by these judgments is not contradictory, the law of the excluded third with respect to counter—judgments is not binding. For example, the propositions “all bodies drown in water” and “no body drowns in water” are both false. Here, besides these two, there is a third possibility. This third possibility is expressed by judgments: “some bodies drown in water”, “some bodies do not drown in water”.

Why, then, in the case of a counter—contrast does the truth need not be expressed in one of two counter—judgments? Why is there a *third* possibility that expresses the truth?

The fact is that the proposition “no body drowns in water” does not only negate the statement expressed in the proposition “all bodies drown in water”. If we were only talking about such a denial, then it would be enough to make the judgment “all bodies drown in water” to contrast the contradictory proposition “some bodies do not drown in water”. But in our case, a denying counter—judgment expresses something more. It not only says that there are bodies that do not drown in water, but states that “no body drowns in water.” It is precisely this universal character of denial that makes the second counter—judgment as false as the counter—counter—

judgment opposite to it. In many cases, two opposing judgments are both false. Since the opposite counterpoint expresses something more in comparison with what is expressed in the case of *contradictory* opposites, then counter—judgments represent the relation of the most extreme opposite, which is only conceivable. No judgment can be more opposite to the judgment "all bodies are drowning in water" than the counter—judgment "no body is drowning in water".

On the contrary, in the case of *conflicting* judgments opposite—more distant, not as extreme as in the case of judgments of *contraries*. So, opposing the judgment "all bodies drown in water", the judgment "some bodies do not drown in water", we express, of course, the opposite, but not unconditional: by the judgment "some bodies do not drown in water", the truth of the judgment "some bodies drown" in water ", but this last proposition is not the opposite of the proposition "all bodies drown in water".

§ 7. Since two counter—judgments cannot be true together, in the case of the truth of one of them, the other will be necessarily false. If, for example, the proposition "all gases can be liquefied in a liquid" is true, then the counter—judgment "no gas can be liquefied in a liquid" will be false. His falsity is based on the law of contradiction, according to which two counter—judgments, as well as two contradictory ones, cannot both be true at once.

But since counter—judgments can be both false, the truth of the other is not at all visible from the falsity of one.

Indeed, the law of the excluded third, which applies to *conflicting* judgments, has no effect on *counter—* judgments. If it is false that "not a single body drowns in water", then this does not mean at all that "all bodies drown in water". The truth is "that some bodies drown, and some do not drown in water. Therefore, the falsity of a general judgment does not mean the truth of a counter—judgment regarding it.

Thus, in the case of counter—judgments, the truth of one means necessarily the falsity of the other, but the falsity of one does not mean at all that the other must necessarily be true. It may turn out to be true, but it may turn out to be false. So, for example, the judgment "not a single planet is inhabited by organisms" is false, but it is invisible from falsehood whether its counter—judgment will be true or false: "all planets are inhabited by organisms".

The judgment "no body has a stretch" is false. But the counter—judgment "all bodies are stretched" is true. The judgment "no man survives to a hundred years" is false. But also a false and counter—judgment: "all people live to be one hundred years old."

Judgment of Judgment

§ 8. The third kind of opposition of judgments is the opposite of *controversial* (or *subcontraditional*) judgments. In this case, *both judgments*, in contrast to the counter ones, *cannot be false at the same time.* It is impossible for, for example, the propositions "some gases can be liquefied in a liquid" and "some gases cannot be liquefied in a liquid" turn out to be both false. If the judgment "some gases cannot be liquefied in a liquid" is false, then this means the truth of the judgment "some gases can be liquefied in a liquid".

Indeed, the word "some" here means some part of the entire class of gases. But since this word does not indicate which part of the class of gases should be thought at the same time, it is possible that in one counter—judgment this word denotes one and the other part of the same class. Therefore, counter—judgments can be both true at the same time.

Comparison of Judgments of Subordination

§ 9. When juxtaposing judgments by submission, both juxtaposed judgments have the same quality: either affirmative or negative. Subordinate judgments differ from each other in *quantity* : one of them is always *general* , and the other is *particular* . This shows that when comparing subordinate judgments, *two* cases are possible:

1. Both assertions are affirmative, one of them is A, the other is I. For example: “all gases can be liquefied in a liquid”, “some gases can be liquefied in a liquid”.

2. Both judgments are negative, one of them is E, the other is O. For example: “no gas can be liquefied in a liquid”, “some gases cannot be liquefied in a liquid”.

The relation of submission is that, having recognized the general judgment as true, we see the truth of a particular judgment having the same material. If it is true that “all gases can be liquefied in a liquid”, then it is even more true that “some gases (that is, at least some of the gases) can be liquefied in a liquid”. If it were true that “no gas can be liquefied into a liquid”, then it would be all the more true that “some gases (that is, some gases) cannot be liquefied in a liquid”.

The comparison of judgments by submission is determined by the following rules:

a) *The truth of a general judgment (A, E) implies the truth of a subordinate private judgment (I, O).*If it is true that “all insects are arthropods,” then it is even more true that “some insects are arthropods.” If it is true that “not a single spider is an insect,” then it is also true that “some spiders are not insects.”

b) *The falsity of a particular judgment (I, O) implies the falsity of the corresponding general judgment (A, E).* If it is false that “some spiders are insects,” then it is also false that “all spiders are insects.” If it is false that “some gases cannot be liquefied in a liquid”, then it is also false that “not a single gas can be liquefied in a liquid”.

c) *The truth of a particular judgment (I, O) does not imply the truth of the corresponding general (A, E).*If some bodies are drowning in water, this does not mean that all bodies are drowning in water. If some bodies do not drown in water, then this again does not mean that no body is drowning in water.

d) *From the falsity of the general judgment (A, E), neither the necessary falsity, nor the necessary truth of the subordinate private judgment (I, O) can be deduced*. Here the question remains open and cannot be resolved from consideration of this judgment alone. It is possible that with further investigation it will turn out that the subordinate judgment will also be false. But it is also possible that it will turn out to be true. If the general judgment “all planets are inhabited by organisms” is false, then it remains unclear whether the subordinate private proposition: “some planets are inhabited by organisms” will be true or false. This judgment will be false if the counter—judgment is true: “no planet is inhabited by organisms.” But it will be true if the counter—judgment “no planet is inhabited by organisms” turns out to be false.

Thus, in the case of submission of judgments:
1) the truth of the *general* judgments mean the indispensable truth of a private judgment, but the truth of a *private* judgment does not yet mean the necessary truth of the *general* ;
2) the falsity of a *private* judgment means the falsity of a *general* judgment as well , but the falsity of the *general* does not yet mean the inevitable falsity of the *particular* .

§ 10. When comparing subordinate judgments, it must be remembered that the subordination relation takes place only where a particular judgment (I), subordinate to a general judgment (A), does not have the *opposite* meaning the meaning of general judgment. Consider from this point of view the judgments: “all gases can be liquefied in a liquid” and “some gases can be liquefied in a liquid”.

Will there be a subordination relationship between these judgments?—It depends on the meaning of the expression “some gases” in the second judgment. If the expression “some gases” makes sense: “some, it is not known which part of the gases”, then in this case a particular judgment (“some gases can be liquefied in a liquid”) will really be subject to a general judgment (“all gases can to be liquefied in a liquid “). In this case, truth A means together truth I.

The same condition leads to the same result if both judgments are negative (E and O). So, the judgment “some gases cannot be liquefied in a liquid” (O) is subordinated to the judgment “no gas can be liquefied in a liquid” (E), provided that the expression “some gases” makes sense: “some, unknown which part of the gases. “ And in this case, the truth of E means together the truth of O.

But if the proposition “some gases can be liquefied into a liquid” (I) it makes sense: “not all gases, but only a part of the gases can be liquefied into a liquid”, then the proposition is not will be subject to the judgment “all gases can be liquefied in a liquid” (A). In this case, the judgment “some (i.e., *not all*) gases can be liquefied in a liquid” will be the *opposite* judgment A (“all gases can be liquefied in a liquid”), so that truth A is incompatible with truth I.

The same meaning of the expression “some gases” leads to the same result if both judgments are negative (E and O). And in this case, the relationship between E and O will be the ratio of the *opposite* , not submission, and therefore the truth of E is incompatible with the truth of O.

The logical square

§ 11. All possible types of opposition and subordination of judgments are easily accessible for viewing with the help of the so—called “logical square”. This is the name of a visual diagram depicting all the relationships between propositions that have the same material but different shapes.

This square is constructed as follows (see Fig. 33). The upper corner on the left is marked with the letter A — a sign of affirmative judgments. The upper corner on the right is marked with the letter E — a sign of generally negative judgments.

The lower corner on the left is marked with the letter I — the sign of partial affirmative judgments, and the lower corner on the right is marked with the letter O — the sign of partial negative judgments.

§ 12. Having placed the signs of quality and the number of judgments on the vertices of the square, we easily notice that the *lateral* sides of the square AI and EO clearly represent the relations of *subordination*.

Indeed, the particular judgment “some planets are inhabited by organisms” (I) is subordinate to the general judgment “all planets are inhabited by organisms” (A). The same is true with respect to E – O judgments: the particular judgment “some planets are not inhabited by organisms” (O) is subordinate to the general judgment “no planet is inhabited by organisms” (E).

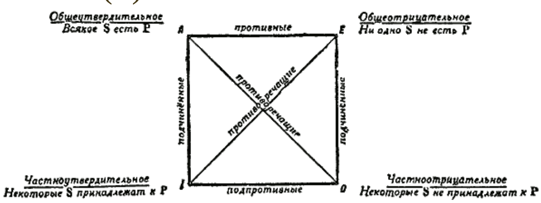


Fig. 33

Considering sequentially all the relationships between judgments A, I, E, O, indicated on the "logical square", we can easily repeat all the rules for matching judgments by opposite and by submission that we have deduced above.

§ 13. We examined three types of transformation of judgments: conversion, transformation, opposition to the predicate. In all these three forms, *only the form of* judgment changes , but its meaning remains the same. Neither conversion, nor transformation, nor juxtaposition to the predicate as a result of transformation gives us any new truth in return for truth expressed in the original form of judgment.

However, there is such a way of transforming the form of judgment, in which, as a result of the transformation, not only one form of judgment is changed, but some new truth is added to the truth already known. This method of transforming the form of a separate judgment is called the *restriction of the third concept.*

Consider, for example, the judgment "the Orion nebula is a galactic nebula" [1]. We ask ourselves: what should be the spectrum of the Orion nebula? Obviously, it should be one of those spectra of galactic nebulae. Let us express this idea in a new proposition: "the spectrum of the Orion nebula is the spectrum of a galactic nebula". Compare this new proposition with the original proposition: "The Orion Nebula is a galactic nebula." It is easy to see that the transformation we made is a transformation of more than just a form of judgment. In the original judgment, it was a question of the "Orion nebula", in the transformed judgment it was not a question of the Orion nebula in the entire content of this concept, but only the " *spectrum* Nebulae of Orion. " In the predicate of the initial proposition, the concept of "galactic nebula" was thought of, in the predicate of the transformed proposition, again, not all the content of this concept is thought, but only the concept of "*spectrum of the* galactic nebula". As a result of the transformation, we got a judgment that gives us some new truth in comparison with the truth of the original judgment. This new truth came about as a result of our introduction to the judgment of a certain third concept—the concept of "spectrum".

But we did not just introduce the new concept of "spectrum" into our judgment. Introducing this new — third — concept, we, firstly, *limited* him, defining with his help the concept of the subject. As a result, the subject of the new judgment was not the old concept of the Orion Nebula, and not just the new concept of the spectrum, but the concept of the spectrum of the Orion Nebula.

Secondly, introducing into the composition of the judgment the new concept of "spectrum", we did not leave the *predicate of* our judgment unchanged . In the transformed proposition, the predicate is no longer the concept of "galactic nebula", but the same new, third concept of "spectrum". However, even here, having become a predicate of a new proposition, the concept of "spectrum" is no longer conceived in all its content: it is *limited* through the concept of "galactic nebula." Thus, the predicate of the new judgment was not the old concept of "galactic nebula" and not just the new — third — the concept of "spectrum", but the concept of "spectrum of the galactic nebula".

Now we see that the transformation of judgment described above is indeed a "limitation of the third concept". This concept is limited twice: becoming a *subject* and becoming a *predicate of a* new judgment. And becoming a subject and becoming a predicate, it is limited by the concepts of the original proposition: in the first case, by the subject of this proposition, in the second case, by its predicate.

Since the transformation by restricting the third concept gives as a result not a simple repetition of the previous thought, but some new truth, this form of transformation of the judgment is transitional from conversion, transformation and contrasting the predicate with various forms of *inference* .

Tasks

1. Having recognized the *first* judgment in each pair of the following judgments *as true* , determine what can be expressed under this condition regarding the truth (or falsity) of the *second* the judgments of the same pair: “all students solved the control problem”, “some students did not solve the control problem”; ”Some fish can fly”, “no fish can fly”; ”Some poets were playwrights”; “some poets were not playwrights”; ”Some rivers flow neither into other rivers, nor into lakes, nor into seas”, “all rivers flow either into other rivers, or into lakes, or into seas”; ”Some spiders are poisonous”, “some spiders are not poisonous”; ”Some bodies have a stretch”, “some bodies have no stretch”; ”All insects are arthropods”, “some insects are arthropods”; ”Some planets have satellites”; “all planets have satellites”; ”Some spiders are not insects”, “not a single spider is an insect”; ”Some guardsmen are order bearers”, “all guardsmen are order bearers”; ”Not a single shooting star is really a star”, ”Some shooting stars are not really stars”; ”No planet shines with its own light,” “all planets shine with its own light.”

2. Having recognized the *first* judgment in each pair of the following judgments as *false* , determine what can be expressed under this condition regarding the truth (or falsity) of the *second* judgments of the same pair: “no atom is divisible”, “all atoms are divisible”; ”No student can solve this problem”, “some students can solve this problem”; ”No student can solve this problem”, “every student can solve this problem”; ”Some planets shine with their own light”, “all planets shine with their own light”; ”All snakes are poisonous”, “not a single snake is poisonous”; ”All composers have absolute pitch”, “some composers have absolute pitch”; ”All composers were pianists”, “some composers were not pianists”; ”All composers were pianists”, “not a single composer was a pianist”; ”Some bodies are unchanging”, “all bodies are changeable”; ”Some bodies are unchanging”, “some bodies are changeable”; ”All engines are internal combustion engines”, “some engines are internal combustion engines”.

3. In which of the following pairs of judgments there is a relation of *subordination* and in which there is a relation of *opposite* : “all planes are three—engine”, “not all, but some planes are three—engine”; ”None of these medicines can help the patient”; “some of these medicines cannot help the patient”; ”Some (not all) students serve”, “all students serve”; ”At least some elements are decomposable”, “all elements are decomposable”; ”All difficulties are surmountable”, “only some difficulties are surmountable”; ”Only some books are worthy of attention”, “all books are worthy of attention”; ”Each shell moves in a parabola upon exiting the gun”, “some shells move in a parabola upon leaving the gun”

CHAPTER IX. INFERENCES

Definition of Inference

§ 1. Some truths are established directly, without any reasoning, by simply discerning what observation shows, or what seems obvious to thought. These are the judgments: “the sky is overcast now”; ”This book is on the shelf”; ”The whole is greater than its part”, etc. The truth of such judgments does not have to be proved, since it is obvious.

But the obvious statements are only a small part of all the truths. In the vast majority of cases, truth is not a position directly visible or taken for granted. Usually, in order to establish the truth, it is necessary to carry out a special study in each case: clearly raise the question, take into account other, previously established truths, collect all the facts and observations necessary to solve the problem, make experiments, consider their results, verify in practice the validity of the conjecture and etc.

Logical thinking is carried out when obvious truths are expressed, and when truths are not obvious, but are obtained in a more complex way. In the latter case, logical thinking takes the form of *reasoning*. A reasoning is a series of judgments that all relate to a certain subject or question and which go one after another in such a way that others necessarily follow or follow from the previous judgments, and the result is an answer to the question posed. That the organism consists of cells, that the area of the triangle is equal to half the product of the base and the height, that Peter I was one of the greatest Russian statesmen — all these and many other judgments are not simply proclaimed as truths, but are justified by special reasoning.

§ 2. Already considering judgment and the various forms of transformation of judgments, we have seen that judgment is rarely thought of separately from other judgments. In order to correctly understand the meaning of a given judgment, we often have to consider not only this judgment, but also other judgments with which it is connected by the relation of opposite or submission.

In many cases, we cannot even be convinced of the truth of a given judgment until we consider its relation to other judgments. Suppose, for example, that we do not know what is the relationship between the concept of "bamboo" and the concept of "plants spikelets." Do bamboos bloom in the same way that rye and wheat bloom, or are spikelets not a form of inflorescence of bamboos?

As long as we consider only the relationship between the concept of "bamboo" and the concept of "flowering spikelets", we cannot answer the question posed and, therefore, cannot say what relation the "bamboos" to "flowering spikelets" are. We will now act differently. Consider, before answering the question, two other judgments: "all bamboos are cereals" and "all cereals bloom in spikelets". Suppose that we have already been convinced of the truth of these two judgments. But if we know that all bamboos are cereals and that all cereals bloom in spikelets, can we say something about the relation of the concept of "bamboos" to the concept of "flowering spikelets"?—Obviously, we can. Based on the fact that all bamboos are cereals and that all cereals bloom in spikelets, we can make the judgment: "all bamboos bloom in spikelets." This judgment will be true. But we did not see the truth of this judgment directly. From the concepts of "bamboo" and "flowering spikelets", we could not immediately see what the relationship between these concepts would be.

This attitude, expressed in the judgment "all bamboos bloom in spikelets," we received through inference, or inference. The connection between the concepts of “bamboo” and “flowering spikelets”, which is not immediately or directly visible, we have *deduced* , that is , we *have* understood through the relationship of each of these concepts to a certain third concept — to the concept of “cereals”. That is why we needed *two* judgments to substantiate the conclusion . In one of them, we examined the relationship between the concept of “bamboo” and the concept of “cereals,” in the other, the relationship between the concept of “flowering spikelets” and the same concept of “cereals.” This concept of “cereals” turned out to be that mediating, or third, concept with the help of which we were able to understand the previously unseen connection between the concept of “bamboo” and the concept of “flowering spikelets”.

§ 3. Having acknowledged that these judgments are true, we must recognize the true judgments arising from them. That logical action by which the truth of these new judgments is revealed is called *inference*. In other words, a conclusion is a form of thinking, consisting in the fact that the truth of a certain judgment is derived from the truth of two or more other judgments.

Judgments from which a conclusion can be drawn and from which, since they are recognized as true, any conclusion necessarily follows, are called *premises* or *premises of* inference. In our example, the premises of inference are the judgments: “all bamboos are cereals” and “all cereals bloom in spikelets”.

Judgment, which is recognized by the true way of inference, that is, by comparing the premises, is called a *conclusion* , or *conclusion* , in the narrow sense of the word. In our example, the conclusion will be the judgment “all bamboos bloom in spikelets.”

Sometimes the whole inference as a whole, that is, all the premises and the conclusion taken together, is also called the conclusion — this time in the broad sense of the word. So, in our example ("all bamboos are cereals, all cereals bloom in spikelets, therefore, all bamboos bloom in spikelets"), the conclusion in the broad sense of the word will be all this conclusion as a whole, that is, its premise and its conclusion.

§ 4. The purpose of the conclusion is to derive a *new* truths from truths that are already known to us. Any true conclusion does not just repeat what we already know from the premises in the conclusion. True inference leads our thought beyond what we know from the premises, attaches a *new* truth to previously established truths . The package "all bamboos are cereals", taken separately, does not contain the idea that "all bamboos bloom in spikelets." This idea is not contained in the premise, since the concept of "flowering with spikelets" is not included in the number of essential signs of the concept of "cereals". Although all cereals bloom "spikelets", I can nevertheless think of the concept of "cereals", without necessarily thinking about this property of cereals. I can formulate the concept of "cereals" through such a group of essential features, which will not include the sign of "flowering spikelets" at all.

And just as well, in the premise "all cereals bloom in spikelets", taken separately, the idea that "all bamboos bloom in spikelets" is not yet contained. This idea is not contained in the premise, since from the premise "all cereals bloom in spikelets" it is not yet clear that the bamboos are among the cereals. Although all bamboos are among the grains, I can nevertheless think of the concept of "cereals", not knowing that bamboos also belong to the number of cereals.

But as soon as we compare both of these judgments — "all bamboos are cereals" and "all cereals bloom in spikelets", their comparison leads us to a *new* truth — to the conclusion that "all—bamboos bloom in spikelets." This conclusion is a different thought in comparison with each of the premises, taken separately. The conclusion is not a simple repetition of the truths that I already thought in premises. The conclusion is not a simple transformation of the form of premises that does not change their logical meaning, such as conversion or transformation.

The conclusion is the extraction of a *new* truth from the truths already recognized earlier and already known.

§ 5. But the conclusion does not simply add the new truth to the truths already established or known. The new truth is derived from the premises in such a way that its adherence to the premises is recognized by us as absolutely *necessary* and *necessary* for our thought.

We may not know that all bamboos are cereals and that all cereals bloom in spikelets, and therefore disagree with someone who tells us that all bamboos are cereals and that all cereals bloom in spikelets. But if we agree that all bamboos are cereals and that all cereals bloom in spikelets, then, agreeing with both of these premises, we can no longer disagree with the fact that all bamboos bloom in spikelets. Consent to the premises here necessarily leads to agreement with the conclusion. The conclusion is not simply attached to the premises as a new thought compared to premises. The conclusion follows from the premises as thought associated with the premises of the necessary logical connection. This connection, firstly, is based on the law of *sufficient reason*. Only that conclusion is true and is accepted as true, which has a sufficient basis in the truth of the premises and in the correctness of the logical course of inference. Secondly, this connection is based on the law of *contradiction* . Thinking of the premise and conclusion, we understand that it is impossible, while agreeing with the premise, not to disagree with the conclusion. If, having agreed that all bamboos are cereals and that all cereals are spikelets, our interlocutor would deny that all bamboos are spikelets, he would thereby show that in this case he contradicts himself, i.e. thinks inconsistently, illogically.

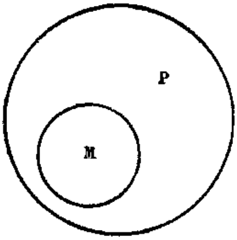
Having agreed that bamboos are cereals and that all cereals bloom in spikelets, but asserting at the same time that bamboos do not bloom in spikelets, our interlocutor would thereby admit that there are cereals that *do not bloom in spikelets* . But this means that he would recognize as true a judgment that *contradicts* the very premise with which he has already agreed and which states that "all cereals bloom in spikelets." Such an interlocutor would immediately assert that "all cereals bloom with spikelets" and that "some cereals do not bloom with spikelets", that is, would violate the law of contradiction.

The logical connection between the conclusion and the premises is based, thirdly, *on the law of the excluded third*. And indeed: if the interlocutor denies that all bamboos bloom in spikelets, then, since, by virtue of the law of the excluded third, except for the judgments "all bamboos bloom in spikelets" and "some bamboos do not bloom in spikelets", no third judgment about the relation of "bamboos" is possible to the "flowering spikelets." But since such a third judgment is impossible, the denial of the truth of the judgment "all bamboos bloom in spikelets" is equivalent to the statement of the truth of the judgment "some bamboos do not bloom in spikelets." However, to acknowledge our premises ("bamboos are cereals", "all cereals bloom in spikelets") and at the same time to admit as true that "some bamboos do not bloom in spikelets" means breaking the law of contradiction.

Thus, the law of contradiction, both in itself and in conjunction with the law of the excluded third, really determines the logical connection between the premises and the conclusion in the conclusion. But this connection is also based *on the law of identity*. The conclusion drawn from the premises could not be true if the terms “bamboos”, “cereals”, “flowering spikelets”, appearing in the conclusion each twice, were not thought in the identical sense, that is, if in the conclusion somewhere the law of identity is violated. If, for example, under “cereals” in one premise one content was thought, and in another — different, then the conclusion about the relationship between “cereals” and “flowering spikelets” from such premises could not be deduced. This conclusion is possible only on the basis of the relationship of each of these concepts to the concept of “cereals” disclosed in the premises. But it is quite obvious that if the concept of “cereals” is not identical in both premises, then it is impossible to establish through this concept any logical connection between the concept of “bamboo” and the concept of “flowering spikelets”.

Thus, all four logical laws of thinking — the law of identity, the law of contradiction, the law of the excluded third and the law of sufficient reason—are applied in all conclusions. Without these laws, inference could not be seen as a logical connection between the premises and the conclusion.

Any correct conclusion reveals for our thought the *necessary* relationship between objects that are thought in premises and in conclusion. Thus, the premise “all cereals bloom in spikelets” expresses the idea that the property of flowering with spikelets is a necessary property of all cereals; therefore, all objects called cereals, are necessarily included in the number of “flowering spikelets” (see. Fig. 34).



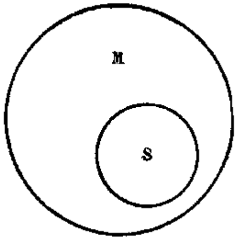


Fig. 34 . . . . **Fig. 35**

In this figure, the volume of the concept of “cereals” is depicted by the circle M, the volume of the concept of “flowering spikelets” by the circle R. From the figure it is clear that all cereals must belong to flowering spikelets, that is, all M must belong to R. Parcel “All bamboos are cereals” expresses the idea that the properties of cereals are necessarily the properties of bamboos; therefore, all objects called “bamboos” are necessarily included in the number of cereals (see Fig. 35).

In this figure, the volume of the concept of “bamboos” is depicted by the circle S, the volume of the concept of “cereals” is shown by the circle M. From the figure it can be seen that all bamboos necessarily belong to cereals, that is, that all S necessarily belong to M. Comparing both of these parcels, we obtain the conclusion: “all bamboos bloom with spikelets.” This conclusion expresses the idea that the property of all cereals to bloom with spikelets is also a property of all bamboos; therefore, all objects called “bamboos” are necessarily included in the number of “flowering spikelets” (see Fig. 36).

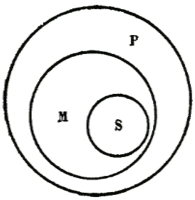
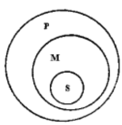


Fig. 36

From this figure it is clear that not only all the cereals need blooming in spikelets, as was evident from the first premise, and that not only all bamboos are necessary, cereals, as was seen from the second premise, but that, in addition, all bamboos need to bloom spikelets. The need for a conclusion necessarily follows from the truth of the premises: since, according to the already clarified relations between the properties of bamboo, cereals and flowering spikelets, the entire volume of the concept of “cereals” (circle M) is included in the volume of the concept of “flowering spikelets” (circle P) and since the whole the volume of the concept of “bamboo” (circle S) is included in the volume of the concept of “cereals” (the same circle M), then the entire volume of the concept of “bamboo” must be included in the volume of the concept of “flowering spikelets” (the entire circle S must be inside the circle P).

If someone, recognizing that “all bamboos are cereals” and that “all cereals are spikelets”, would at the same time deny that “all bamboos are spikelets”, this would be tantamount to as if someone recognizing that the circle M fits all inside the circle P and that the circle S fits all inside the circle M, he would at the same time deny that the circle S all fits inside the circle P. A person who thinks in this way would be in conflict with his own thought: agreeing with the premises, he would think of the circle S entirely *inside the* circle P (see Fig. 36a); at the same time, denying the conclusion, he would think circle S outside circle P (see Fig. 37).



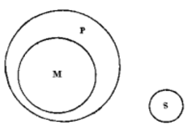


Fig. 36a . . . **Fig. 37**

§ 6 . Since the conclusion 1) gives a *new* thought in the conclusion compared to the thoughts expressed in the premises, and 2) reveals the *need for a* connection between the premises and the conclusion, inference is a very important form of logical thinking. Where we do not immediately see the connection between the two concepts, we can find this connection *through the third* concept, if we only know in what relation this third concept stands for each of our two concepts, the relationship between which we are trying to clarify. It is this problem that is solved by inference. The two concepts, the relation between which is not directly visible, are connected by inference by means of the third concept, knowing the relation of this third concept to each of them individually.

It is especially important that the connection between concepts, revealed by inference, is a *necessary* connection . If the premises are true and if during the inference we did not make any logical mistake, then the conclusion will always be necessary true. The conclusion does not reveal such a connection between premises and conclusions, which may be true, but may not be true. The conclusion reveals the *need for a* connection that exists between the premises and the conclusion. He who is convinced of the truth of the premises must agree, he cannot but agree with the truth of the conclusion.

This property of inference — the logical necessity of any correct conclusion derived from true premises — makes inference an important link in *proof* and *rebuttal,* in all kinds of *disputes* and *discussions* . Inference is a powerful means of persuasion. So, having received in the conversation or in the dispute the consent of the enemy with the premises, we can easily force him to agree with the conclusion, as soon as we show that the premises received by him necessarily compel the agreement with the conclusion as well. Considering the previously proved theorems as premises of inference, we can show that the new theorem, which we undertook to prove, is nothing but a conclusion necessary from the truth of these premises, etc.

In view of the importance of inference for logical thinking, logic systematically considers all forms of inference. Logic explores what types of inferences exist, what value each of them represents for knowledge, what is the structure of each form of inference, according to what logical rules we make inferences, and what logical errors are possible in inferences.

Division of Inferences into Syllogistic and Non—Syllogistic

§ 7. In the practice of logical thinking, there are various types of inferences. To distribute conclusions by type, it is necessary to proceed from an analysis of premises, i.e. judgments.

We already know that any proposition is a statement of some relationship between the concepts of subject and predicate. Depending on the type of relationship between the subject and the predicate, all judgments are divided: 1) into judgments about the property belonging to the subject; 2) on judgments on the belonging of an object to a class of objects (or one class to another class of objects); and 3) on judgments on the relations of magnitude, space, time, cause and action, strength, kinship, etc.

Any inference is based on a consideration of the relationship between the concepts of premises. Since premises in their logical form are judgments, it is obvious that the relations between the concepts included in the premises should, in general, be the same as in judgments.

It follows that the conclusions, as well as judgments, are divided into types depending on the type of relationship existing between the concepts included in the premises of the judgment.

The first group of inferences are inferences, the premises of which express relations of *belonging*(a) the properties of an object and (b) an object to a class of objects (or one class to another class of objects). An example of the conclusion that one class of objects belongs to another class is the conclusion considered by us: “all bamboos are cereals, all cereals bloom in spikelets, therefore, all bamboos bloom in spikelets.” The conclusions, the premises of which express the relations of belonging of objects to the class of objects or a class of another class, are called *syllogistic* , or *syllogisms* . The name “syllogism” was introduced into logic by the ancient Greek philosopher Aristotle (384–322 BC) and comes from the word συλλογισμός, meaning “conclusion”.

Second the group of inferences consists of inferences whose premises express not relations of belonging, but relations of magnitude, space, time, cause and action, strength, kinship, etc. An example of such a conclusion: “Moscow is east of Smolensk, Kazan is east of Moscow, therefore Kazan is east of Smolensk “. In this conclusion, the premises do not express the relation of belonging, but the relative position of objects in space.

The conclusions, the premises of which express the relations of magnitude, space and time, causes and actions, forces, etc., are called *non—syllogistic*.

§ 8. The difference between syllogistic and non—syllogistic conclusions depends on the difference between premises, i.e., between judgments that substantiate the conclusion. In judging the belonging of an object to a class of objects, the relationship between the content of the subject and the content of the predicate naturally determines the relationship between the *volumes of the* subject and the predicate. All judgments of this kind are connected not only with operations of *definition of a* concept, by means of which the *content of* concepts of a subject and a predicate is clarified , but also with operations of *division of a* concept, by means of which generic concepts are divided into specific, and specific are included in the generic.

In accordance with this, syllogisms, i.e., conclusions whose premises represent judgments about the belonging of an object to a class of objects, also give conclusions regarding the belonging of an object to a class of objects. In syllogisms, such relations between concepts in terms of content are considered, from which relations between the same concepts in *volume* can be immediately and easily derived.

Therefore, the relations between concepts in *content* , expressed in the premises of the syllogism and justifying its conclusion, can easily be represented by circles or other figures, the mutual position of which represents the relationship between the *volumes* the same concepts. We could verify this by looking at an example of syllogism: “all bamboos are cereals, all cereals bloom in spikelets; therefore, all bamboos bloom in spikelets.”

But syllogisms make up only part of all kinds of inferences. There are conclusions, the forms of which cannot be reduced to the forms of syllogisms. These conclusions do not address the question of whether a known class of objects belongs to another class of objects. In these conclusions, questions are solved about the relationship between objects in magnitude, in position in space, in simultaneity or sequence in time, in reason and action, in strength, in kinship, etc. Already in the premises of these conclusions, the object of thought is not the relation of belonging subject to a class of subjects, but other types of relationships. In accordance with this and in the conclusions of these conclusions, the object of thought is not relations of belonging of the object to the class of objects, but relations of magnitude, position in space, time, etc.

But that is why relations between *volumes* are not characteristic of non—syllogistic conclusions concepts included in the premises and conclusions. True, in the case of non—syllogistic conclusions, as in the case of syllogisms, the well—known relationship between the *content of the* subject and the *content of the* predicate in the premises and conclusion necessarily determines the well—known relationship between the *volumes of* these concepts. So, in our example of a non—syllogistic conclusion in the premise “Moscow lies east of Smolensk”, the relationship between the concepts “Moscow position in longitude” and “Smolensk position in longitude” also determines the relationship between *volumes* both concepts. According to this relation, the entire scope of the concept of “Moscow” is included as part of the scope of the concept of “all cities east of Smolensk”. Similar relations of volumes are also expressed by the second premise and conclusion of our conclusion “Kazan lies east of Moscow”, “Kazan lies east of Smolensk”.

But although, thus, in the case of non—syllogistic inferences, the relations between the content of concepts included in the premises and in conclusion determine the relations between their *volumes as well* , these relations in non—syllogistic inferences do not answer the question that is solved in the conclusion.

When we say that “all bamboos are cereals”, this proposition answers the question of the relationship between the volumes of the concepts of “bamboos” and “cereals”. Here we really think that the class of objects belongs to another class.

On the contrary, the judgment “Moscow lies east of Smolensk” does not answer the question of the relationship between the volumes of the concepts “Moscow” and “cities lying east of Smolensk". True, from the judgment "Moscow lies east of Smolensk" it indisputably follows that the scope of the concept of "Moscow" is part of the scope of the concept of "all cities east of Smolensk". But it is not this relationship between the volumes of concepts that constitutes the answer to the request of our thought when we affirm that Moscow lies east of Smolensk. Even having transformed this judgment into a judgment about the relationship between volumes, we, under the guise of a relationship between volumes, think of a relationship according to its position in space. The essence of this relationship does not depend on whether the concept of "Moscow" is included in the broader class of "all cities east of Smolensk."

Saying "Moscow lies east of Smolensk," I essentially do not think of Moscow's attitude to the class of cities east of Smolensk. In this case, a judgment is made not about Moscow's attitude to a whole class of other cities, but about the position of *one* Moscow city in longitude relative to another — *also the only one* — the city of Smolensk.

Simple Categorical Syllogism

§ 9. The purpose of the syllogism, like any other inference, is to obtain from the premises of a *new* judgment, or conclusion. In this case, the relation conceivable in the conclusion between the subject and the predicate of inference is not established directly. This relationship is not directly visible from either the first or second premises, taken separately. This attitude is clarified for thought only after comparing both premises of the syllogism. Comparing the premises, we consider the relation of the subject and the predicate of inference to a certain third concept. It is only through the relation of this third concept to the subject of inference and to the predicate of inference that the relation between the subject and the predicate of inference is clarified.

For the convenience of analysis, we denote all the components of the syllogism by special signs. For this purpose, we write a syllogism, placing one premise under another, and the conclusion, or conclusion, under the second premise. Separate the conclusion from the premises with a horizontal line.

Example syllogism:

All frogs are amphibians.
All amphibians are vertebrates.

All frogs are vertebrates.

Consider first the conclusion, or conclusion. In it, the concept of a predicate is denoted by the letter P, the concept of the subject by the letter S. Since the premises express relations of belonging, the conclusion also expresses the relation of belonging. Therefore, all the relations between the concepts included in the premises and the conclusion, being the relations between the content of the concepts, will be at the same time the relations of their volumes. Obviously, the volume of the predicate in the output is *larger than the* volume of the subject (the volume of the concept of "vertebrates" is larger than the volume of the concept of "amphibian").

On this basis, the inference predicate (P) is called the *larger* concept, or the greater term of the syllogism, and the inference subject (S) is called the *lesser* term. That premise, which includes the *larger* term (P), is called *greater premise of* syllogism. In our example, the larger term is the concept of "vertebrates," the smaller term is the concept of "frogs," the larger premise is the judgment, "all amphibians are vertebrates." That premise, which includes the *smaller* term (S), is called the *lesser premise of the* syllogism. In our example, the smaller premise is the judgment "all frogs are amphibians."

The conclusion "all frogs are vertebrates" is really justified by these premises. Having recognized that "all amphibians are vertebrates" and that "all frogs are amphibians", we cannot but admit that "all frogs are vertebrates".

How justified is this conclusion?—In a smaller premise, we established the relation of a smaller term to a certain third concept — to the concept of "amphibian". A smaller premise established that "all frogs are amphibians," that is, the entire scope of the concept of "frogs" is fully included in the scope of the concept of "amphibians". In a larger premise, we have established the relation of this same third concept to a larger term — to the concept of "vertebrates". A larger premise found that "all amphibians are vertebrates," that is, the entire scope of the concept of "amphibians" is fully included in the scope of the concept of "vertebrates". As a result, it was possible to establish — through the third concept (the concept of "amphibians") — the relationship between the concept of "frogs" and the concept of "vertebrates": since all frogs are among the amphibians, and all amphibians, in turn, are among the vertebrates, then all frogs should also be among the vertebrates. Or otherwise: since all amphibians are included in the number of vertebrates, and all frogs are in the number of amphibians, then all frogs must be included in the number of vertebrates.

The third concept, through which the conclusion clarifies the relationship between the smaller and the larger terms, is called the *average term* syllogism.

As can be seen from the example, the middle term is included in each of the premises, but is not included in the conclusion, or conclusion, of the syllogism. It is easy to understand why this is so. The goal of syllogism, as we already know, is to clarify the relationship between the two concepts of S and P. The average term appears in the syllogism not because the average term interests us in itself. It appears because only through the relation of the average term to S and P can the directly invisible relation between S and P be clarified. But the clarification of the relation of the average term to S and P is achieved already in the premises of the syllogism: a larger premise reveals the relation of the average term to P the smaller one to S. As soon as the task of clarifying these relations is completed, as soon as the relationship between S and P has become clear from the relationship of each of them individually to the middle term, the average term ceases to be the subject of our thoughts. Our thought is no longer directed to the middle term, but to the relationship between S and P, which was clarified using the middle term. Therefore, only S and R. appear in the conclusion, or in the conclusion, of the syllogism.

Let us designate the middle term with the letter M. Then our syllogism can be represented by the following scheme, or, as they say in logic, a “figure”:

M — P		S — M
S — M	or	M — R
———		———
S — P		S — R

As can be seen from the example and from its scheme, the order of the packages does not play any role: a larger package can be the first, and a smaller one the second and vice versa. The conclusion, i.e., the logical connection between the subject and the predicate, does not depend on the order of premises in the syllogism.

This circumstance must be remembered so as not to associate the names “larger premise” and “smaller premise” with the order in which the parcels follow one after another. Regardless of this order, only the premise that includes the *larger* term, i.e., the predicate of inference (P), will be *larger* , and less than only the premise that includes the *smaller* term, i.e., the subject of inference (S).

Syllogisms can have a different structure of premises, and therefore the very conclusions in them can stand depending on different rules. Logic establishes all these rules and studies all varieties of syllogisms.

§ 10. The first group of syllogisms is the so—called *simple categorical* syllogisms. So—called syllogisms in which the conclusion is obtained from two premises and in which both premises are categorical judgments.

Considering the simple categorical syllogisms encountered in the practice of thinking, one can notice that the arrangement of concepts, or terms, in the premises of these syllogisms can be different.

Consider the following syllogism:

All amphibians are vertebrates.		M — R
All frogs are amphibians.		S—M
————————————		———
All frogs are vertebrates.		S—P

In it, the middle term in the larger premise is the subject, and in the smaller—the predicate.

A syllogism in which concepts, or terms, are arranged in this way is called a syllogism of the *first figure*.

In our example of the syllogism of the first figure, a smaller premise ("all frogs are amphibians") reveals that the entire volume of class S is included as part of the larger volume of class M (see Fig. 38).

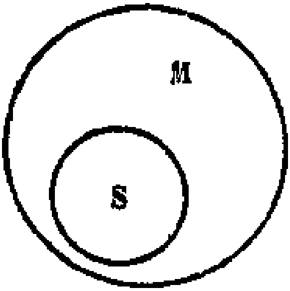


Fig. 38

A larger premise ("all amphibians are vertebrates") reveals that this larger class M volume is all included as part of the even larger class P volume (see Fig. 39).

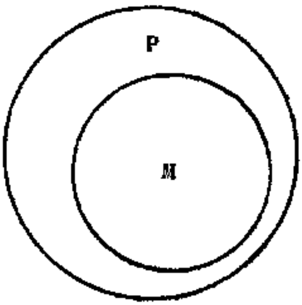


Fig. 39

Comparing these relations of concepts found out from the premises, we establish in the conclusion ("all frogs are vertebrates") that the class S with the *smallest* volume belongs to the class P with the *largest* volume (see Fig. 40).

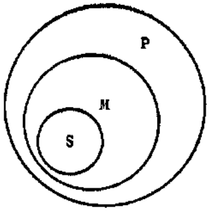


Fig. 40

§ 11. We now consider another example of syllogism:

All stars shine with their own light.
No planet shines its own light.

Not a single planet is a star.

This conclusion is a syllogism. In it, the conclusion, or conclusion ("no planet is a star"), is obtained from two premises. In these premises, the relation of the subject of inference ("planet") and the predicate of inference ("star") to the third or middle concept ("body shining with its own light") is established. It is through the relation of the middle concept to the concepts of "planet" and "star" that the relation of the latter between them is revealed.

And indeed: a larger premise ("all stars shine with their own light") establishes that the entire volume of class P is included in the volume of class M (see Fig. 41).

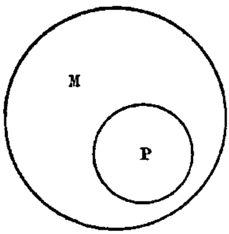


Fig. 41

A smaller premise ("no planet shines its own light") establishes that the class does not belong to class M, that is, the entire volume of class S is entirely *outside the* volume of class M (see Fig. 42).

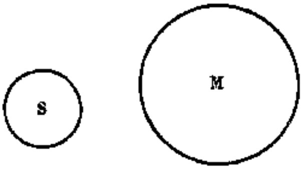


Fig. 42

Comparing these relations of concepts that have emerged from the premises, we conclude ("no planet is a star") that the class S does not belong to the class P, that is, the entire volume of the class S is outside the entire volume of the class P (see Fig. 43).

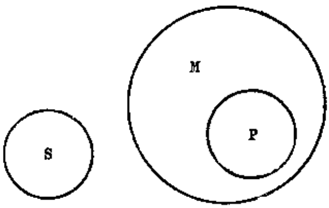


Fig. 43

Looking closely at the arrangement of terms in the premises and in the derivation of this syllogism, we note that this arrangement

P—M
S—M
———
S — P

differs from the arrangement of terms in the syllogism of the first figure:

M—P
S—M
———
S — P

Namely: in the second syllogism, the middle term in both premises—larger and smaller—is a *predicate* . A syllogism with this arrangement of terms is called a syllogism of the *second* figure.

§ 12. Consider the third example of syllogism:

All platypuses are animals laying eggs.
All platypuses are mammals.
———
Some mammals are animals laying eggs.

And this conclusion is syllogism. And in it, on the basis of the relation established in two premises, the concept of “mammals” and the concept of “animal laying eggs” to the third concept (“platypus”) establishes the relation of the subject to the predicate in conclusion.

A larger premise (“all platypuses are animals laying eggs”) establishes that class M belongs to class P, i.e. that the entire volume of class M is included as part of the volume of class P (see Fig. 44).

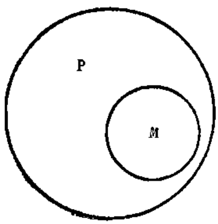


Fig. 44

A smaller premise ("all platypuses are mammals") establishes that class M belongs to class S, that is, the entire volume of class M is included as part of the volume of class S (see Fig. 45).

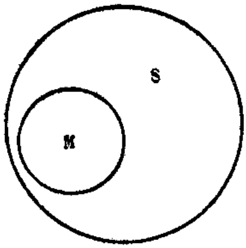


Fig. 45

Comparing these relations of concepts, which were found out from the premises, we conclude ("some mammals are animals laying eggs") that some part of class S belongs to class P, that is, the volume S in some part coincides with volume P (see. Fig. 46).

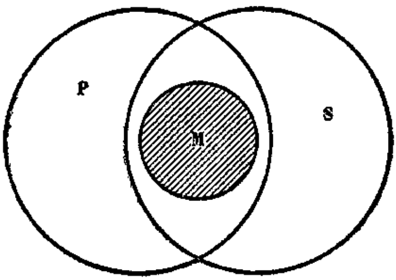


Fig. 46

Indeed, since the entire volume M is entirely located both inside the volume S and inside the volume P, all those parts of the volume S that are occupied by the volume M will be at the same time parts of the volume P. And vice versa: all those parts of the volume P, which are occupied by volume M, will at the same time be parts of volume S.

Consider the arrangement of terms in the last syllogism:

M—P
M—S (III)
―――――
S — P

Here, the arrangement of the terms differs from their location in the syllogisms of the first and second figures:

M — P			P — M	
S — M	(I)	and	S — M	(II)
―――――			―――――	
S — P			S — P	

Namely: in the third syllogism, the average term in both premises is the *subject*. A syllogism with this arrangement of terms is called a syllogism of the *third* figure.

The differences between the three figures of a simple categorical syllogism are of interest not only because the terms in the premises of these syllogisms are placed in different ways. The different arrangement of terms in the premises is associated with a different relationship between the content and volume of the concepts included in the premises and in the conclusions. And indeed, on whether, for example, the middle term is the subject parcels or its predicate depends distributed middle term in the premises, i.e. the possibility of thinking in the medium term.. *The whole* amount or *only in part* its volume. The same is true for the larger and smaller term. In turn, the different value of the syllogism figures for logical thinking and knowledge depends on the relationship between the content and volume of concepts included in the premises and conclusions, and therefore the different role that each of the figures plays in the evidence and reasoning.

§ 13. In order to clarify the role of each figure, that is, the nature of the conclusions that can be obtained through this figure, it is necessary to become familiar with the varieties of figures, or *modes*.

Comparing the various conclusions made on the same figure, we notice that the syllogisms of the same figure can differ in quality and in the number of premises and conclusions.

Compare two syllogisms:

All cereals are monocotyledonous plants.	No cereal is a dicotyledonous plant.
All bamboo are cereals.	All bamboo are cereals.
———	———
All bamboo are monocotyledonous plants.	Not a single bamboo plant is a dicotyledonous plant.

Both of these syllogisms are syllogisms of the *first* figure, since in both the middle term is the subject in the larger and the predicate in the smaller premise. But at the same time, there is a difference between the two syllogisms of the first figure. It consists in various quality of sendings and conclusion. In the first syllogism, both premises and the conclusion are general affirmative judgments. Scheme of this syllogism:

A
A
—
A

In the second syllogism, the larger premise is a general negative judgment, the smaller one is affirmative, and the conclusion is general negative judgment. Scheme of this syllogism:

E
A
—
E

Compare two more syllogisms:

All mushrooms are spore plants.	No planet is a star.
Some flowerless ones are mushrooms.	Some luminaries are planets.
———	———
Some flowerless are spore plants.	Some luminaries are not stars.

Both of these syllogisms are also the syllogisms of the *first* figure, since in both the middle term is the subject in the larger and the predicate in the smaller premise. But at the same time, there is a difference between the two syllogisms of the first figure. It consists in various quality and quantity of packages and output. In the first syllogism, both premises and conclusions on quality are affirmative. In terms of quantity, the big premise is general judgment, the smaller one is particular, the conclusion is also particular. Scheme of this syllogism:

A
I
—
I

In the second syllogism, the larger premise is a general negative judgment, the smaller one is a private affirmative, the conclusion is a private negative judgment. Scheme of this syllogism:

$$\frac{\begin{matrix}E\\ I\end{matrix}}{O}$$

Comparing the quality and number of conclusions in all four examples of syllogism of the first figure above, we see that in the first example, the conclusion is affirmative ("all bamboo are monocotyledonous plants"), in the second — negative ("not a single bamboo is a dicotyledonous plant") , in the third— private affirmative ("some colourless— pore plants"), in the fourth—private negative ("some luminaries are not stars").

Varieties of syllogisms of the same figure, due to different quality and number of premises and conclusions, are called *modes* (from the Latin word, "modus", meaning "method", "view").

§ 14. So, among the conclusions of a simple categorical syllogism, the conclusions of all possible types of quality and quantity can be found: A, E, I, and O. But we already know that judgments of different quality and quantity have different applications in knowledge and different values for knowledge. Therefore, when studying the syllogisms of all three figures, the question of which particular modes each syllogism figure can give, in other words, what the conclusions of this figure in terms of quality and quantity can be of great interest.

To answer this question, it is necessary first of all to investigate whether all theoretically possible modes, i.e., whether all combinations of premises differing only in quality and quantity, are capable of giving the right conclusions.

The study shows that not every theoretically possible modus, i.e., not every combination of quality and quantity in the premises of syllogism, gives the correct conclusion.

Consider, for example, judgment:

All students are required to take exams.
All graduate students are not students.

In these judgments there are three concepts located according to the scheme of the first figure of a simple categorical syllogism. The term "students" in one of the premises is a subject, in another—a predicate. In one premise, the relation of the terms "students" to one concept is established, in another, the relation of the same term to another concept.

So, the arrangement of terms in judgments seems to correspond exactly to the scheme of the first figure:

M—P
S—M

The first judgment will be affirmative, the second—all negative.

Schematically, the quantity and quality of these judgments will be as follows:

A
E

But although the arrangement of terms in this case seems to meet the conditions of the first figure, the correct conclusion from these two premises is *impossible*. From the fact that “all students are obliged to take exams”, and from the fact that “all graduate students are not students”, it cannot be deduced in any way as a necessary conclusion that, for example, “graduate students are not obliged to take exams.” Although the first premise clarifies the attitude of “students” to “persons required to take exams”; and the second is the attitude of “students” to “graduate students,”—these relations are not such that it was clear from them what the attitude of “graduate students” should be like to “persons obliged to take exams.” As can be seen from the figure (see Fig. 47), here *three* cases are logically possible—with these premises.

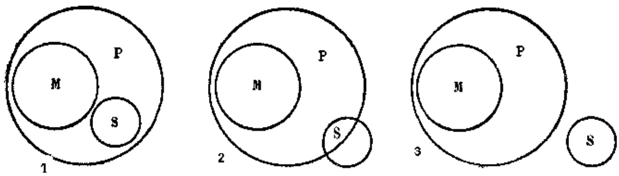


Fig. 47

Not being students (M), graduate students (S) 1) can *all* belong to the number of persons obliged to take exams (P), 2) can belong to the number of these people *only in some part of it,* and 3) may *not belong* to the number of these persons.

So, some modes, for example mode AE of the first syllogism figure, are impossible. This means that the quality and quantity of packages in these modes do not give a basis for the correct logical conclusion. Therefore, in order to answer the question of which modes each of the three figures of a simple categorical syllogism gives, it is necessary first of all to find out the conditions or rules that the premises and the terms included in these premises must satisfy in order to make the conclusion really possible. It turns out that there are rules common to all syllogism figures. In any simple categorical syllogism, whatever its figure, whatever the mode of this figure, all the rules common to all syllogisms must be satisfied. Violation of at least one of them makes the conclusion erroneous.

In addition to the rules common to all syllogism figures, there are also rules that are *special* rules for each syllogism figure separately. These rules are obligatory for all modes of a given syllogism figure and are not obligatory for modes of other figures.

§ 15. There are ten rules common to all figures of a simple categorical syllogism. Of these ten general rules, two determine the number of terms and the number of judgments that make up the syllogism. Two other rules determine the necessary conditions for the distribution of terms in the premises and in the conclusions of the syllogism. The remaining general rules determine the necessary relationship between quality and the number of premises and the quality and number of conclusions (conclusions) of the syllogism.

Rules Defining the Number of Terms and the Number of Judgments in Syllogism

The first of the general rules is that there *should be three terms in the syllogism — no more and no less*. If there are only two terms, then the conclusion cannot give anything new and will be reduced to a simple repetition of one of the premises. For example, “bamboos are cereals”, “cereals are cereals”, therefore, “bamboos are cereals”. If there are *four* terms, then the conclusion is impossible, since in one of the premises the relation of the subject to one term is established, and in the other — the relation of the predicate to another term. There is no mediating term through which the relation or connection between the subject and the predicate in the derivation could be established. For example, in the premises “all laws are published in official publications” and “universal gravitation—the law”, the concept of publication in official publications is put in relation to the concept of law in the *political* sense , and the concept of universal gravitation is related to the concept of the law of *nature*. Since the word “law” means two different concepts here, in our premises there were not three, but four terms, the term of the subject (“universal gravitation”) turned out to be in no way connected with the term predicate (“publication in official publications”), and the conclusion , that is, a judgment that would establish a connection between the concepts of “universal gravitation” and “publication in official publications” turned out to be impossible.

§ 16. *The second* general rule is formulated as follows: *in a syllogism it cannot be less and cannot be more than three judgments.* This rule follows from the very essence of syllogism. As we already know, the purpose of syllogism is to clarify the relationship between two concepts from the already known relationship of each of them individually to the same third concept.

This shows, firstly, that *in a syllogism there should be at least three judgments.* Indeed, in one of them (the smaller premise), the relation of the concept S to the mediating third concept of M. is revealed. In the other (the larger premise), the relation of another concept is revealed — P to the same mediating third concept of M. Finally, in the third proposition (conclusion or conclusion) syllogism), it turns out what relation of the concept of S to the concept of P necessarily follows from the relationship of each of them individually to M.

True, in many cases it may seem as if the syllogism does not consist of three, but only of two, and even of one judgment. Thus, in the conclusion "bamboos, like all cereals, bloom in spikelets", the syllogism is expressed through one complex sentence. In the conclusion "all cereals bloom in spikelets, therefore, all bamboos bloom in spikelets" the syllogism is expressed through two sentences. There could be many such examples.

However, in all these and similar cases, the grammatical form of the utterance misleads us. We already know that the grammatical forms of sentences do not always coincide with logical forms of thinking. The same holds true in our examples. In fact, the syllogism in these examples consists of three propositions. However, part of these judgments—due to the speed of thinking or the desire for brevity and conciseness of expression — only implies, remains unexpressed in the form of three separate sentences expressing three separate judgments. And yet, each of these syllogisms can be—without any change in its logical meaning — expressed in the usual and obligatory form of three judgments for all syllogisms: two premises and conclusions. So, the abbreviated syllogism is "bamboos, like all grains, bloom with spikelets "easily unfolds into a complete syllogism:" all bamboos are cereals, all cereals bloom with spikelets, therefore, bamboos bloom with spikelets". The second syllogism of our example is easily reduced to the same complete and obligatory form of three propositions for all syllogisms: "all cereals bloom in spikelets, therefore, bamboos bloom in spikelets".

But in the syllogism, secondly, *there can be no more than three propositions.* It has already been proved above that there must be at least three judgments in syllogism. From these mandatory three propositions, the conclusion establishes the desired relation between S and P, the larger premise is the relation between M and P, the smaller is the relation between M and S. The question of whether any other statements can be included in the syllogism apart from the three, it reduces to the question of whether, in addition to the three syllables required for each syllogism, two combinations of two terms from S, M, P are possible, any other combinations of two terms from the same three terms S, M and P. But there can be no more than three such combinations. Therefore, in a simple categorical syllogism there can be no more than three propositions.

Distribution Rules for Terms in Premises and Conclusions of Syllogism

§ 17. The third general rule is formulated as follows: *for the conclusion to be possible, the middle term (M) must be distributed in at least one of the premises.*

So, from the packages

Some mammals are aquatic animals.
All seals are mammals.

no conclusion can be drawn about the relationship of seals to aquatic animals. In fact, the middle term here is the term "mammals". This term is not distributed either in a larger or a smaller premise, i.e., it is not meant in these premises in its entirety. In the larger premise, it is not distributed, since it is a predicate of *affirmative* judgment (see Ch. I, § 25), in the smaller—since it is the subject of a *private* judgment (see Ch. VI, § 25).

Since the average term in both packages is not distributed, then in each package we have in mind some uncertain exactly part of its volume. In this situation, it is quite possible that in one premise we are talking about one, and in another premise— about some other part of the volume of the average term (see. Fig. 48).

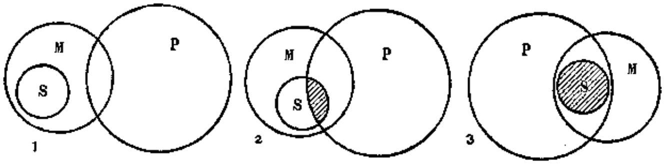


Fig. 48

The figure shows the relationship of concepts expressed by premises. The scope of the concept of “mammals” is depicted by the circle M, the scope of the concept of “seals” by the circle S, the scope of the concept of “aquatic animals” by the circle P. From the figure it can be seen that the premises leave us in the unknown about the relationship between the concepts S and R. Since the premises it is not known which part of volume M is volume S and which part of volume P is volume M, *three* possibilities remain open : 1) the *entire* volume S is *outside the total* volume P; 2) the volumes of S and P *partially* coincide; 3) the volume S is *entirely included* as a part in the volume of R. But this means that the connection between the subject and the predicate of inference, which should have been revealed through their relationship to the middle term, cannot be established: in fact, there is not even a middle term, but there are two meaning in one word concepts, of which one, possibly, marks one part of the volume, and another.

§ 18. The fourth general rule is formulated as follows: *if larger or smaller terms are not distributed in premises, they cannot be distributed in output.* So, from the premises “all great poets have a strong imagination”, “all great poets are impressionable people” it cannot be deduced that “all impressionable people have a strong imagination”. Here, only a particular conclusion will be correct: “some impressionable people have a strong imagination.” Indeed, the concept of “impressionable people” — the smaller term of our syllogism — is not distributed in the premise. In the judgment “all great poets are impressionable people” the concept of “impressionable people” is not conceived in its entirety. It is quite obvious that in the conclusion, where the concept of “impressionable people” becomes a subject, there is no sufficient reason to take this concept in its entirety (see Fig. 49).

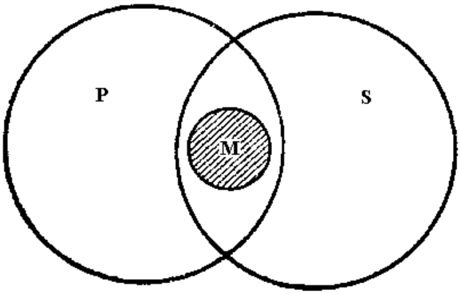


Fig. 49

The figure shows the relationship of concepts expressed by premises. The scope of the concept of “great poets” is represented by the circle M, the scope of the concept of “people with a strong imagination” “is represented by the circle P, the scope of the concept of” impressionable people “by the circle S. It can be seen from the premises that the whole part of the volume S certainly enters into the volume P, which is occupied by volume M. Therefore, the premises give the correct conclusion: “*some* S belong to volume P”. But we are not entitled to derive more from this from the premises. Only that part of the volume S that coincides with M is reliably known from the premises that it is included in the volume P.

And since this part, equal to M, does not exhaust either the entire volume S or the entire volume P, it follows that not all S, but only some S belong to the volume P.

Another example: from the premises "all students must take exams" and "graduate students are not students", it cannot be concluded that "graduate students should not take exams." In fact, in the conclusion "graduate students should not take exams", a larger term as a predicate of *negative* judgment would be distributed. But in the larger premise — "all students must take exams"—the larger term as a predicate of affirmative judgment (cf. chap. VI, § 25) is not distributed. It is clear that, without being distributed in the premise, it cannot be distributed in the output (see Fig. 50).

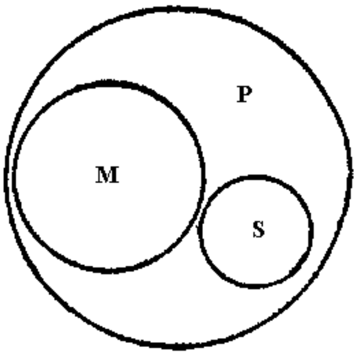


Fig. 50

The figure shows the relationship of concepts expressed by premises. The scope of the concept of "students" is represented by the circle M, the scope of the concept of "persons required to take exams" is represented by the circle P and the scope of the concept of "graduate students" by the circle S. It can be seen from the figure that we are not entitled to conclude that the volume S will need to be outside the volume P Since the term P is not distributed, the volume M is only some part of the volume P. Therefore, it is entirely possible that the entire volume S, which we know from another premise that it is not included in the volume M, will still be entirely *inside* volume P — like the other, along with M, part of this volume (1). In our example, the way it is: graduate students, not being students, still belong to the number of persons required to take exams.

Rules Determining the Relationship Between Quality and the Number of Premises and Conclusions of Syllogism

§ 19. The fifth general rule is formulated as follows: *if both premises are negative, then no conclusion can be drawn from them.* Thus, from the premises "whales are not fish" and "dolphins are not fish", no conclusion can be drawn about the attitude of dolphins to whales. And indeed, both premises are negative. It can be seen from them that the entire volume of the larger and the entire volume of the smaller term are outside the entire volume of the average term: not a single whale and not a single dolphin are among the fish. But knowing this, we still *do not know anything* about the relation of volumes of larger and smaller terms to each other: they can stand outside each other, they can partially coincide, and they can be one inside the other (see Fig. 51).

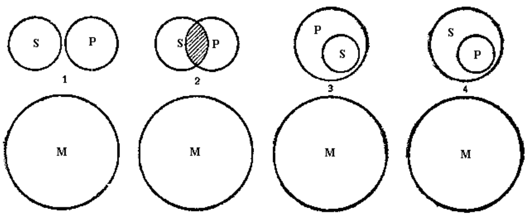


Fig. 51

The figure shows the relationship of concepts expressed by premises. The volume of the concept of “fish” is represented by the circle M, the volume of the concept of “dolphins” is represented by the circle S, the volume of the concept of “whales” by the circle P. From the figure it can be seen that we are not entitled to make any conclusion about the necessary ratio of the volume S to the volume P From the fact that the entire volume S is outside the volume M and the whole volume P is also outside the volume M, it is not yet clear how much the volume S will be to the volume P. The figure shows that *four* possibilities remain open here : the entire volume S is outside the whole volume P; 2) the volumes of S and P partially coincide with each other; 3) the volume S is entirely included as part of the volume P; 4) the volume P is entirely included as part of the volume S.

§ 20. The sixth general rule is formulated as follows: *if the conclusion from these premises is generally possible and if one of the premises is negative, the output will also be negative.*

Consider the syllogism:

No cereal is a spore plant.

Wheat is cereal.

Wheat is not a spore plant.

Here, one of the premises is negative, and the other is affirmative. This means that the volume of one of the terms included in the output *is outside the* volume of the average term, and the volume of the other term included in the output is *part of the* volume of the average term (see Fig. 52).

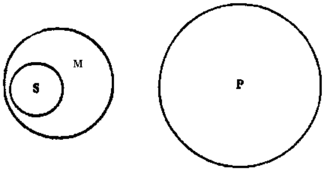


Fig. 52

The figure shows the relationship of concepts expressed by premises. The volume of the concept of “cereals” is depicted by the circle M, the volume of the concept of “wheat” by the circle S, the volume of the concept of “spore plants” by the circle P. It can be seen from the figure that since the entire volume M is outside the volume P (larger premise), then the volume S, which enters as a whole as part of the volume M (the smaller premise), is all outside the volume P (output).

§ 21. The seventh general rule of syllogism is formulated as follows: *from two affirmative premises you can never get a negative conclusion.* Indeed, a negative conclusion is obtained with such a relationship between the subject and the output predicate, when the entire volume of the predicate (P) is outside the entire volume of the subject (S) or at least outside some part of the volume of the subject. For this, in turn, it is necessary that the entire volume P be outside the entire volume of the average term (M). Then, even provided that the volume S appears to belong in some part to the volume M, the entire volume P will be outside if not all of the volume S, then at least outside some part of the volume S, i.e., the conclusion will be negative (see fig. 53).

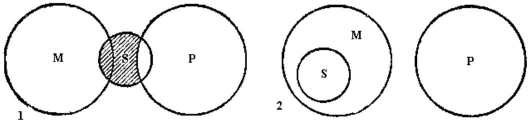


Fig. 53

The figure shows the relationship between the concepts of syllogism, providing a negative conclusion. It can be seen from the figure that, in any case, the volume P should be all outside the entire volume of M. As for the ratio of the volume S to the volume M, the conclusion can be negative even if S enters M only in a known part of its volume (1), and — all the more so—in the case when S enters M in its entire volume (2). In the first case, the conclusion can be private negative, in the second—the conclusion will always be negative.

So, the volume P must be all outside the entire volume M, so that the conclusion can be negative. But this means that one of the premises of the syllogism (greater) should be negative.

On the contrary, if both premises are affirmative, the withdrawal predicate (P) cannot in any way be in such a relation to the output subject (S), in which the entire volume P could be outside all or at least outside some part of the volume S (see Fig. 54).

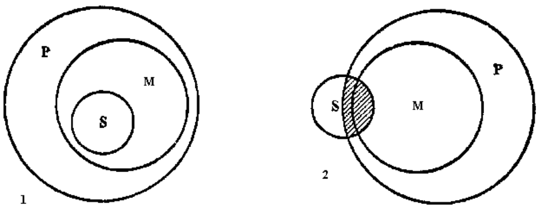


Fig. 54

The figure shows the relationship between S and P in the case when both premises are in the affirmative. It can be seen from the figure that in this case, the conclusion is only affirmative: general affirmative (1) and private affirmative (2).

So, a negative conclusion can never be obtained from two affirmative premises.

§ 22. The eighth general rule of the syllogism is formulated as follows: *from two particular premises on any figure of the syllogism it is impossible to get the correct conclusion.* Indeed, if both premises are partially affirmative (I, I), then this means that not a single term is distributed in them. Thus, in the premises “some birds are grain—eating” and “some aquatic animals are birds” not a single term is distributed. The terms of the subject are not distributed as subjects in *private* judgments, the terms of the predicate are not distributed as predicates of *affirmative* judgments expressing the submission of concepts. Since the middle term must be distributed in at least one of the premises and since this condition cannot be satisfied with two private premises, the conclusion from two private premises is impossible (see Fig. 55).

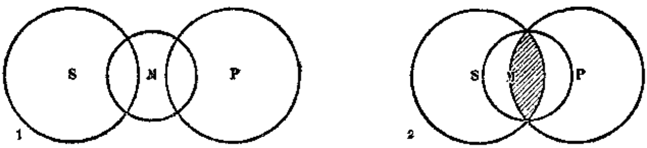


Fig. 55

The figure shows the relationship of concepts in two partial affirmative premises. The parcels confirm that the volume S is included in the volume M by a known part, and the volume M is included in the volume P by a known part. But since it is not visible from the parcels which part of its volume includes S in M and which is M in P, they remain open two possibilities: 1) volume S is included in volume M and volume M is included in volume P in such a way that not a single part of volume S is found to belong to volume P; 2) the volumes S, M, and P are so related to each other that some part of the volume S appears to belong to the volume P. In the first case, the output will be negative, in the second it will be partially affirmative. Since it is not visible from the premises which of both possibilities should take place in each individual case, it is impossible to deduce from two partially affirmative premises.

But a conclusion is impossible even if one of the two private premises is affirmative and the other is negative (I, O). Consider the premise “some birds—animals, nests,” and “some animals, nests, are not predators.” In such premises, one term, namely the predicate, of the negative premises is distributed. But we know that if one of the premises is negative, then the conclusion can only be negative. Assume that the conclusion is private negative. In this case, at least two terms should be distributed in the premises of the syllogism: *medium* , as in any syllogism, and *larger* , since, being a predicate of *negative* conclusion, the larger term is distributed in the output, and therefore should be distributed in the premise. But since only one term is distributed in our premises, the conclusion is impossible.

§ 23. The ninth general rule is formulated as follows: *if one of the premises is private and if a conclusion is possible at all, then it can only be private.* If both premises are affirmative and one of them is general and the other is private (A, I), then one term — the subject of the general affirmative premise — will be distributed. But for the conclusion to be general, it is necessary that the terms have two terms distributed: the middle one, as in all syllogisms, and the smaller one, since the smaller term cannot be distributed in the output if it is not distributed in the package. But since in our case only one term is distributed in the premises, the conclusion is possible only in particular. So, from the premises “all fish are vertebrate animals” and “some aquatic animals are fish” one can only obtain a particular conclusion: “some aquatic animals are vertebrate animals”.

If, of the two premises, one is affirmative and the other negative, and one of them is private (IE, EI, OA, AO), then two terms will be distributed in the premises: the subject of general judgment and the predicate of the *negative* . However, in this case, the conclusion cannot be general. And indeed, with one negative premise, the conclusion can only be negative. Since our premises are IE, EI, OA, AO, the conclusion from them can only be negative. Thus, our general conclusion, if it were possible, should have been negative. But since both the subject and the predicate are distributed in the general negative conclusion (the subject as the subject of the *general*, the predicate as the predicate of the *negative* judgments), then they must be distributed in the premises. In addition, the middle term should also be distributed in one of the premises. So, in order for the conclusion from our premises to be general, there must be as many as *three* terms distributed in the premises . And since only two terms are distributed in our premises, a general conclusion from them is impossible.

§ 24. The tenth rule, common to all figures of the syllogism, is formulated as follows: *if a larger premise is private and a smaller premise is negative, then a conclusion is impossible.* Consider, for example, the premises: “some guardsmen are order bearers”, “not a single fighter of the N—th unit is a guardsman”. According to a larger premise, the relation between the middle term M (“guardsmen”) and the larger term P (“order bearers”) is such that part of the volume M is included in the volume P (see Fig. 56).

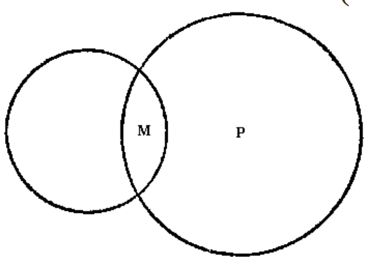


Fig. 56

According to a smaller premise, the relation between the smaller term S ("fighters of the N—th unit") and the middle term M ("guards") is such that the entire volume S is entirely outside the entire volume M (see Fig. 57).

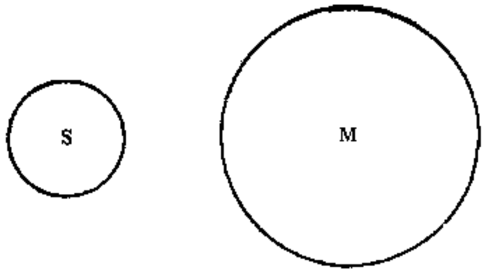


Fig. 57

Let us now compare both premises and see what can be deduced from them about the attitude of the "fighters of the N—th unit" to the "order—bearers" (S to P). What is known from the premises about the relations between the terms M, P and S leaves open three possible relations between S and P (see Fig. 58).

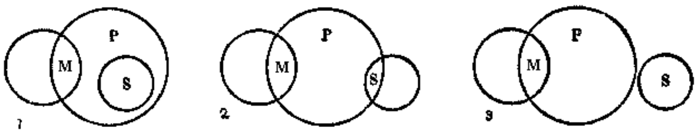


Fig. 58

The first of them is that, being entirely outside the volume M, the volume S all enters the volume R. In this case, not being guardsmen, all the fighters of the N—th unit can be order bearers. The second is that, being entirely outside the volume M, the volume S, with its known part, enters the volume R. In this case, not being guards, *some* fighters of the N—th unit can be order bearers. Finally, the third possibility is that, being entirely outside the volume M, the entire volume S is also outside the entire volume P. In this case, not belonging to the guards, not a single fighter of the N—th unit at the same time also to the order bearers. Since the parcels do not show which of these three possibilities should take place, under the indicated conditions (when the larger premise is private and the smaller is affirmative), a conclusion is impossible.

§ 25. From what has been said it is clear that syllogistic conclusions of different quality and quantity require different conditions for the distribution of terms in the premises.

To obtain a private affirmative conclusion (I) it is enough if only one middle term is distributed in the premises.

In order to obtain a general affirmative conclusion (A), in addition to the average term in the premises, a smaller term must also be distributed, since it will be distributed as a subject of general judgment in the conclusion.

To obtain a private negative conclusion (O), in addition to the average term in the premises, a larger term must also be distributed, since it will be distributed as a predicate of negative judgment in the output.

Finally, to get a *negative* Inference (E), in addition to the middle term in the premises, the smaller and larger terms should be distributed: smaller, since it will be distributed in the output as a subject of general judgment, and larger, since it will be distributed in the output as a predicate of negative judgment.

§ 26. The ten rules set forth must not be violated in any syllogism, whatever his figure, whatever the mode of his figure. Any violation of them destroys the possibility of a conclusion, leads to a logically erroneous conclusion.

It is this obligatory nature of all the rules considered for each syllogism that explains why some modes are impossible, that is, why correct conclusions are impossible with some combinations of quality and number of premises. *All those modes are impossible in which the quality and quantity of the* *premises is such that with this quality and quantity, at least one of the rules of the syllogism will be violated.*

Why, for example, the mode AE of the first figure considered by us turned out to be impossible:

All students are required to take exams.
Not a single graduate student is a student.

Why cannot one conclude from these premises, for example, that “not a single graduate student is obliged to take exams”? Because this conclusion would violate the fourth rule common to all syllogisms. According to this rule, a larger term cannot be distributed in output unless it is distributed in a larger premise. In the conclusion, “no graduate student is required to take exams”, a larger term, as in any negative judgment, would be distributed. Therefore, it would have to be distributed in a larger premise. But in our example, the bigger premise is the judgment “all students are required to take exams.” This premise is a general affirmation judgment. And in an affirmative proposition expressing the subordination of the concept of the subject to the concept of a predicate, as we know, the term predicate is not distributed. Not being distributed in the package, this term cannot be distributed in conclusion. Therefore, the conclusion here is incorrect, and mode AE in the first figure is impossible.

Thus, not all arithmetically possible modes, i.e., not all arithmetically possible combinations of quality and number of premises, justify the correct conclusions. Of the total number of all possible modes, all should be excluded in which the quality and quantity of the packages do not comply with the ten rules outlined.

§ 27. But this is not enough. In addition to mods, which should be excluded as not complying with the rules *common* to all syllogism figures, all those modes that do not correspond to the *special* rules of each figure separately should also be excluded. Therefore, it is necessary to consider these rules.

The special rules of each figure can all be inferred from the basic rules of syllogism. But these same rules can be deduced from the nature of the conclusions that are obtained from each of the figures of the syllogism, i.e., from the nature of the tasks for which each figure is applied.

The First Figure and Its Special Rules

§ 28. The first figure of a simple categorical syllogism is applied in deciding the question of the subordination of one concept to another. In the syllogisms of the first figure, we learn from the conclusion that the concept of S is either subordinate or not subordinate to the concept of P. In turn, the submission (or disobedience) of the concept of S to the concept of P can be either complete or partial. In the case of the complete submission of the concept of S to the concept of P, the conclusion will be general affirmative (A), in the case of a partial affirmative (I). In the case of a complete absence of the subordination of the concept of S to the concept of P, the conclusion will be general negative (E), in the case of a partial absence of the relationship of subordination, the conclusion will be partial negative (O) (see Fig. 59).

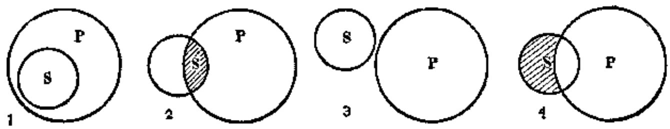


Fig. 59

The figure shows four possible relationships between the volumes of concepts S and P, which can be clarified through syllogisms of the first figure. In all these four cases, the relationship between the *volumes of the* concept of S and P. is depicted . But these relations between the volumes represent only the direct result of the relationship between the *content of the* concept of S and the *content of the* concept of R.

So that the concept of S can be subordinated to the concept of P, in other words, so that the volume of S can be included as part of the volume of P, it is necessary that the content of the concept of P be part of the content of the concept of S. Only knowing that all the essential features of the concept of P are among the essential features of the concept of S, we can argue that the volume S is part of the volume P. Knowing from the lesser premise (S — M) about the belonging of the subject S to the known class M and knowing from the greater premise (M—P) that all objects of this class belong to the known property P , we can conclude in the conclusion of the first figure that property R belongs to the subject S. So, knowing that all bamboos are cereals (the smaller premise) and that all cereals have the property of blooming spikelets (the larger premise), we conclude from the first figure that we that bamboos also have the property of blooming spikelets.

A particularly important feature of the first figure is the way in which the conclusion is established in it. As in any syllogism, in the syllogism of the first figure, the relation of the subject of the derivation to its predicate is not directly visible. This relation is established through the relations of the subject and the predicate of inference to some third concept.

But these relationships are here *subordinate* relationships*:* the submission of the concept of M to the concept of P is established by the larger premise, the submission of the concept of S to the concept of M by the smaller premise. As a result, the concept of S is not only subordinate to the concept of P, but the whole movement of thought in the syllogism of the first figure turns out to be a movement from the most general to the least general. So, knowing that all amphibians are vertebrates and that all frogs are amphibians, we conclude from the first figure that all frogs are vertebrates. We started by looking at the most general class—vertebrates, found in it as part of its volume the class of amphibians, and finally, after considering the class of amphibians, we found in it as part of its volume the class of frogs. In other words, we found that all the essential features of the vertebrate class belonging to the amphibian class must also belong to the frog class (see Fig. 60).

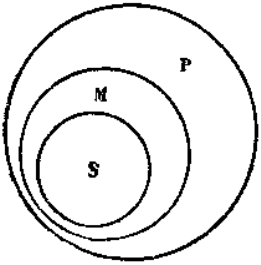


Fig. 60

This feature of the first figure determines the scope of its application. The first figure of a simple categorical syllogism is used in all operations of thinking, where a well—known general rule or law can be applied to particular cases.

In turn, this application of the first figure determines its special rules.

§ 29. The first of these rules is that the *smaller premise must be an affirmative proposition.* This rule is necessary, since from a smaller premise we learn that an object (subject of inference) belongs to the same class, the general property of which is revealed in the larger premise. This rule is derived from the general basic rules of syllogism. Indeed, if the smaller premise in the first figure were negative, then the conclusion, according to the sixth general rule, would also be negative. That would mean that the larger term is as a predicate of the *negative* judgments would be distributed. But, being distributed in the conclusion, the larger term should have been distributed in the larger premise. However, in our case this is not possible. Indeed, since we assumed that the smaller premise is negative, and since the neglect of the smaller premise, the larger premise must be affirmative, the larger term as a predicate of affirmative judgment expressing subordination of S and P cannot be distributed. So, with a negative smaller premise, a conclusion on the first figure is impossible.

§ 30. The second special rule of the first figure is that a *larger premise must be a general judgment.*

Indeed, if the larger premise in the first figure were private, then the average term as a subject of *private* judgments would not be distributed in a larger premise. But at the same time, it would not be distributed in a smaller premise. In fact, the smaller premise of the first figure, according to the special rule of the first figure just proved, must certainly be affirmative. And since the average term is a predicate in it, then as a predicate of affirmative judgment expressing the relation of subordination S and P, it will not be distributed. Thus, if the larger premise of the first figure were private, it would mean that the middle term would not be distributed in any of the premises. But this is unreal. Therefore, a larger premise must be shared.

This rule is necessary, since if it were violated, a larger premise could not express the general law in the application of which the conclusions of the first figure consist.

§ 31. Now it’s easy to establish which modes are able to give the correct conclusion on the first figure. To do this, we exclude from the list of all arithmetically possible modes, firstly, those by which the conclusion is impossible due to the rules common to all figures, and, secondly, those by which the conclusion is impossible due to the special rules of the first figure. After this exception, obviously, only the correct modes of the first figure will remain.

Since there are two premises in syllogism, and since each of them theoretically can have any quality and quantity, that is, it can be general affirmative, partial affirmative, general negative and particular negative, it is obvious in the first figure (as well as in the second and third) sixteen modes are possible arithmetically:

AA	SHE	HE	OA
AE	YES	IE	OE
AI	NO	II	HI
TO	IT’S THE	I	NO

We exclude all modes in which the quality and quantity of the packages are such that, according to the rules common to all the figures and to the rules specific to the first one, the conclusion is impossible. Firstly, all modes in which both premises are negative will disappear: EE, EO, OE, OO. Secondly, all modes in which both premises are private will disappear: II, IO, OI, OO. Thirdly, according to the *special* rules of the first figure, all modes in which the larger premise is private will disappear: IA, IE, OA. Fourth, according to the *special* rules of the first figure, all modes in which the smaller premise is negative will disappear: AE, AO.

Only *four* will be left as a result the modus of the first figure: AA, EA, AI, EI, in which the number and quality of the packages do not contradict either the general or the syllogism rules that are special for the first figure.

In mode AA, a smaller premise establishes that the entire class S belongs to class M, and a larger premise confirms that the *whole* class M belongs to class P. This relation of terms gives grounds to state that the *whole* class S belongs to class P in the conclusion. Thus, the conclusion is confirmed by mode AA (A), and the whole structure of a modus can be designated AAA.

Example: “All amphibians are vertebrates, all frogs are amphibians, therefore, all frogs are vertebrates.”

In modus EA, the smaller premise establishes that the entire class S belongs to class M, and the larger puts the entire class P out of the entire class M. This ratio of terms gives grounds to exclude the entire class S from the whole class P. Thus, according to modus EA, the conclusion it turns out to be generally negative (E), and the entire structure of the mode can be designated EAE.

Example: “No planet is a star, all asteroids are the planet, therefore, no asteroid is a star.”

In modus AI, the smaller premise establishes that some S belongs to the class M, and the larger premise indicates that the whole class M belongs to the class P. This relationship between the terms gives reason only for the partial affirmation (I), since a smaller term that is not distributed in the premise cannot be distributed in output. The entire structure of this modus can be designated AII.

Example: “All fish are vertebrates, some aquatic animals are fish, and therefore some aquatic animals are vertebrates.”

In modus EI, the smaller premise establishes the belonging of some S to class M, and the larger puts the whole class P outside the entire class M. Based on this relation of terms in deriving the syllogism from the whole class P, the very “some” whose affiliation to M are established by the smaller premise . In other words, the conclusion is partial negative (O), and the entire structure of the mode can be denoted by EIO.

Example: “Not a single mushroom reproduces by seeds, some plants are mushrooms, therefore, some plants do not propagate by seeds.”

§ 32. So, all four modes of the first figure, remaining after the exclusion of impossible modes, give the correct conclusions. Comparing the quality and quantity of the correct conclusions of the first figure, we note that conclusions of all kinds of quality and quantity are possible according to the first figure: general affirmative (mode AAA), general negative (mode EAE), partial affirmative (mode II) and partial negative (mod EIO). With this ability to give conclusions of any quality and quantity, the first figure is different from all the others.

Even more important is the ability of the first figure to justify the affirmative conclusion (mode AAA). As we will see later, *no modus of any other figure gives a general affirmative conclusion.* The value of the AAA modus of the first figure is extremely high. With the help of this modus, a general law expressing the *positive* property of a wider class of objects can be applied to an entire class or category of objects. Thus, the laws of celestial mechanics, discovered by Newton, and formulated in a general way, can be applied in the study of the movements of not only planets, but also of orbital binary stars [1] .

Particularly widespread is the application of the first figure (namely, AAA modus) in mathematical proofs and in solving mathematical problems. The so—called direct proofs of theorems representing affirmative statements are carried out in the vast majority of cases by this mode.

Let us consider as an example of the application of syllogisms the solution of a simple geometric problem.

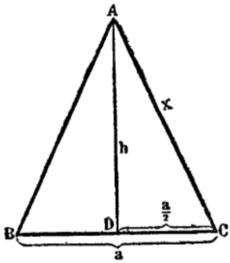


Fig. 61

In the isosceles triangle *ABC*, the base *BC* = *a* and the height *AD* = *h are* known . What will the *speaker* side be equal to ? We draw an isosceles triangle *ABC* (see. Fig. 61), we denote elements known to us by the letters *h* and *a* . Denote the unknown side of the speaker by the letter *x* . From geometry it is known *that in any isosceles triangle its height divides the base in half*. Triangle *ABC—isosceles*. *Therefore, in it the height AD, omitted from the top of the acute angle A, divides the base a in half*. Therefore, *DC* = *a* / 2. Now consider the triangle *ADC*. In it, the *AD* side is known by the condition of the problem and is equal to *h* , the *DC* side has just been determined and is equal to *a* / 2 , and the angle *ABC* is straight, since the *AD* side is the height of the triangle *ABC*. *In any right triangle, the square of the hypotenuse is the sum of the squares of the legs*. *Therefore, in the right—angled triangle ADC, in which the hypotenuse is AC = x, and the legs are AB = h and DC = a / 2,* $x^2 = h^2 + (a / 2)^2$. Solving the quadratic equation, we obtain: $x = \sqrt{(h^2 + (a / 2)^2)}$.

Let’s consider those parts of our reasoning which are emphasized in italics. In each of them we are talking about a different subject, but the very train of thought is the same. In the first part of the argument in italics, it is proved that in this triangle *ABC, the* height divides the base in half, in the second it is proved that the desired side of the *speaker* can be found as the hypotenuse of the right triangle *ADC* . But in the first and in the second part, the provable propositions are established using syllogisms. In the first part of the premises, that “in any isosceles triangle, its height divides the base in half” and that “a given triangle *ABC* is isosceles”, we concluded that “therefore, in this triangle *ABC*, height *AD* divides the base in half.”

In the second part of the argument, after it was found that $DC = a / 2$ and that the triangle *ADC* is right—angled, we conclude as follows: “Since in any right—angled triangle the square of the hypotenuse is equal to the sum of the squares of the legs and since the triangle *ADC* is right—angled, then in it the square of the hypotenuse is equal to the sum of the squares of the legs “, or” $x^2 = h^2 + (a / 2)^2$ ”.

Reasoning is also a syllogism.

According to AAA modus, a court is usually concluded in a properly set trial. The establishment of the fact of a crime forms a smaller premise here: “S — M”. The law defining the punishment for a crime of this composition forms a larger premise: “M — P”. The verdict of the court, which determines the measure of punishment prescribed by law for the proved crime, forms the conclusion: “S — P”.

Inference by the AAA mode of the first figure of the syllogism is constantly applied in the practice of everyday thinking. This modus is used everywhere where, on the basis of known knowledge or a position of *general* importance, *special* or *particular* methods suitable for achieving the goal. So, knowing the general property of fertilizers to increase productivity and knowing that apatites are one of the types of fertilizer, the manager uses apatites in agriculture.

In order to facilitate the memorization of correct modes, each correct mode is denoted by a special artificial, i.e., specially invented, Latin word in which the first vowel means the quality and quantity of the larger premise, the second vowel means the quality and quantity of the smaller premise, and the third vowel means the quality and amount of output. The names of the mods of the first figure are as follows:

Barbara, Celarent, Darii, Ferio.

The Second Figure and Its Special Rules

§ 33. We turn to the consideration of the *second* figure of a simple categorical syllogism:

P — M
S — M
———
S — P

The conclusion of the second figure establishes that objects of class S cannot belong to class P, since they do not have properties that belong to objects of class P and which are certified in the premises.

Consider the following examples:

All heroes are able to subordinate the personal to the public.	No star has a quick visible movement relative to other bodies.
No egoist is able to subordinate the personal to the public.	All planets have fast visible motion relative to other bodies.
———————	———————
No egoist is a hero.	No planet is a star.

These examples represent two varieties of the second syllogism figure. In the *first* example, a larger premise confirms that the well—known property M belongs to *all* objects belonging to class P, and a smaller premise establishes that objects of class S do not have property M. From this relation of terms it follows that no object of class S can be included in the class of objects R.

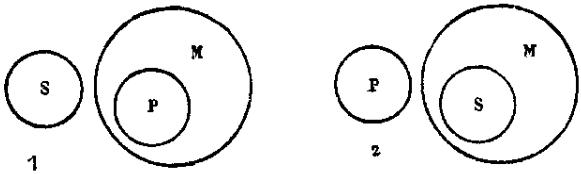


Fig. 62

In the *second* example, a larger premise confirms that not a single object of class P has property M, and a smaller premise establishes that all objects of class S have property M. From this relation of terms it follows that not a single object of class S can be included in the class of objects P (see Fig. 62).

The figure shows the relationship between the concepts in both of our examples of the second figure. It can be seen from the figure that in both examples the conclusion leads *to the exclusion of the* class of objects S from the class of objects P and vice versa.

But this exclusion of the volume of one concept from the volume of another is possible only because, as can be seen from the premises, the essential features of both concepts that form their *content* turned out to be incompatible.

In the first case (1), a larger premise confirms that all the essential features of the concept of M are included as part of the essential attributes of the concept of P and therefore the entire volume P is part of the volume of M. A smaller premise confirms that the essential features of the concept of S are incompatible with the essential features of the concept of M But since all the essential features of the concept of M are among the essential features of the concept of P, then, being incompatible with the essential features of M, the essential features of S are, moreover, incompatible with the essential features of P. And it follows that the entire volume S is outside the entire volume P.

In the second case (2), a larger premise confirms that the essential features of the concept of P are incompatible with the essential attributes of the concept of M and therefore the entire volume P is outside the entire volume of M. A smaller premise confirms that all the essential features of the concept of M are included as part of the essential features of the concept of S and therefore the entire volume S is part of the volume M. But since all the essential features of M are among the essential features of S, being incompatible with the essential features of M, the essential features of P will also be incompatible with the essential features of S. And this means that the entire volume S will be outside the total volume R. The common thing for both examples is that the conclusion in them is to exclude the item from the class on the basis of the difference between the properties of the item and the properties of the class established by the premises.

§ 34. This particular value of the second figure determines its special rules. According to the first of these, a *larger premise should be a general judgment.* Indeed, it is possible to exclude the object S from the class of objects P, based on the properties of the object S, only if all objects of class P have a property opposite to that of the object S.

To exclude the class of planets from the class of stars, based on the property of the planets to have fast visible motion relative to other bodies, it is necessary to know that all stars have the opposite property of planets: they do not have fast visible movement relative to other bodies. To exclude the class of egoists from the class of heroes, based on the inability of egoists to subordinate the personal to the public, it is necessary to know that all heroes have the opposite property of egoists: they are able to subordinate the personal to the public.

§ 35. According to the second rule, special for the second figure, *one of the premises must be negative.* In the absence of a negative premise, by means of which the incompatibility of the property of an object and the properties of objects of a class is determined, there will not be sufficient reason to exclude an object from the class. But which of the premises—larger or smaller—should be negative, the rule does not indicate. So, in our first example, the smaller premise is negative, while the larger is yes. In the second example, on the contrary, the larger premise is negative, while the smaller is affirmative. Indeed, the exclusion of an object from the class can be based both on the fact that the object S does not possess the property M, which belongs to all objects of class P, and that no object of the class P has the property M, which necessarily belongs to the object S. B in the first case, the smaller premise will be negative, in the second case, the larger one.

According to the second figure, *only negative conclusions* can be obtained . This trait follows from the main purpose of the second figure, which consists in the fact that in the conclusion the object S is *excluded* from the class of objects P.

Negative conclusions can be obtained not only from the second figure. We have already seen above that of the four possible correct mods of the first figure, two (Celarent and Ferio) also give negative conclusions: general negative and particularly negative. On the other hand, in the future we will be convinced that negative conclusions are possible in the third figure as well.

The peculiarity of the second figure, distinguishing it from the rest, is not at all that only one second figure is capable of giving negative conclusions. The peculiarity of the second figure is, firstly, that *according to the second figure, no other conclusions are possible except negative ones*. A negative conclusion is not just one of the possible cases of syllogism of the second figure. A negative conclusion is the *main goal of* any syllogism of the second figure. The task of this figure is that, by establishing the incompatibility of the essential features of the concepts S and P, to show that the volumes of these concepts are mutually exclusive.

Therefore, the negation expressed by the modes of the second figure is *different* from the negation expressed by modes, for example, the first figure. This difference is another feature of the second figure. Indeed, in the negative modes of the first figure, a negative conclusion is obtained as a negative answer to the question of whether class S belongs to class R. But the question itself does not have a negative, but a positive meaning: we are interested in the fact that S belongs to P; the Barbara and Darii modes find out that the relation of this affiliation takes place, the Celarent and Ferio modes, that the relation of this affiliation is not present.

On the contrary, in all modes of the second figure, without exception, the task of the conclusion is precisely the proof of the incompatibility of the essential features of the concepts S and P, and, consequently, the separation of the volumes of these concepts. Here (of course, if the conclusion is justified) there can be no question of an affirmative result: the conclusion can only be negative.

Thus, the difference between the negative modes of the first figure and the negative modes of the second figure expresses the difference in our interest. In some cases, we are interested in a positive result, and negation is only a discovery that in this case a positive result, no matter how desirable it is, is still impossible. This is the case with the negative modes of the first figure.

In other cases, on the contrary, we are interested in a negative result, and the question is only about the conditions and the completeness of the denial itself. This is the case with all the modes of the second figure.

§ 36. Both special rules of the second figure may also be deduced from the rules common to all figures of the syllogism. The rule according to which one of the premises must be negative is easily inferred from the distribution of terms. If both premises were affirmative, the middle term would appear as a predicate of *affirmative* judgment, expressing the subordination of the concept S to concept P, in both premises unallocated, and the conclusion would be impossible.

The rule that a larger premise cannot be *private* also follows from the distribution of terms. Indeed, according to the first special rule of the second figure, one of the premises in this figure must be negative. This means that the conclusion, according to the sixth common rule for all syllogisms, will be negative. But in negative conclusions, a larger term (as a predicate of negative judgment) is always distributed. Being distributed in the conclusion, the larger term, according to the fourth general rule, should be distributed in the larger premise. According to the conditions of the second figure, the larger term in the larger premise is the subject. But the term subject is distributed only in general judgments. So, a larger premise cannot be private.

§ 37. All possible correct modes of the second figure are set in the same way as the modes of the first figure. Having excluded from the sixteen arithmetically possible modes all the modes that contradict the general rules of all the figures and the special rules of the second figure, we get four correct modes of the second figure: EA, AE, EI, AO.

In the EA mode, the conclusion, as it is easy to show from the distribution of terms, will be negative (E), and the whole structure of the mode can be denoted by EAE.

Example: “Not a single fat is soluble in water; all alcohols are soluble in water; therefore, no alcohol is fat.”

In EA mode, the conclusion is also generally negative (E), and the entire structure of the mode can be denoted by AEE.

Example: “All insects are tracheal breathing, not a single spider is tracheal breathing; therefore, no spider is an insect.”

In the EI mode, the conclusion is partial negative (O), and the entire structure of the mode can be denoted by EIO.

Example: “Not a single plant with a rhizome is annual, some violet plants have a rhizome; therefore, some violets are not annual plants.”

In the AO mode, the conclusion is also partially negative (O), and the entire structure of the mode can be denoted by AOO.

Example: “All hot solids give a continuous spectrum, some nebulae do not give a continuous spectrum; therefore, some nebulae are not red—hot solids.”

Conditional names of the modes of the second figure:

Cesare, Camestres, Festino, Varoso.

Comparing the conclusions possible in the second figure, we see that all of them can really only be negative: generally negative or particularly negative.

It does not follow, however, that the negative conclusions, the only ones possible in the second figure, have no value for knowledge.

It has already been shown that the modes of the second figure are used in those cases when the object of our interest is precisely negation, not affirmation. But such cases are not rare. Both in practical activity and in the activity of scientific knowledge, our interest is directed towards clarifying not only what connects, but also what separates. The establishment of distinction, heterogeneity, and incompatibility is often of the greatest interest, both practical and theoretical.

On the other hand, negative conclusions, which are of little interest in themselves, can in some cases be used as a *means of preparing a positive solution to the problem* . Many complex tasks are solved by *sequential exclusion* those cases in which the desired solution cannot be found until, finally, they reach the only remaining case representing a positive solution. In studies of this kind, an exception is made on the basis of negative conclusions up to the second figure. Suppose that when examining a gaseous substance, we ask ourselves whether sodium is in the composition of this substance. Knowing that the spectrum of gaseous substances containing sodium in its composition has a characteristic bright yellow line, and having established that the test substance does not give this line in the spectrum, we conclude from the second figure (Camestres mode) that there is no sodium in the studied substance.

Another example. If we know that in this mixture there can be only some of the substances m, k, n, 1, p, but we don't know which ones, then one of the ways to solve the problem is that, based on the negative conclusions on the second figure the impossibility of the presence, for example, of substances k, 1, p, we conclude that the composition of the mixture includes m and n.

The Logical Course of Inference in the Syllogisms of the First and Second Figures

§ 38. The logical course of inference in the syllogisms of the *second* figure differs significantly from the course of inference in the syllogisms of the *first* figure.

In the syllogisms of the first figure, the conclusion goes from a *group of* objects to *individual* objects. And indeed: the greater premise in the syllogism of the first figure is a judgment on a whole group of objects. But at the same time, the predicate of this proposition is not only the predicate of the whole group, but also the predicate of *each of its members separately*. Therefore, having established in a smaller premise that any object is in fact one of the members of the group, we can attribute to this separate subject the definition of the whole group.

On the contrary, in the syllogisms of the *second* Inference figures are based on a comparison of predicates, or, what is the same, on a comparison of the definitions of the subjects of both premises. Comparison of this reveals that both definitions stand against each other in relation to the logical opposite and that the subject of one definition cannot be identical with the subject of another. Therefore, the establishment of the logical opposite of two predicates appears in the syllogisms of the second figure as the basis for the assertion that the subject of one of them cannot be the subject of the other. Therefore, all the conclusions of the second figure can only be negative.

The Third Figure and Its Special Rules

§ 39. The third figure of a simple categorical syllogism:

$$\begin{array}{c} M — P \\ M — S \\ \hline S — P \end{array}$$

The conclusions of the third figure apply everywhere where the subject of our interest is the *knowledge of the private*. The area of interest in the private is extremely vast. It would be wrong to think that the private can interest us only as a means of knowing the general. Of course, in some cases, the particular attracts our attention precisely as such a means. To the knowledge of the general, we go through the knowledge of the particular. In these cases, we take advantage of the fact that the general reveals its properties, manifesting itself in the particular. So, we want to know the properties of a tree in general, of every tree. But we don't see the "tree at all," we only see particular cases or varieties of the tree — this oak, this birch, this spruce, etc. Studying the properties of oak, birch, spruce, we clarify the properties of not only these particular species, but also the properties of the tree in general.

However, except for the cases when cognition of the private is only a stepping stone to cognition of the general, there are many cases when the quotient is the subject of our interest and knowledge no longer as a way of knowing the general, but in itself, i.e., as a *particular*. I may be interested not in those properties of oak, from which it is clear that oak is only a case, or species, of a tree, but precisely in its properties that distinguish oak from all other trees: birches, firs, pines, maples, etc.

When our thought moves from the particular to the general in such a way that interest in the particular is only a step towards the knowledge of the general, we apply various forms of the so—called inductive inferences. These forms will be considered by us in their place (see chap. XI).

When the subject of our thought is private in itself, and not as a means of knowing the general, we use the various modes of the *third figure of the syllogism.*

Examples of syllogisms of the third figure:

All cetaceans are mammals.	Not a single spider is an insect.
All cetaceans are aquatic animals.	All spiders are arthropods.
———	———
Some aquatic animals are mammals.	Some arthropods are not insects.

In the first example, the big premise confirms that all M belong to class P, the smaller one — that all M belong to class S (see Fig. 63).

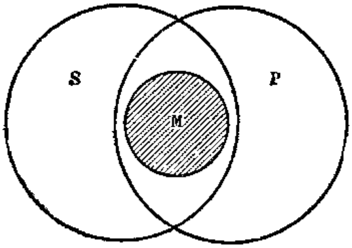


Fig. 63

The figure shows the relationship between the concepts in the premises. It can be seen from the figure that the whole volume M is included as part of both volume P and volume S. But since it is not visible from the premises which part of volume P and which part of volume S is occupied by volume M, we cannot state in the conclusion that *all* S belong to P; we can only say that *some* S belong to R. Namely: the common part of S and P will be the part of the volume of each of these concepts that is occupied by the volume M.

In the second example, the larger premise states that not one M belongs to the number R. The smaller premise states that all M belong to S (see Fig. 64).

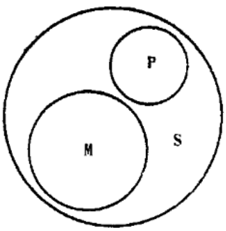


Fig. 64

The figure shows the relationship between the concepts in the premises. The figure shows that the entire volume of class M is outside the entire volume of class P and that the same entire volume of class M is included as part of the volume of class S. Since, being all arthropods, spiders are not insects at the same time, this implies the conclusion is that some of the arthropods (spiders) are not insects: some S do not belong to R.

And in both examples of the third figure, the conclusion is *private:* in the first example it is partially affirmative, in the second it is private negative.

Often the third figure is used to prove the partial compatibility of two concepts, which for some reason it is customary to think of as if they are completely incompatible. Let someone think that no mammal lays eggs. Believing in this way obviously affirms the complete incompatibility of the concepts of "mammal" and "ovipositing." His thought can be expressed through the general judgment "no mammal is an ovipositor." To refute this general judgment, it is enough to prove the truth of the *private* judgment that contradicts it . Such a particular proposition will obviously be the proposition "some mammals are ovipositing." This judgment can be deduced from the third figure of the syllogism:

All platypuses are egg—laying.
All platypuses are mammals,

Some mammals are egg—laying.

Since a judgment that contradicts a general judgment will always be *private*, and since *partial* compatibility of concepts is established in a *private* judgment, the conclusions of the third figure, which can be used either to refute general judgments through contradictory particular ones or to prove partial compatibility of concepts, can *only* be *private*.

§ 40. A special rule of the third figure follows from these tasks. This rule is formulated as follows: a *smaller premise must be in the affirmative.* Indeed, if the smaller premise of the third figure were negative, then the conclusion would also have to be negative. But this means that a larger term, like a predicate of negative judgment, should have been distributed in the conclusion. However, in order to be distributed in the output, the larger term must be distributed in the larger premise. Since we assumed that the smaller premise is negative, the larger should be affirmative. But since in the third figure the larger term is a predicate, then as a predicate of affirmative judgment expressing the subordination of the concept S to the concept of P, it cannot be distributed, and, therefore, the conclusion about the third figure in the case of negativity of a smaller premise is impossible.

§ 41. Eliminating from the sixteen arithmetically possible modes of the third figure all modes that contradict the general rules of all figures and the special rule of the third, we obtain six modes of the third figure: AA, EA, IA, AI, OA, EI.

In mode AA, the conclusion is partially affirmative (I), and the entire structure of the mode can be denoted by AAI.

Example: “All whales are mammals, all whales are aquatic animals, therefore, some aquatic animals are mammals.”

In EA mode, the conclusion is Partially Negative (O), and the entire structure of the mode can be denoted by EAO.

Example: “No mushroom has chlorophyll, all mushrooms are plants, therefore, some plants do not have chlorophyll.”

In mode IA, the conclusion is private (I), and the entire structure of the mode can be denoted by IAI.

Example: "Some planets have satellites, all planets revolve around the sun, therefore, some bodies revolving around the sun have satellites."

In the AI mode, the conclusion is partially affirmative (I), and the entire structure of the mode can be denoted by AII.

Example: "All beavers are aquatic animals, some beavers build houses for themselves, therefore, some animals who build houses for themselves, water animals."

In the OA mode, the conclusion is private negative (O), and the entire structure of the mode can be denoted by OJSC.

Example: "Some planets do not have satellites, all planets revolve around the sun, therefore, some bodies orbiting the sun do not have satellites."

Finally, in the EI mode, the conclusion is also partially negative (O), and the entire structure of the mode can be denoted by EIO.

Example: "Not a single graduate student is a student, some graduate students are required to attend lectures, therefore, some persons required to attend lectures are not students."

The conventional names of the six modes of the third figure are as follows: Darapti, Felapton, Disamis, Datisi, Bocardo, Ferison.

Thus, all three figures of a simple categorical syllogism give a total of fourteen correct modes. Other modes in these figures are impossible, that is, they cannot be the basis for the correct conclusion.

The Logical Course of Inference on the Third Figure

§ 42. The conclusions of the third figure have features in the very logical course of the conclusion that distinguish them from the conclusions of the first and second figures. From the conclusions of the second figure, in which the logical course of the conclusion is based on comparing the *predicates of* both premises, the conclusions of the third figure differ in that, as in the conclusions of the first figure, the *subjects of* both premises are compared.

Consider the conclusion:

All beavers are aquatic animals.	M — R
All beavers are mammals.	M — S
————————	————
Some mammals are aquatic animals.	S—P

The affiliation of a part of mammals to aquatic animals is derived from the fact that all beavers have been found to be in aquatic animals and mammals.

At the same time, the conclusions of the third figure are different from the conclusions of the *first* figure. In the conclusions of the first figure, the logical course of the conclusion is that, having established in a lesser premise that an object belongs to a known *group of* objects, we transfer to a separate object, conceivable in a smaller premise, a predicate that characterizes the group as a whole. This transfer is based on the fact that the predicate of a larger premise is not only the predicate of the whole group as a whole, but also the predicate of each of its members separately.

Consider the syllogism:

All amphibians are vertebrates.

All frogs are amphibians.

All frogs are vertebrates.

Having established the affiliation of frogs to amphibians in a smaller premise and having established in a larger premise that belonging to vertebrates is a property not only of the whole amphibian group as a whole, but also of each member of the amphibian group, we can attribute to all frogs belonging to vertebrates.

In the conclusions of the third figure, the logical course of the conclusion is different. Although the conclusions of this figure by general premises substantiate a particular conclusion, the meaning of the conclusion is not only to express a predicate with respect to some members of the group. When from the premises “all beavers are aquatic animals”, “all beavers are mammals” deduce that “some mammals are aquatic animals”, the meaning of this conclusion is not only to ascribe a certain part of mammals to aquatic animals. The meaning of the conclusion to the predicate “aquatic animals” to indicate not only as a predicate to a subject, “some mammals”, but also as a possible predicate or determination archaeological. To group *new* what we learn from this syllogism is not in the idea that some of the mammals are aquatic animals. This, in essence, we already know from the premise “all beavers are aquatic animals.” The new thing that we learn from this syllogism is the idea that *mammals can be* aquatic animals, in other words, that belonging to aquatic animals is a possible characteristic of the *entire group of mammals,* although in reality this characteristic can always be applied, as can be seen from the conclusion of the syllogism, only to *some* members of the mammalian group. In other words, the new, delivered by this syllogism, consists in the idea that a group of mammals as a *whole,* as a *group* characterised by the fact that some members of this group, such as beavers, can be aquatic animals.

The fact that the conclusion of the syllogism of the third figure can only be a *private judgment* does not in any way contradict the fact that the conclusion of the third figure is, in essence, a conclusion about a *group of objects* generally. The private nature of these conclusions shows only that the possibility of assigning the predicate of conclusion to the whole group is limited to some, definitely not defined part of the group: although belonging to aquatic animals is a possible belonging to the whole group of mammals and although in this sense it can be said that the subject is in conclusion the group of mammals as a whole is itself—nevertheless, this characteristic of the whole group remains here incomplete and insufficient: we do not know from the conclusion *which* part of the mammals are aquatic animals.

The use of syllogisms of the third figure *to refute* erroneous judgments about the group proves the truth of what was said. So, the statement "atomism is incompatible with the doctrine of the possibility of freedom" can be opposed by the following syllogism of the third figure as a refutation:

Epicurus was an atomist.

Epicurus claimed the possibility of freedom.

Next, some atomists have argued for freedom.

In this syllogism, the subject of the conclusion "some atomists asserted the possibility of freedom", despite the particular nature of the conclusion, is precisely the group as a whole: the *entire* group of atomists is characterized as such within which, as part of it, individuals who allow the possibility of freedom *can* be found.

The Fourth Figure and its Special Rules

§ 43. The fourteen correct modes considered were established by the founder of the science of logic, the ancient Greek philosopher Aristotle (384–322 BC). Already the next successors of Aristotle’s logical work drew attention to the fact that in the first figure, in addition to the four modes indicated by Aristotle, five more are possible. These moduses are possible if the middle term is a predicate in the larger premise and the subject in the smaller. (In the Aristotelian first figure, the middle term is, on the contrary, the subject in the larger premise and the predicate in the smaller.)

500 years after Aristotle, the scientist Galen singled out the correct modes, resulting from this arrangement of terms, in a new—fourth—figure.

Scheme of the fourth figure:

P — M
M — S
———
S — P

Although the fourth figure is theoretically possible and gives five correct modes, the conclusions on the fourth figure are not found in actual thinking. The artificiality of the fourth figure is that the position of the smaller and larger terms in *deducing the* position of these terms in the *premises.* Therefore, you can not come up with a single example of the conclusion on the fourth figure, which would not be artificial.

For instance:

All seals are pinnipeds.	M — R
No pinnipeds eat fish.	P — M
———————	———
No fish eat a seal.	M — S

Here, of course, the conclusion on the first figure would be natural:

No pinnipeds eat fish.	M — R
All seals are pinnipeds.	S — M
________________	______
No seals eat fish.	S—P

In view of the perfect artificiality of the fourth figure, we note only its most important features without a detailed examination and derivation of them.

Conclusions on the fourth figure can be partly affirmative, generally negative, and particularly negative. The fourth figure (as well as the second and third) does not give general affirmative conclusions. The general conclusion on the fourth figure can only be negative. With the assertion of a larger premise, the smaller premise in the fourth figure should be common. If one of the premises is negative, the large premise in the fourth figure should be common.

The correct modes of the fourth figure are: AAI, AEE, IAI, EAO, EIO. Their artificial names are Bramantip, Camenes, Dimaris, Fesapo, Fresison.

Thus, given the possibility of an additional five modes of the fourth figure, we get only nineteen correct modes of simple categorical syllogism.

Reduction of all Figures of a Simple Categorical Syllogism to the First Figure

§ 44. Each of the figures with all its modes is independent and has its own special field of application. But since the relation between the smaller and the larger terms, which constitutes the conclusion, is determined by the relations between all three concepts of syllogism, and since these relations can be revealed in a different order—depending on which concept we begin the consideration—the conclusion made on some figure of the syllogism can be made on any other (unless the quality and quantity of the conclusion contradict this). Such a change in the conclusion drawn from any figure of the syllogism to the conclusion drawn from another figure is called a *reduction.*

In logic, the rules for reducing all figures to the *first* figure are set in detail—in view of the importance of the conclusions that the first figure, especially the Barbara modus, have in scientific and everyday thinking.

Usually the conclusions of the third figure are reduced to the conclusions of the first figure by *reversing* one of the premises.

For example, the conclusion of the third figure

All whales are mammals.	M — R
All whales are aquatic animals.	S — M
———————————————	————
Some aquatic animals are mammals.	S—P

can be changed to the output of the first figure. To do this, leaving the larger premise unchanged, we draw the smaller premise: “all whales are aquatic animals.” The appeal of a general affirmative proposition expressing the subordination of the concept S to the concept P gives, as you know, a particular affirmative proposition: “some aquatic animals are whales.” Now we will connect the large premise left unchanged with the smaller one reversed:

All whales are mammals.
Some aquatic animals are whales.

In the premises of these terms, they are located according to the scheme of not the third, but the first figure:

$$\frac{\begin{array}{c} M — P \\ S — M \end{array}}{S — P}$$

The conclusion of the first figure (mode Darii) will be: "some aquatic animals are mammals." As you can see, the conclusion is the same as in the first case was made on the third figure (modus Darapti).

§ 45. There is a more complicated method of reduction. This method is used to reduce some conclusions on the second and third figure to the conclusion on the first.

Consider the syllogism:

All planets revolve around the sun.	P — M
Some luminaries do not revolve around the sun.	S — M
———	———
Some luminaries are not planets.	S—P

This syllogism, as can be seen from the arrangement of terms, is a conclusion on the second figure (modus Varoso). To bring it to a conclusion on the first figure, we will reason as follows. Suppose that the conclusion of our conclusion is false, that is, suppose that all luminaries are planets. Let us leave the larger premise unchanged and add to it, as the smaller premise, the judgment "all the stars are planets," that is, a judgment *contrary to the* conclusion:

All planets revolve around the sun.
All luminaries are planets.

These premises form the premises of the correct conclusion on the first figure. The very conclusion is, obviously, mode Barbara:

All planets revolve around the sun.	M — R
All luminaries are planets.	S — M
————————	————
All luminaries revolve around the sun.	S—P

Let us now compare our new conclusion with the premise of sending the initial syllogism: "some luminaries do not revolve around the sun." Obviously, this conclusion contradicts the smaller premise.

From this, of course, we conclude that our assumption that "all the stars are planets" is false, since it contradicts one of the premises we have accepted. But this means that a judgment must be true that contradicts the assumption made, that is, a judgment: "some luminaries are not planets."

So, we were convinced of the truth of the conclusion of the second figure by reducing this conclusion to the conclusion of the first. This reduction was necessary in order to be convinced of the absurdity of a judgment contrary to the conclusion.

This technique of information is called "reductio ad absurdum"—"leading to absurdity". Through this technique, the conclusions on the first figure are reduced: 1) the modus Varoso of the second figure and 2) the modus Bocardo of the third. The letter r in the names of these modes shows that in them the reduction to the conclusion on the first figure is achieved by reductio ad absurdum. The letters B, C, D, F in the names of the modes of the second and third figures show that, after mixing the modes, these transform into the Barbara, Celarent, Darii, Ferio modes of the first figure, respectively. The letters s and p appearing in the names of the modes of the second and third figures after the vowels indicate that, for information, the premise indicated by these vowels should be reversed. At the same time, the letter s indicates that the amount of the parcel remains the same during handling, and the letter p indicates that when handling the general parcel becomes private.

For example, when reducing the Cesare modus of the second figure, looking at the name of the Cesare mode, we immediately see that after the reduction we should get the Celarent modus of the first figure (this is indicated by the letter C in the word Cesare), that the reduction itself must be done by reversing the larger premise (this is indicated by the letter s, placed after e, the sign of the larger premise) and that the larger premise remains after the appeal is common (this is clear from the fact that after e is not p, as). Indeed, the conclusion on the second figure of the Cesare modus

Spore plants have no flowers.

Cereals are plants that have flowers.

Cereals are not spore plants.

comes to the conclusion on the first figure of the Celarent modus:

Plants with flowers are not spore plants.

Cereals are plants that have flowers.

Cereals are not spore plants.

The reduction is achieved here by reversing the larger premise: “spore plants have no flowers.” As a negative judgment, the larger premise after the appeal remains general: “plants with flowers are not spore.”

§ 46. Since the conventional names of mods include indications of the quality and quantity of premises and conclusions in the correct conclusions, as well as indications of the methods for reducing the conclusions of the second, third and fourth figures to the conclusions of the first, in order to conveniently remember and review all mods and their features, a Latin poem was invented listing all these names by individual figures. Here it is:

Barbara, Ceiarent, and Darius, I beat *the former* ;
Cesare, Camestres, I make haste, baroque, *of the second;*
The third Darapti, Disamis, then we Datisi,, Felapton and
Bocardo, the Ferison, *it has; the fourth ‘he adds,*
moreover, Bramantip, Camenes, Dima, Fesapo, Fresison.

The Axiom of Syllogism and Its Two Formulas

§ 47. We examined all the figures and all the correct modes of syllogism. We have seen that subject to the well—known rules to which the premises and the relations between the terms included in the premises must obey, the premises lead to the correct conclusions. This means, in other words, that, having recognized such premises as true, we cannot but recognize as true those conclusions that are justified by premises.

Although in different figures, and inside the same figure in its various modes, the methods for substantiating conclusions turn out to be, as we have seen, different, yet in all syllogistic conclusions there is a common ground for all of them, by virtue of which, having acknowledged the premises, we must recognize true and the conclusions arising from them.

This common ground for all syllogisms is expressed in the following formula: *“The sign of the sign of some thing is the* *sign of the thing itself; that which contradicts the attribute of a certain thing contradicts the thing itself."* This formula expresses in the most general form the logical connection of the concepts S, M and P, on which the conclusion is based and which makes this conclusion necessary. Consider, for example, syllogisms:

All halides are found in the form of salts.	No spore plant reproduces by seed.
All chloride compounds are halogens.	All mushrooms are spore plants.
________________________	________________________
All chloride compounds are found in the form of salts.	Not a single mushroom reproduces by seed.

In the first of these syllogisms, a larger premise establishes that salt is a sign of halogen. A smaller premise establishes that the sign of belonging to halogens is a sign of chloride compounds. From both premises it can be seen that the sign of belonging to salts turned out to be a sign of the sign of some thing. Hence the conclusion follows that belonging to salts is at the same time a sign of the thing itself, or that "all chloride compounds belong to salts."

In the second syllogism, the smaller premise clarifies that "belonging to the spore" is a sign of a thing called "mushrooms". A big premise finds out that "seed propagation" contradicts this attribute of a thing. From this it follows that, being in conflict with the sign of a thing, the sign of "propagation by seeds" is in conflict with the thing itself, that is, "not a single mushroom reproduces by seeds".

A formula expressing a common ground for all syllogisms is called the *axiom of syllogism* . This name shows that the rule expressed by the axiom of syllogism is not proved. It is obvious and underlies all syllogistic conclusions.

The axiom of syllogism expresses the essence of syllogism. All the syllogism rules set forth above, which relate to the terms of syllogism, to the quality and quantity of premises, to the quality and quantity of the conclusion, are nothing but the various applications of the axiom “the sign of the sign of some thing is the sign of the thing itself”.

§ 48. But this is not enough. The axiom of syllogism also expresses the significance that logical laws of thought have for syllogisms: the law of identity, the law of contradiction, the law of the excluded third, and especially the law of sufficient reason.

Indeed, the predicate P expressed about M turns out to be in syllogism the foundation that determines all the consequences arising from it: only a conclusion that has sufficient basis in the premises can be correct; a sufficient basis for judging whether a sign belongs to an object is that the sign expressing the property of the object is, as can be seen from the premises, a sign of the sign of the object itself.

Further. Any attempt to violate the law of contradiction and the law of the excluded third, when thinking of syllogism, that is, an attempt, in agreement with the premises, to deny the conclusion necessary from these premises is an obvious violation of the axiom of the syllogism. If the sign of the sign of a certain thing is a sign of the thing itself, then it is impossible to simultaneously recognize the premises, i.e., to acknowledge that we are dealing with the sign of the sign of a certain thing, and to deny the conclusion, i.e., to assert that, being a sign of the sign of a certain thing, this sign is not at the same time a sign of the thing itself.

Finally, the axiom of syllogism is incompatible with the violation of the law of identity. Any violation in the syllogism of the law of identity, that is, any attempt to think a second time in premises or in conclusion, is no longer a concept of P, but some other concept of P_1, not the concept of M, but some other concept of M_1 and not the concept of S, but some other concept of S_1 , obviously, would mean a violation of the axiom of syllogism. Indeed, any substitution in the syllogism, for example, instead of the concept P of some other concept P_1, would mean the impossibility, having agreed that we are dealing with a sign of a sign of a certain thing, to assert that this sign is a sign of this very one, and not some other stuff.

§ 49. The axiom of syllogism in the form we have examined expresses the significance that the *content of* our concepts has for thinking . This axiom expresses that the necessary connection of concepts, revealed by syllogism, is the connection between concepts *in their content* , that is, in their *essential* attributes: “the sign of the sign of a thing is a sign of the thing itself”.

But since the relation between their volumes is also determined by the relationship between concepts, the axiom of syllogism can be expressed in another form, highlighting the relationship *between the volumes of* concepts included in the premises and in the conclusion of the syllogism.

In this form, the axiom of syllogism is formulated as follows: *“Everything that is affirmed with respect to a whole genus or species must be affirmed with respect to everything subordinate to that genus or species, and everything that is denied with respect to a whole genus or species must be denied with respect to everything subordinate to that genus or species.”*

§ 50. Each of the above two formulations is an expression of the axiom of syllogism. The first reveals the necessary relationship between the *content of* concepts that make up the premise, and the *content of* concepts that make up the conclusion. The second reveals the necessary relationship between the *volumes of* the same concepts.

If the purpose of the syllogism is to establish that the class belongs to the class, then the logical basis for the conclusion is expressed as the *second* the formula: “Everything that is affirmed regarding a whole genus or species, etc.”

But although, thus, the second formula, just as correctly as the first, expresses the axiom of syllogism, and in some cases (where the relations of *volumes* are a special subject of interest) even deserves preference, the first formula is the main one.

Indeed, the relationship between the volumes of concepts established in the conclusion of the syllogism itself is based, as we have repeatedly seen in this, on the relationship between the same concepts *in content*.

The Truth Conditions of Syllogistic Conclusions

§ 51. Until now, when considering syllogistic conclusions, we have always assumed that the premises on the basis of which the conclusion is drawn are true. If these premises are true, we reasoned, and if the relations between the concepts in these premises correspond to the conditions of correct conclusions, then the conclusions themselves must be true.

In the practice of thinking, this condition is far from always fulfilled. Not always the premises from which they conclude are truly true.

If one of the premises or both are false, then, even after exactly fulfilling all the inference rules defined by its figure, mode, distribution conditions for terms in the premises, etc., we generally cannot get the correct conclusion.

Consider the following conclusion:

All plants contain chlorophyll.
All mushrooms are plants.
____
All mushrooms contain chlorophyll.

In making this conclusion, we obviously believe that both premises are true. If they really were both true, then, since we did not violate a single general and not a single special rule of syllogism figures, our conclusion in the conclusion would also be true.

In reality, however, the larger premise is false in substance. Chlorophyll is not all plants. This means that in the larger premise we thought not the relationship between the content of concepts, and therefore, not the relationship between the volumes of concepts that actually exists. In fact, only a part of plants has essential characteristics of plants possessing chlorophyll, and therefore only a part of plants is included in the category of plants having chlorophyll. Truth would be expressed by the premise “some plants have chlorophyll.” Combining it with another true premise “all mushrooms are plants”, we would get a system of premises: “some plants contain chlorophyll”, “all mushrooms are plants”, from which no conclusion can be made about mushrooms, as the average term (“plants”) is not distributed in any of the packages, and the relationship between S and P remains too vague to deduce. Instead, we, having allowed the false premise “all plants contain chlorophyll”, also received the false conclusion: “all mushrooms contain chlorophyll”.

It can be seen from the foregoing that the *first necessary condition for the correct syllogism is the truth of the premises on which the conclusion is based, in essence their content.* If the premises are false, then they cannot be a sufficient basis for a conclusion that is correct in content.

Thus, even the most exact observance of the logical rules of syllogism does not yet ensure the truth of the conclusion in itself. Observance of the rules of syllogism gives a true conclusion only if the premises are true, that is, if the premises correspond to real facts.

All syllogism assumes (albeit sometimes erroneously) *truth* parcels; with this in mind, we have the right to mentally distract ourselves from the question of the truth of premises and focus only on the question of the logical connection between premises and conclusions, i.e., the question of whether this conclusion follows from the data (and always assumed true) premises.

The conclusions considered from this point of view may be right or wrong. They will be correct if the parcels satisfy all the general and special conditions of the figures necessary to obtain a conclusion from them. They will be wrong if the conclusion is made contrary to these conditions.

Logical Errors Encountered in Syllogisms

§ 52. Some of the logical errors of an incorrect conclusion, which are especially common in the practice of thinking, deserve to be especially noted.

One of the most common mistakes here is that, judging by the first figure, they conclude with a negative smaller premise.

All students are required to take exams.
Graduate students are not students.
————————————
Graduate students are not required to take exams.

The conclusion is clearly erroneous. The conclusion can be negative only if the larger term is distributed in the larger premise. But in the larger premise, he, as a predicate of an affirmative proposition expressing the subordination of the concept of S to the concept of P, is not distributed. Therefore, the conclusion here is logically impossible.

But if it is logically impossible, then why is such a mistake *actually* possible ? —One of its sources is a misinterpretation of the meaning of a larger premise. If, having heard that “all students are required to take exams”, we will interpret this provision in the sense that “only students are required to take exams”, then our conclusion will take the following form:

Only students are required to take exams.
Graduate students are not students.
———————————
Graduate students are not required to take exams.

Recognising these premises as true, we made the correct conclusion from them, i.e., the conclusion here necessarily follows from the accepted premises. The mistake here is not that we ignored the well—known rule about the distribution of the larger term distributed in the output, but that, having misinterpreted the meaning of the larger premise, we received a premise that was essentially false, and therefore received a false conclusion.

§ 53. The second mistake encountered in the practice of syllogistic conclusions is that they conclude from the second figure of two assertions.

Example:

All fish have fins.
This animal has fins.
———————————

This animal is a fish.

The conclusion here is clearly erroneous. Since the average term in both premises is a predicate of an affirmative judgment expressing the submission of concepts, it is not distributed in any of the premises. Therefore, no conclusion is possible here. Both “fish” and “this animal” are included in the scope of the concept of “animals with fins.” But since from the premises it is not known which part of this volume includes “fish” and which part is “this animal”, the relation of “this animal” to “fish” remains completely unclear; it is possible that “this animal” is “fish”, and it is possible that it is not “fish”.

However, in such a case, the mistake usually consists not so much in violating the well—known rule about the distribution of the average term, but in misinterpreting the meaning of the larger premise. Who, having heard the judgment “all fishes have fins,” will understand it in the sense of “only fishes have fins,” he will obviously draw the following conclusion:

All animals with fins are fish.
This animal has fins.
———————————
This animal is a fish.

In this conclusion, the conclusion would have to be true if both premises were true. But the greater premise is false, and therefore the conclusion is also false.

§ 54. A third mistake, often encountered in the practice of conclusions, is called the “quadrupling of terms” (quaternio terminorum). It consists in drawing a conclusion from two premises, which include not three, but four terms. An example of such an error:

All combustion produces ash and ash in the remainder.
All oxidation is combustion.
———
Any oxidation produces ash and ash in the residue.

Since the connection between the concepts included in the conclusion is not immediately visible, it can be established only through the third concept, the relation of which to larger and smaller terms would be known from the premises. But in our example, this connection cannot be established: here the premise does not establish the relation of a larger or smaller concept to the third concept, but establishes in one premise the relation of a larger term to the third concept ("combustion" in the *chemical* sense, that is, a process which is not necessarily accompanied by the appearance of ash and ash), and in the other—the relation of a smaller term to the fourth concept ("burning" in *everyday* unscientific sense, meaning a process in which ash and ash are always obtained in the residue). It is not surprising that, without being interconnected through the third concept in the premises, larger and smaller terms cannot be related in the conclusion.

And here the basis of the error is not so much in violating the rule on the number of terms included in the syllogism as in the ambiguity of the word "combustion", which has not one meaning, but two, expresses two concepts.

The mistake here is that packages that have a structure

M_1—P
S—M_2

we, due to the insufficient distinction between M_1 and M_2, take for premises having the structure of ordinary syllogism:

M — R
S—M

Mistakes are possible not only with respect to the mean, but also with respect to the larger and smaller terms.

From the foregoing, we see that errors encountered in syllogisms rarely consist in violating only the rules of the logical connection between premises and terms. In the final analysis, the basis of the error of the conclusion is usually the falsity of the premises, which are accepted as true.

Tasks

1. Determine which of the following reasoning will syllogisms and which—non—syllogistic inferences:

"Since *a* larger *b* and *b* is equal to *c* , then, consequently, *a* greater *with* "; "Mont Blanc below Elbrus, Elbrus below Stalin's peak, therefore, Mont Blanc below Stalin's peak"; "Lermontov was the predecessor of Leo Tolstoy, Leo Tolstoy was a contemporary of Chernyshevsky, therefore, Lermontov was the predecessor of Chernyshevsky"; "Since a biologist must be able to master a microscope, and Ivanov does not own a microscope, then Ivanov is not a biologist"; "Since all equilateral triangles are equiangular and since the triangle *ABC*— equilateral, then, therefore, the triangle *ABC* is equiangular"; "Botvinnik as a chess player is stronger than Smyslov, Smyslov as a chess player is stronger than Ragozin, therefore, Botvinnik as a chess player is stronger than Ragozin"; "None of the European mountains is higher than Elbrus, Everest is higher than Elbrus, therefore, Everest is not among the European mountains."

2. Having considered the following syllogisms, determine: a) they are right or wrong in terms of the logical connection between the premises and the conclusion; b) if they are correct, then according to what figure the conclusion is drawn in them; c) if they are incorrect, then which of the rules common to all syllogisms, and which of the rules special for individual figures, are violated in them:

"All strong chess players know the theory of a chess game well, Nikolaev is not a strong chess player, therefore, Nikolaev does not know a theory of a chess game"; "Everyone who knows how to play hockey is a skater, Sergeyev is not a skater, therefore, Sergeyev is not able to play hockey"; "White nights are observed not south of the parallel of Poltava, Kiev is located not south of the parallel of Poltava, therefore, white nights are observed in Kiev"; "Some plants reproduce by spores, all ferns are plants, therefore, all ferns reproduce by spores";""All ferns are spore, all horsetails are spore, therefore, some horsetails are ferns"; "All the heroes of the Soviet Union are awarded the Order of Lenin, comrade N was awarded the Order of Lenin, therefore, comrade N is a Hero of the Soviet Union"; "Heavy bombers are not single—engine, Mikhailov's plane is single—engine, therefore, Mikhailov's plane is not a heavy bomber"; "All arthropods are invertebrates, all spiders are arthropods, therefore, all spiders are invertebrates"; "At all the rivers of our hemisphere, flowing from north to south, the right bank is mountainous and the left bank is low, the Dnieper river is one of the rivers of our hemisphere flowing from north to south, therefore, the right bank of the Dnieper is mountainous and the left bank is low" ; "In all ancient Indian manuscripts, words do not separate from one another, in this manuscript words do not separate from one another, therefore, this manuscript is ancient Indian"; "Gas fountains are a sign of close oil production, in the village N a gas fountain has clogged, therefore, near the village N there are oil births; "All the great scientists are thoughtful people, all the great scientists are scattered people, therefore, some scattered people are thoughtful people"; "All planets have fast visible motion, not one planet is a star, therefore, some stars do not have fast visible motion."

CHAPTER X. TYPES OF SYLLOGISMS

Conditional Syllogism

§ 1. In addition to simple *categorical* syllogisms, there are also *conditional* and *dividing* syllogisms.

In simple categorical syllogism, both premises and conclusions are categorical judgments. As in any conclusion, in a simple categorical syllogism the conclusion will be true provided that not only the course of the conclusion is correct, but both premises themselves will be true judgments.

Since the premises of a simple categorical syllogism are categorical judgments, their truth is not made dependent on any conditions other than those that are in the very subject of thought. These conditions are not advanced by our thought and are not marked in the very form of judgment.

But a syllogism is also possible if the truths expressed by its premises are dependent on conditions that are immediately indicated in the premises themselves and noted in the very form of judgment.

Consider, for example, the conclusion:

If the angle inscribed in the circle is based on the diameter, then such an angle is straight.
This angle *DIA is* based on the diameter.
———————————————
This angle *DIA* is a straight line.

This conclusion is a syllogism. It clarifies in the conclusion the relationship between the two concepts (the concept of " *ACB* angle " and the concept of "right angle"). This relation is revealed through the relation of each of both concepts to the third concept (the concept of “an angle inscribed in a circle based on the diameter of the circle”).

Like any conclusion from two premises, this conclusion is a simple syllogism. However, unlike a simple *categorical* syllogism, where both premises are *categorical*, in our example, the syllogism has a different structure.

One of the premises of our syllogism—the second—is a categorical judgment. This premise establishes the relation of belonging to the angle of the *DIA* to corners based on the diameter of the circle. This attitude is conceived here as something that has already been established and is not dependent on any conditions.

On the contrary, the first premise of our syllogism is a *conditional* proposition . In the premise of this, the belonging of an angle to right angles is expressed not unconditionally, but as such a relation, which takes place provided that the inscribed angle is based on the diameter of the circle. This condition is immediately indicated, and the condition is already noted by the very form of the premise, which is a conditional judgment.

Comparing the two premises and finding (from the second premise) that the general condition indicated in the first premise in the second premise takes place, we conclude—this time categorically, and not only conditionally—that this *DIA* angle is straight.

Such a syllogism of two premises, in which at least one of the premises is a conditional proposition, is called a *conditional syllogism.*

§ 2. In conditional syllogism, at least one of the premises is conditional. As for the other premise, it can be either conditional or categorical.

If another premise of conditional syllogism is also a conditional proposition, then such a syllogism is called purely conditional.

Consider, for example, the following conclusion:

If the earth rotates around an axis, then when it rotates on the surface of the earth, centrifugal force should develop.
If centrifugal force develops when the earth rotates, then the same body on the surface of the earth should weigh less near the equator than near the poles.
————————————————————————
If the earth rotates around an axis, then the same body on its surface should weigh less near the equator than near the poles.

This conclusion is a purely conditional syllogism in which the relation of S to P is derived from the relationship of these concepts to the concept of M (“centrifugal force developing during the rotation of the earth”).

In contrast to the previous one, in this conclusion, firstly, not one condition is conditional, but *both* premises, and secondly, the *conclusion is* also a *conditional* judgment . In conclusion, not only the well—known relation is affirmed, but the dependence of this relation on some other relation is indicated: the property of bodies located on the earth’s surface to weigh less near the equator than near the poles is made in conclusion dependent on a certain condition — on the rotation of the earth around the axis. The condition is right there, in the conclusion itself, indicated.

On the other hand, the dependence of the relation conceivable in the conclusion on the condition formulated in the very conclusion is not directly established. Already in the first premise, the rotation of the earth around the axis is conceived as a condition, namely, as a condition for the development of centrifugal force.

However, it is not yet clear from the first premise that the presence of this condition signifies the truth of the relationship that is conceived in the conclusion. From the first premise it is only visible that if there is rotation of the earth around the axis, then centrifugal force should develop.

The second premise states that if centrifugal force takes place, then bodies located on the surface of the earth should weigh less near the equator than near the poles.

Comparing both premises, we find that if there is a condition specified in the *first* A premise must have a relationship not only depending on this condition, expressed in the first premise (the development of centrifugal force), but also a relation expressed in the *second* premise (lower body weight near the equator than near the poles).

The need for a relationship between the condition indicated in the first premise and the relation expressed in the second is evident from the fact that the relation depending on the condition of the first premise (the development of centrifugal force) is at the same time a condition in which there is a relation expressed in the second premise (lower weight of bodies on the surface of the earth at the equator)[1].

In general terms, the entire structure of a purely conditional syllogism can be represented by the following formula:

If A is B, then C is D.
If C is D, then E is F.
———————————
If A is B, then E is F.

§ 3. The difference between a purely conditional syllogism and a categorical one is not that the relation of concepts disclosed by conditional syllogism is supposedly devoid of necessity (which is characteristic of the relation of concepts disclosed by categorical syllogism). Conditional syllogism, like categorical syllogism, reveals the *necessary* connection between the concepts of inference. The relation conceivable in concluding a conditional syllogism is an absolutely necessary relation, if only the premises are true. But the very provisions of the premises, which necessarily determine the position of the conclusion, will not necessarily be true.

The fact that A is B is not necessary: A may be B, but may not be B. But as soon as the situation is established: “if A is B, then C is D”, it is established with it that “C *it* must be D “, if only A really has B.

In other words, in conditional syllogism, the conditional is by no means the relation that is conceived in its conclusion. Conditionally, that is, it is not necessary for thinking, only that position, which is indicated in conditional premises as conditional on the truth of the conclusion. On the contrary, the connection between this condition and the relation that follows from it is a necessary connection: since a condition exists, it is necessary to have what is due to it.

§ 4. Conditional syllogisms play a large role in everyday and in scientific thinking. They are applied wherever the question is posed about the consequences that necessarily arise from conditions that are assumed by us as theoretically possible or created by us in practice. The designer, commander, economist, business executive, mathematician, astronomer, etc. use conditional syllogisms at every step, by means of which, knowing the necessary connection existing between the known condition and the resultant from it, and also knowing that the fulfillment of this condition (in thoughts or in practice) is in our power, they conclude that in our power there will also be consequences necessary from the indicated conditions.

Conditionally Categorical Syllogism

§ 5. A purely conditional syllogism expresses the necessary connection between the conditions specified in the conditional premises and the conclusion. At the same time, however, in purely conditional syllogisms none of the premises confirms that at least some of the conditions noted in them exist in reality. Therefore, the conclusion in a purely conditional syllogism cannot be a categorical, but only a conditional proposition.

But another kind of simple conditional syllogism is also possible. A conditional syllogism is possible in which not only the condition necessary for a known position to be true is clarified, but it is also established that, since this condition does occur, the situation necessary due to this condition is actually true.

Consider, for example, the conclusion.

If the triangle *ABC is* rectangular, then the square of its side lying against the right angle should be equal to the sum of the squares of its two other sides.
Triangle *ABC* — Rectangular.
———————————————————————
In a triangle *ABC, the* square of its side lying against a right angle equals the sum of the squares of its two other sides.

Inference is a simple *conditional* syllogism, since one of its premises is conditional judgment. However, in contrast to a purely conditional syllogism, in which both premises are conditional, in this conclusion *only one* of the premises is conditional , the other is a *categorical* judgment . This premise establishes that the position that was conceived in the conditional premise as a condition of the truth of some other position and which was not yet known whether it actually takes place, actually takes place.

Since it can be seen from the categorical premise that the condition indicated in the conditional premise is indeed fulfilled, the conclusion establishes that the consequence of this condition does occur.

Conditional syllogism of such a structure is called conditionally categorical.

§ 6. The conditionally categorical syllogism, in turn, has two varieties, or two modes.

The first modus of conditionally categorical syllogism has a structure, an example of which we have just examined. In this modus, as in any conditionally categorical syllogism, one of the premises is a conditional proposition, the other is categorical.

The part of the conditional premise that establishes a known position as a necessary result of a certain condition is called a *consequence*. The part of the conditional premise, indicating the very condition on which the truth of the investigation depends, is called the *basis*.

Example:

If the low tide comes at low tide, ships at that port go to sea.
In a shallow port, the ebb tide came.
———
Ships in the shallow port overlook the sea.

In this example, the categorical premise confirms that the basis that was only put forward by the thought in the conditional premise is not only an assumption, but a real fact.

The combination of both premises provides a basis, based on which we are entitled to make not only conditional only, but a *categorical* conclusion about the consequence that was only supposed in the conditional premise.

The whole course of the conclusion in this case is that, having recognized the *foundation* as true , we must also recognize the *consequence* as true . The truth of the foundation is established by a categorical premise, the consequence caused by the foundation is conditional.

A conditionally categorical syllogism of this kind is called the *"affirming mode"*. His Latin name is "modus ponens".

In general, the structure of the ponens modus can be expressed by the following formula:

If A is B, then C is D.
But A is B.
———————————
Next, C is D.

It does not at all follow from this formula that any conclusion that can be obtained by the ponens modus will *always* be an *affirmative* proposition. The conclusion can be in some cases, as in the above example, affirmative, in others—negative.

The quality of the conclusion in the syllogism mode ponens depends on the quality of the conditional premise. If the conditional premise is affirmative, that is, if the relation put in it depending on a certain condition as a consequence of it is a positive relation, then the conclusion will be affirmative.

But if the conditional premise is negative, that is, if the relation set in it depending on a certain condition as its consequence expresses negation, then the conclusion will be negative.

Example:

If the sun is not visible in the sky, then the coniferous forest does not smell of tar.
The sun is not visible in the sky.
____
Next, coniferous forest does not smell of tar.

Here we have the correct syllogism modus ponens. The conclusion of this syllogism is negative.

It would also be wrong to think that the categorical premise in the syllogism of the ponens mode should always be affirmative. The significance of the categorical premise in this modus is not at all that it expresses a statement without fail. A categorical premise in the ponens modus is intended to certify that a condition that is indicated by a conditional premise and on which some consequence depends on the basis is true. But whether this condition itself will be affirmative or negative — it depends on the *content* conditional parcel. If the conditional premise indicating the basis for the investigation is negative, then the categorical premise, certifying that this condition holds, will also be negative. This is exactly the case in the example discussed above.

A characteristic feature of the affirming mode is *in the course of thought from foundation to effect.* This modus is used wherever it is necessary to deduce from the presence of a basis the existence of a consequence caused by this basis. This modus is used not only to state the provisions that are necessary following from known conditions. It is also widely used in all kinds of disputes, evidence. One of the means of convincing the truth of a well—known thesis is to prove that this thesis is only a necessary result of the fact that some proposition put forward earlier as a condition for the truth of a thesis is not only something conceivable by us, but also a real fact.

§ 7. The second modus of conditionally categorical syllogism represents a different train of thought.

Consider the conclusion:

If this substance is sodium, then the spectrum of its incandescent vapours will give a bright yellow line.
The spectrum of incandescent vapours of the substance does not give a bright yellow line.
———
Next, this substance is not sodium.

This syllogism is also conditionally categorical, since one of its premises is conditional, and the other is categorical judgment. But the conclusion in this syllogism is not the same as in the case of the approving mode. Here the categorical premise establishes that the investigation, which in the conditional premise was made dependent on the basis indicated in it, does not actually take place. The absence of an investigation gives the right to deny the existence of a foundation, if we know from the conditional premise that if there is a basis, the investigation must also be obtained. In other words, the conclusion here is that, denying the *consequence* , it is necessary to deny its *foundation*.

A conditionally categorical syllogism of such a structure is called a "denying mode". His Latin name is "modus tollens".

In general terms, the formula of this modus is as follows:

If A is B, then C is D.
But C is not D.
———
Next, A is not B.

It does not follow from this formula that any conclusion that can be obtained by the tollens modus will always be a negative judgment. The conclusion, as in the ponens modus, can be negative in some cases and affirmative in others.

The quality of the conclusion in the tollens mode is *opposite to the quality of the conditional premise:* if it is negative, it will be affirmative, if it is affirmative, it will be negative.

Consider, for example, syllogism:

If the moon in its circulation around the earth never passed through the line connecting the centres of the earth, the moon and the sun, then solar eclipses could never have been observed on the earth.
But solar eclipses are sometimes observed on earth.
————————————————————
Next, in its circulation around the earth, the moon sometimes passes through a line connecting the centres of the earth, the moon and the sun.

In this syllogism, the negative basis of the conditional premise causes a negative consequence. But a categorical premise denies the consequence. Therefore, the conclusion is denied in the conclusion of the syllogism. And since this ground itself expresses negation, the negation of negation gives a statement in conclusion.

Another example:

If the observed luminary is a planet, then its spectrum will be the reflected spectrum of the sun.
But the spectrum of the observed luminary is not the reflected spectrum of the sun.
————————————————————
Next, the observed luminary is not a planet.

In this syllogism, the affirmative basis of the conditional premise determines the affirmative effect. But a categorical premise denies the consequence. Therefore, the conclusion is denied in the conclusion of the syllogism. And since this basis makes a statement, the negation of the statement gives a denial in the conclusion.

It can be seen from these examples that the categorical premise in the syllogism of the tollens mode does not have to be negative at all. The purpose of the categorical premise in this modus is to deny the consequence of the conditional premise. In cases where this consequence is a negative judgment, its denial, confirmed by a categorical premise, will be an affirmative judgment.

Errors Possible in Conditionally Categorical Syllogism

§ 8. Modus ponens and modus tollens are two unique modes of conditionally categorical syllogism by which the correct conclusion can be obtained. *The truth of the foundation always logically implies the truth of the effect as well.* And in the same way, *falsity and foundation always logically follow from the falsity of the investigation.*

On the contrary, the *falsity of the foundation does not in itself give the right to affirm the falsity of the investigation.*

So, consider the conditionally categorical syllogism:

If Ivanov is a student, then he is required to take exams.
Ivanov is not a student.
Ivanov is not required to take exams.

This syllogism will obviously be logically erroneous. In fact: if the judgment expressed by the conditional premise were true, then it would follow from it that the scope of the concept of “Ivanov” is included in the scope of the concept of “persons required to take exams”. But precisely because the scope of the concept of “Ivanov” is, as can be seen from the conditions of the premise, *only part of the* volume of the concept of “persons required to take exams”, we learn from a categorical premise that the scope of the concept of “Ivanov” is not included in the scope of the concept of “students”. We have no right to conclude that Ivanov is not required to take exams. Indeed, Ivanov may be required to take exams on membership of high school students, graduate students, etc.

§ 9. Another type of logical error, possible in conditionally categorical syllogism, arises in the case when they try to conclude *from the truth of the investigation to the truth of the foundation.*

For example: conditionally categorical syllogism:

If it rained at night, then the grass should be wet.
The grass is wet.
———————————————
Next, it rained at night.

Of course, it will be logically erroneous, since the conclusion in it does not necessarily follow from the premises. And indeed, the grass could be wet not because it rained at night, but because the dew fell at night

The inference from the truth of the investigation to the truth of the foundation is erroneous in view of the fact that the same effect can be caused not by one single, but by many reasons.

§ 10. In some cases it may seem that the correct conclusion from the truth of the investigation to the truth of the foundation is still possible.

Consider, for example, syllogism:

If Ivanov does not know chemistry, he cannot successfully conduct physiological studies.
Ivanov successfully conducts physiological studies.
———
Next, Ivanov knows chemistry.

In this example, the output is correct. It might seem that this syllogism proves the possibility of a correct conclusion from the *truth of the* investigation to the truth of the foundation. In fact, this syllogism is an example of a conclusion from the *falsity of the* investigation to the falsity of the foundation.

In fact, the quality of judgment is determined, as we know, by more than one grammatical form of judgment, that is, not by the presence or absence of a negative particle in it before the predicate of the sentence. The quality of judgment is determined by the ratio of the logical meaning of the utterance to the logical meaning of the whole argument. In relation to the judgment "cannot conduct physiological research successfully", the judgment "successfully conducts physiological research" is, of course, a negative judgment, despite the absence of a grammatical form of denial. Therefore, the second premise in our example ("Ivanov successfully conducts physiological research") expresses the idea not of truth, but of the falsity of the foundation. It follows that our syllogism is a conditionally categorical syllogism from the falsity of the investigation to the falsity of the foundation, and the conclusion in it is logically correct.

There are cases when the conclusions from the truth of the investigation to the truth of the foundation and from the falsity of the foundation to the falsity of the investigation, being logically erroneous, still lead to conclusions that are true in content. Thus, a person who concludes that it was raining at night, on the grounds that the grass was wet in the morning and that the grass is always wet after rain, makes a conclusion that, in fact, regardless of the way it is bred, it may turn out to be true. It will be true provided that it really rained at night. Similarly, a person who knows that all students are required to take exams, and concludes that Ivanov is not required to take exams, since Ivanov is not a student, makes a conclusion that can actually turn out to be true. It will be true provided that Ivanov, for example,

But although in both of these examples the conclusion turned out to be true in content, this coincidence of the conclusion with reality is completely random, and the very syllogisms by which these conclusions were drawn remain, of course, erroneous.

Indeed, for a syllogism to be correct, that is, infallible as *a syllogism, it is* not enough that the conclusion of the syllogism be true in content. It is necessary, moreover, that this conclusion, true in its content, *really follows from the premises of the syllogism*, that is, that, having recognized the premises as true, we could not help but recognize the conclusion as true.

But precisely this condition is not fulfilled in both of our examples. The fact that after the rain the grass is wet, and the fact that the grass was wet in the morning, should not be *necessary*, though in this case the grass is wet just because of the rain fallen during the night. From the fact that all students are required to pass examinations, and from the fact that Ivanov did not have a student should not be *necessary*, if Ivanov is not obliged to pass examinations. But where there is no *necessary* connection between the concepts justifying the conclusion, there is no syllogism either.

Not all true content judgments are conclusions of syllogism. The conclusion of a syllogism can only be such a true proposition, in which the relationship between the subject and the predicate necessarily follows from the relations established in the premises of the relations of each of these concepts to a certain third concept. Where there is no such need, there the syllogism can only be erroneous. This is precisely what he is in every conclusion from the truth of the investigation to the truth of the foundation or from the falsity of the foundation to the falsity of the effect.

But if this is so, then why, in certain cases, an erroneously obtained conclusion can still turn out to be true in its content?

This is possible because the same phenomenon can be caused not only by one, but by many reasons. The fact that the rain that fell at night could be the cause of the humidity of the grass is nothing impossible. This fact *could* happen. And if in this case the possibility coincided with reality, if it really rained at night, then the conclusion obtained as a result of a logical error will accidentally coincide with reality.

However, from this random coincidence, a logical error in no way ceases to be an error: what is only possible was thought in syllogism as not only possible, but also necessary.

Simple Dividing Syllogism

§ 11. Simple syllogisms, in addition to categorical and conditional, can also be *dividing* . A simple dividing syllogism is called a syllogism, in which one of the two premises is a separation judgment. Another premise may be either categorical, conditional, or also dividing.

Consider, for example, the conclusion:

Binary stars are either optical binaries or physical binaries.
Vega’s binary star is not a physical binary.
————————————
Vega’s binary star is an optical binary.

Inference is a simple syllogism. In it, the relationship between the concepts that make up the conclusion is established through the third concept—the concept of physical binary stars. At the same time, unlike a simple *categorical* syllogism, in which the premises are categorical, and unlike a simple *conditional* syllogism, in which at least one premise is conditional, in our example one of the premises is a *separate* judgment . In general, the structure of this syllogism is expressed by the formula:

And there is either B or C.
But A is not C.
————————————
Next, A is B.

Another example of a simple dividing syllogism:

Each luminary is presented to the observer either as a luminous point (i.e., a star), or as a luminous disk.
If the luminary is presented to the observer as a luminous disk, then it is a planet.
————————————
Next, each luminary is presented to the observer either as a star or as a planet.

And this syllogism is a simple dividing one. And in it one of the premises is a separation judgment. But unlike the previous example, in this case the second premise is no longer a categorical, but a conditional proposition. The conclusion in this syllogism is also a separative judgment.

In a general form, the structure of the syllogism in this case can be represented by the following formula:

And there is either B or M.
If A is M, then A is C.
———
Next, A is either B or C.

Finally, a third example of a simple dividing syllogism:

Each star is a star or planet.
Each planet is either internal or external.
———
Next, each star is either a star, or an inner planet, or an outer planet.

This syllogism is also a simple dividing one. Unlike both previous ones, in it both premises are separate judgments. The conclusion in it, as in the previous example, is also a separative judgment. In general form, the entire structure of this syllogism can be represented by the scheme:

And there is either B or M.
M is either C or D.
———
Next, A is either B, or C, or D.

§ 12. Let us now compare all three varieties of simple dividing syllogism represented by our three examples. It can be seen from the comparison that an essential feature of this form of syllogism is the *presence of a dividing premise* . This premise provides an exhaustive listing of all species, which includes a generic concept. From the premise of this it is clear that any object of this kind must necessarily belong to one of these types.

In turn, another premise either excludes all these species, except for one, or expresses some position about one of these species. The conclusion of a dividing syllogism depends on which species are excluded by another premise, and also depending on whether the position expressed on one of the species is conditional or dividing. This conclusion either confirms in categorical form the belonging of the subject to the species that remained after the exclusion of all the others (the first example), or enumerates in a dividing form the types of the generic concept obtained not only from the results of division given in the dividing premise, but also a new, conditional or dividing relationship expressed by the second premise (second and third examples).

A simple dividing syllogism, in which the second premise is conditional, is called *conditionally dividing*. A simple dividing syllogism, in which the second premise is dividing, is called *purely dividing*. In a purely dividing syllogism, both premises and conclusions are separate judgments. A simple dividing syllogism, in which the second premise is categorical, is called *dividing categorical*.

Dilemma

§ 13. A special case of conditionally dividing syllogism forms a *dilemma* . This is the name of conditionally dividing syllogism, in which the conditional premise provides for dependence on the basis of not one, but *two* consequences. These consequences, or members of a division, are called *alternatives*. Another premise in this case, as in all conditionally dividing syllogisms, is dividing.

For instance:

If the enemies in the environment surrender, they will be spared, and if they continue to resist, they will be destroyed.
But enemies who are surrounded can only either surrender or continue to resist.
___
Next, enemies that are surrounded will be either spared or destroyed.

In everyday speech, the term "dilemma" is used in a different sense. The dilemma in such a speech is the need to choose between two alternatives or exit routes, each of which will lead the selector to *undesirable* consequences for him; as a result of the dilemma, a painful indecision arises: "you go to the left — you lose the horse, you go to the right — you yourself will disappear."

Since a conditional proposition, like any proposition, can be true or false, the relation between each base and its effect, which is confirmed by the conditional premise of the dilemma, can be either true or false, i.e., not true. If the relation between the ground and its consequences, which is stated in the conditional premise of the dilemma, is false, then the conclusion of the dilemma will also be false.

For instance:

If fighter A, who committed a valiant act, committed it on his own initiative, then he is a hero, and if he committed it by order, then he is a person capable of heroic actions.
But fighter A could act only on his own initiative, or by order.
___
Next, fighter A is either a hero or a person capable of heroic actions.

In this example, the conclusion of the dilemma is false. Its falsity is due to the falsity of the conditional premise. Indeed, the relation expressed in this premise between the foundation and the second consequence is false: a fighter who has committed a valiant act by order may be a hero no less and even greater than he committed the same act on his own initiative.

§ 14. The dilemma can lead to a conclusion in another way. Moreover, the conditional premise, as in any dilemma, states that if a certain situation exists, then one of two consequences, or one of two alternatives, necessarily follows from it. On the contrary, the separation premise confirms that none of these effects actually exists. From this, it concludes that the position justifying both consequences, or both alternatives, is false.

For instance:

If the star is a variable, then it must be either an eclipsing variable or a physical variable.
The Vega star is neither an eclipsing variable nor a physical variable.
———
Next, the Vega star is not at all a variable star.

And with this form of the dilemma, a logical error is possible. It consists in the fact that in a conditional premise without sufficient reason it is stated that only two alternatives follow from the condition adopted in this premise, while in reality their number may turn out to be large. For instance:

If the liquid is fat, then it can be of either mineral or animal origin.
Provencal oil is not of mineral or animal origin.
———
Next, olive oil is not fat.

This dilemma is erroneous. In fact, from the base, expressed in the conditional premise, there are not two, but three consequences: in addition to mineral and animal fats, there are also vegetable fats. The omission of one of the division members in the conditional premise led to an erroneous conclusion. In fact, olive oil, of course, is fat, but vegetable.

Separately Categorical Syllogism

§ 15. In comparison with conditionally dividing and purely dividing syllogisms, categorical separation—syllogism has a special purpose. While through the first two forms of dividing syllogism we learn from its conclusion that the subject must belong to any one of the types indicated in the conclusion, through dividing—categorical syllogism we learn to *which* particular species this one should or cannot belong subject.

There are two varieties, or two modes, of separation—categorical syllogism.

Consider the conclusion:

The inscribed angle can be either sharp, or straight, or blunt.
The inscribed angle, based on the diameter, is neither sharp nor obtuse.
———————————————————
The inscribed angle, based on the diameter, is a straight line.

The conclusion is a categorical syllogism. In it, the separation premise indicates which of the mutually exclusive properties may belong to the item. The categorical premise denies everything — each separately — the properties indicated in the separation premise, except for one. The conclusion confirms that the subject belongs to the only property that has not been excluded in the categorical premise.

The separation—categorical syllogism of such a structure is called “modus tollendo ponens,” that is, a mode that “claims to deny.” In fact, the fact that the categorical premise denies leads, in conclusion, to the assertion of a property that has not been denied in the categorical premise and which was indicated in the separation premise in the full list of all possible properties of the object.

The formula of the tollendo ponens modus was already developed by us by considering the first example of dividing syllogism:

And there is either B or C.
But A is not C.
———————
Next, A is B.

§ 16. Another modus of separation—categorical syllogism is opposite to the previous one.

Here is an example of it:

The orbits of comets are either ellipses, or parabolas, or hyperbolas.
The orbit of Halley’s comet is an ellipse.
———————
The orbit of Halley’s comet is neither a parabola nor a hyperbole.

In this syllogism, the separation premise indicates which of the mutually exclusive properties may belong to the subject. A categorical premise establishes which of these properties really belongs to the subject. The conclusion is that none of the other properties can belong to it.

The separation—categorical syllogism of such a structure is called modus ponendo tollens, that is, a mode that “claims to deny." Indeed, the fact that the categorical premise of this modus claims to be truly belonging to the subject leads, in conclusion, to the denial of all other properties belonging to the same genus, but excluding the assertion.

Errors Possible in Separately Categorical Syllogism

§ 17. The modus tollendo ponens and the modus ponendo tollens are the only two modes of separation—categorical syllogism by which the correct conclusion can be drawn. With inferences on the modes of separation—categorical syllogism, *two* logical errors are possible . The first of them is caused by the ambiguity of the grammatical form and the ambiguity of the separative meaning of the judgments caused by it.

Consider, for example, syllogism:

Success in the piano is due to either zeal or giftedness.
Nikolaev's successes in the piano game are due to giftedness.
———
Nikolaev's successes in the piano game are not conditioned by zeal.

In this example, the conclusion was modeled on ponendo tollens. However, the conclusion here turned out to be logically erroneous. The cause of the error is the ambiguity of the union "or", which can have both a separating and non—separating meaning. This means that the predicates enumerated in the sentence and separated from each other by the "or" union can either exclude each other as members of the division, or they may turn out to be compatible with each other. In the first case, the union "or" will have a separating meaning, in the second it will not have a separating meaning.

In our example, the predicates listed in the premise do not necessarily exclude each other as members of a division, but they may also be compatible with each other. Success in a piano game can be explained not only by zeal and not only by giftedness, *individually* taken, but also *by the combined* action of both of these qualities. Therefore, from the fact that Nikolaev's successes are due to giftedness, it's impossible to deduce that Nikolaev's zeal did not influence these successes: both qualities could work together.

The not strictly dividing sense of the union "or" prevents the logical correctness of the conclusion only with inference by the ponendo tollens mode, since only in this mode the conclusion speaks of incompatibility of properties.

On the contrary, when inference by modus tollendo ponens, the strictly dividing meaning of the word "or" does not prevent the conclusion from being correct, since the conclusion does not mean incompatibility of properties, as happens in the ponendo tollens modus, but claims that the object has the only remaining property.

Example:

Nikolaev's success in the piano game is due to either giftedness or zeal.
Nikolaev's success in the piano game is not due to giftedness.

Nikolaev's successes in the piano game are due to zeal.

In this example, the union "or" in the separation premise may, as in the previous example, have not strictly separation meaning. Nevertheless, the conclusion in this example is logically correct. This is explained by the fact that in this case the inference is modeled tollendo ponens. The result of the inference here is the assertion behind the subject of that unique property, which, among all the properties of the genus, has not been excluded by a categorical premise. Since this remaining property, *in any case*, must belong to the subject, the ability of this property to combine with another property (namely, this does not mean the strictly separating sense of the word “or”) does not make a mistake.

§ 18. The second mistake, possible in separation—categorical syllogisms, arises because the division of the generic concept into species, which underlies the properties listed in the separation premise, may turn out to be *incomplete* . In this case, we do not have the right to conclude that the property indicated among the other properties of the genus by the separation premise and remaining the only one with the exception of the properties rejected by the categorical premise must necessarily belong to the subject.

Consider, for example, syllogism:

Fats come from either mineral or animal origin.
Rose oil is not of animal origin.
———
Rose oil is of mineral origin.

This syllogism is, of course, erroneous. The mistake lies in the fact that the separation premise does not provide a complete, non—exhaustive list of properties of a certain kind (in this case, in relation to origin) that may belong to the subject. In addition to fats of mineral and animal origin, vegetable fats are also possible. Therefore, having ascertained by means of a categorical premise that rose oil is not of animal origin, we still have no right to claim that it is of mineral origin. Indeed, the truth is that rose oil is of vegetable origin.

Incomplete division in the separation premise leads to a logical error in the conclusion only upon inference by the tollendo ponens modus. With a gap in the members of the division, it can always happen that the only property remaining by exception is not at all that property that should belong to the object. A property that may be missing a property in a separation premise may be due to incomplete division.

On the contrary, for the correct conclusion by the ponendo tollens mode, the completeness of the division of the defining properties that may belong to the subject and which are listed in the separation premise does not matter. For the correct conclusion by the ponendo tollens modus, it is not the completeness of division that is important, but the incompatibility of its members. Indeed, asserting, in a categorical premise, that some property belongs to an object, the syllogism modus ponendo tollens denies a number of other properties in the conclusion because of their incompatibility with the property being claimed. Since in this modus the categorical premise establishes which property of all the properties of a certain kind belongs to an object, it is quite obvious that the negation of a number of properties incompatible with it remains true even if not all properties are listed.

For instance:

Bodies orbiting the sun are either planets, or asteroids, or comets.
Eros is an asteroid.
―――――――――――――――――――――――――――――――――
Eros is neither a planet nor a comet.

In this syllogism, the conclusion is logically correct. True, the logical division, carried out in the separation premise, is not complete, since it missed some members: planetary satellites, meteorites of the solar system, etc. However, all these omissions do not impede the correct conclusion. Since Eros turned out to be an asteroid on the basis of a categorical premise, and since the concepts of the other members of the division indicated in the separation premise are incompatible with the concept of an asteroid, the conclusion excluding the asteroid Eros from the number of planets and comets remains in any case true, although in it not all types of bodies of the solar system are named, to which Eros cannot belong.

Abbreviated syllogisms

§ 19. In mathematical reasoning and proofs, they usually strive to ensure that no link in a series of logically interconnected thoughts is omitted. Therefore, syllogisms usually enter the proofs of the mathematical sciences in their full form: both premises and conclusions.

In other sciences, in art and especially in everyday thinking, it is far from always necessary to reproduce in thought and express in speech all links of evidence, all parts of the conclusion. Therefore, along with complete syllogisms, that is, those in which *all the* premises and conclusions are clearly and fully expressed , *abbreviated* syllogisms are often found . This is the name of syllogisms in which either the premise or the conclusion is missing.

These omissions are easily explained. In the thinking of an educated person, there is not only a lot of accumulated individual truths, but also a lot of accumulated knowledge about the logical connections between individual truths. Therefore, when conducting a well—known reasoning or proof in cases where there is reason to think that the reader or listener knows these truths and the logical connections between them just as they are known to the speaker himself, some premises, and sometimes even the conclusion itself, can be omitted without damage to clarity and persuasiveness of thought.

§ 20. The abbreviated syllogisms are called *entimem*—from the Greek word ἐν θυμῷ, meaning “in the mind.” The name of this shows that one of the premises of the syllogism is not expressed, but is implied by the speaker.

An example of an entimem: “A coward is cowardly, since cowardice is a property of all egoists.”

Here, as you can easily see, it is omitted, but the lesser premise is implied: “a coward is an egoist.” In full form, this syllogism would have the following form:

All egoists are naive.
The coward is an egoist.
———————————
The coward is cowardly.

Usually not a smaller, but a *larger* package is skipped . This is explained by the fact that the larger premise in most cases is a *general* judgment and therefore often (although, of course, far from always) expresses the truth or thought, widely known, easily implied.

Such is the entimema:

This star is a planet, as it is rapidly changing its position among other stars.

A *big* premise is missing here : “All the stars that are rapidly changing their position among other stars are the essence of the planet.” This situation is so well known that it is possible—without the risk of being misunderstood or unconvincing—to immediately switch from a smaller premise to a conclusion.

Finally, sometimes a *conclusion is* skipped in syllogism. This happens in cases where the conclusion is quite obvious and when, having expressed both premises, they allow the listener or interlocutor to draw a natural conclusion.

For instance:

All patriots must fulfill their military and civil duty.
You are a patriot ...

Epiherem

§ 21. Sometimes each of the two premises of the syllogism is an entimem, that is, an abbreviated syllogism. Such a syllogism with entimematical premises is called an "epicheirem" (from the Greek word ἐπιχέιρημα — "inference").

For instance:

Lies cause distrust, since they are statements that are not true.
Flattery is a lie, since it is a deliberate perversion of truth.
———
Flattery causes distrust.

Here we have a syllogism with two premises and a conclusion. But each of the premises of this syllogism, as it is not difficult to verify, is an entimem. The first premise in full form represents a typical syllogism:

Any statement that does not correspond to the truth causes distrust.
Lying is a statement that is not true.
———
Lies cause distrust.

But the second premise in the above example is also an entity. In full form, it represents a syllogism:

Every deliberate perversion of truth is a lie.
Flattery is a deliberate perversion of truth.
———

Flattery is a lie.

Complex Syllogisms

§ 22. In scientific thinking, syllogisms are rarely used alone. Usually scientific reasoning, where syllogisms are included in its structure, represents a more or less long chain of consecutive conclusions connected among themselves by logical necessity.

A sequence of syllogisms connected into a logically coherent argument or proof is called *polysyllogism*, or complex syllogism.

In complex syllogism, the conclusion of the preceding syllogism is the premise of the following. A syllogism, which provides the basis for sending a subsequent syllogism, is called *prosillogism*. A syllogism, in which the premise is the conclusion of the previous syllogism, is called an *episillogism*.

For instance:

No one capable of self—sacrifice is selfish.
All generous people are capable of self—sacrifice.
Not a single magnanimous is selfish.
All cowards are selfish.
———————————————
No coward is magnanimous.

Here we have *two* syllogisms: the first figure of a simple categorical syllogism (in Celarent mode) and the second figure of a simple categorical syllogism (in Cesare mode). The first of these is the *prosillogism*:

No one capable of self—sacrifice is selfish.
All generous people are capable of self—sacrifice.

——————————
Not a single magnanimous is selfish.

The second of these is *episyllogism*:

Not a single magnanimous is selfish.
All cowards are selfish.
——————————
No coward is magnanimous.

In general, a complex syllogism is represented by the formula:

All MR
All R — M
——————
All R — P
All S — R
——————
All S — P

In a separate form, the formulas of prosillogism and episillogism will be the following:

All M — P prosillogizma All R — M ——————— All R — P . Formula All R —

P syllogism All S — R ——————— All S – P

Sorit

§ 23. A syllogism is possible, representing a combination of a complex syllogism with a shortened syllogism. Like all other complex syllogisms, several syllogisms enter into this syllogism as its parts. At the same time, as in the shortened syllogisms in it, some premises are omitted.

For instance:

All types of peas are moths.
All moths are dicotyledonous.
All dicotyledons are flowering.
────────────
All types of peas are flowering.

This conclusion is a complex syllogism consisting of two syllogisms in which some premises are omitted. Having restored the premises missed in this complex syllogism and enclosing them in parentheses, we obtain the following two syllogisms:

All moths are dicotyledonous.
All types of peas are moths.
────────────
(All types of peas are dicotyledonous.)

All dicotyledons are flowering.
(All types of peas are dicotyledonous.)
────────────
All types of peas are flowering.

A complex syllogism of such a structure is called *sorite* (from the Greek word σωρὸσ—heap). Sorit can consist of several or even many syllogisms. In general, the structure of sorite is expressed by the formula:

All A — B
All B — C
All C — D
All D — E
———
All A — E

The same formula for the restoration of omitted parcels takes the form:

All B — C
All A — B
———
All A — C

All C — D
All A — C
———
All A — D

All D — E
All A — D
———
All A — E

In Sorit, each concept is included in the premise twice: the first time as the predicate of the premise, the second time as the subject of the next premise. The exception is the first and last concepts, i.e., the subject and the predicate of the conclusion.

In the chain of syllogisms from which the Sorit is composed, each syllogism plays the role of a premise for following it, and thus there is a prosillogism regarding this latter. At the same time, every syllogism, starting from the second, is an episillogism relative to the previous one.

§ 24. Sorit is applied in cases where it is necessary to sequentially review a long chain of subordination units.

Therefore, the premises and conclusions that are part of the syllogisms from which the litter is composed are omitted in such a way that in the remaining premises the thought sequentially moves either from the concept of the subordinate to the subordinate, i.e., to the concept that encloses in its volume the entire volume of the subordinate, or conversely, from the concept of subordinate to subordinate. For this purpose, in addition to missing some premises and conclusions, the premises are rearranged in the syllogism with which the mess begins.

If they want to observe the order of submission, passing from the concepts of subordinates to the concepts of subordinates, then in the syllogisms of which the Sorit is composed, smaller premises are omitted. Sorit, in which the smaller premises of the syllogisms included in this sorit are omitted, is called *Aristotelian*. The example of sorite considered by us above is an example of exactly Aristotelian sorite.

If they want to observe the subordination order, moving from the concepts of subordinates to the concepts of subordinates, then larger premises are omitted in the syllogisms from which the Sorit is composed. Sorit, in which the larger premises of the syllogisms included in this Sorit are omitted, is called, by the name of the logic that described this Sorit, as *Gokleevsky*.

An example of a Gokllenian sorite:

All optical instruments are physical instruments.
All astronomical tubes are optical instruments.
All refractors are astronomical tubes.
All apochromats are refractors.
___
All apochromats are physical devices.

It can be seen from the example that in the case of the Goklenite sorit, the transition along the levels of submission is the *reverse of the* transition that takes place in the Aristotelian sorit: while in the Aristotelian sorit, the thought goes all the time from the subordinate concept to the concept of the subordinate, in the Goklelevsky sorit, the thought goes from the subordinate concept to concept subordinate.

Having restored the premises omitted in our example and enclosing them in brackets, we obtain the following series of syllogisms:

.	All optical instruments are physical instruments.
	All astronomical tubes are optical instruments.
	___
	All astronomical tubes are physical instruments.

.	(All astronomical tubes are physical instruments.)
	All refractors are astronomical tubes.
	___

	All refractors are physical instruments.

.	(All refractors are physical devices).
	All apochromats are refractors.
	All apochromats are physical devices.

In the conclusion of the Goklenievsky sorite, as well as in the conclusion of Aristotelian, the concept that has the smallest volume obeys the concept that has the largest volume. Usually in sorites all parcels are affirmative. There can be only one private package in Sorit, and moreover, it must be the first. There can also be only one negative premise and, moreover, it should be the last.

Tasks

I. In the following conditional categorisms, determine their type, modus, examine whether the syllogism will be correct, and if it is erroneous, then indicate what the logical error is:

1) "If a thick spruce forest grows in the forest, then there will be less insects in this forest; if there are fewer insects in the forest, the number of songbirds living in this forest will decrease; therefore, if a thick spruce forest grows in the forest, then there will be less songbirds in this forest"; 2) "If the patient recovers, then his temperature drops; the patient's temperature has not decreased; therefore, the patient does not recover"; 3) "So that the shadow from the earth, approaching the surface of the moon during lunar eclipses, is round, it is necessary that the earth has the shape of a ball; a shadow from the earth, approaching the surface of the moon during lunar eclipses, is round; therefore, the earth has the shape of a ball" 4) "If there are signs that a person suffering from malaria is approaching a seizure, then he needs to take quinine; there are signs that a malaria sufferer is approaching; therefore, he needs to take

quinine"; 5) "If the enemy does not surrender, an order will be given to destroy him; the enemy does not give up; consequently, an order for its destruction will be given"; 6) "If the earth had not once been covered by the sea, then strata consisting of shells of marine animals could not have been found in it; but strata consisting of shells of marine animals are found everywhere in the earth; therefore, the land was once covered by the sea"; 7) "If the apples on the apple tree have ripened, then they must crumble; apples crumbled on the apple tree; therefore, they are ripe. " but strata consisting of shells of marine animals are found everywhere in the earth; therefore, the land was once covered by the sea" ; 7) "If the apples on the apple tree have ripened, then they must crumble; apples crumbled on the apple tree; therefore, they are ripe. "but strata consisting of shells of marine animals are found everywhere in the earth; therefore, the land was once covered by the sea"; 7) "If the apples on the apple tree have ripened, then they must crumble; apples crumbled on the apple tree; therefore, they are ripe."

II. In the following dividing syllogisms, determine their type, mode, examine whether the syllogism will be correct, and if it is erroneous, then indicate what the logical error is:

1) "Each telescope is either a refractor or a reflector; each reflector is either metallic or mirror; therefore, each telescope is either a refractor, or a metal reflector, or a mirror reflector"; 2) "Each plant belongs to either higher or lower; if the plant belongs to the lower, it absorbs substances with its surface; therefore, each plant is either a higher or a suction substance with its surface"; 3) "Bacteria are either spherical (cocci), or cylindrical (sticks), or crimped (vibrios); tuberculosis bacteria do not belong to cocci or vibrios; therefore, tuberculosis bacteria belong to the bacillus"; 4) "If the ferns are heterogeneous, then the ferns are bisexual, and if the ferns are equally spore, the ferns are dioecious; but ferns are only either equipotent or heterogeneous; therefore, ferns can only be bisexual or dioecious"; 5) "Vertebrates are either mammals, or birds, or fish, or amphibians; a lizard, being a vertebrate, is neither a mammal, nor a bird, nor a fish; therefore, the lizard is an amphibian "; 6) "Mammals are either marsupials or prenatal; kangaroo — marsupial mammal; therefore, the kangaroo does not belong to the fetus"; 7) "Victory in a race competition is determined either by natural data or by training; Sergeyev's victory in the race is due to training; consequently, Sergeyev's victory in the race competition is not conditioned by natural data "; 8) "Victory in a race competition is determined either by natural data or by training; Sergeyev's victory in the race is not due to natural data; therefore, Sergeyev's victory in the race is due to training."

III. Imagine the following abbreviated syllogisms in full, restoring the parts missing in them:

1) "Since all frogs in the larval state breathe with gills, the frogs cannot belong to

reptiles"; 2) "Like all snipers, Sokolov had a firm hand and remarkably sharp vision"; 3) "Without being a mathematician, you will not solve this problem"; 4) "Since corn is cereal, it belongs to monocotyledonous plants"; 5) "Like all egoists, a coward cannot be magnanimous."

IV. Present the following sorites in full form, restoring the premises and conclusions omitted from them:

1) "All chameleons are lizards, all lizards are scaly, all scaly ones are reptiles, all reptiles are vertebrates, therefore, all chameleons are vertebrates"; 2) "All conifers are seed, all pine are coniferous, all cedars are pine; therefore, all cedars are seed "

CHAPTER XI. NON—SYLLOGISTIC CONCLUSIONS. INDUCTION AND ITS TYPES

Non—Syllogistic Conclusions

§ 1. All the conclusions we examined in chapters IX and X were syllogistic [1]. In all these conclusions, the goal of the conclusion is to establish in advance an invisible relationship between two concepts—the subject and the predicate of the conclusion. Since the relation is not visible in advance, it is deduced from the relation established by the premises of each of these concepts to some third concept,

In syllogisms, all these relationships are *affiliation*. Both premises and the conclusion establish the relation of belonging of a known sign or group of signs to a known concept. But the same belonging of a sign to a concept means the belonging of an object or species possessing the property that is thought in the sign to a well—known class of objects. In other words, the relations between concepts are thought of in syllogisms not only as relations *in content* , but also — and even mainly — as relations between concepts *in their volume*, as an object belonging to the class of objects.

Although all relationships between concepts in terms of volume that are conceivable in syllogisms are always determined by relations between the same concepts in terms of their content, the subject of the question in these conclusions is usually the relationship between volumes. Suppose I think of syllogism: “All the platypuses are mammals, all the platypuses are egg—laying, therefore, some egg—laying are mammals.” The relationship between the concepts of “oviparous” and “mammals”, which is established in the conclusion of this syllogism, is based, of course, on the fact that the platypuses belong, firstly, all the essential signs of oviparous and, secondly, all the essential signs of mammals. From this belonging of essential signs of oviparous and mammals to all platypuses in the conclusion of the syllogism it is deduced.

But although the relation of belonging that is conceivable in the conclusion of this syllogism is based on the relation between concepts *in* terms *of their content, the* main subject of interest or request for thought in this case is not the relation between the contents of concepts in itself, but *the relation between the volumes of concepts* , which is based on the relation between them in content.

And indeed, in concluding the syllogism, it is not directly stated that the essential signs of the ovipositors belong to a certain part of the mammals, but that some part of the volume of mammals is included as part of the volume of the ovipositors.

On this feature of syllogisms is to highlight the relationship between concepts *by their volume*—the usual explanation of the general and special rules of syllogisms is based, an explanation of the differences between the figures of a simple categorical syllogism, between modes, etc.

§ 2. But, as has been shown above, all possible types of inferences are not exhausted by syllogisms. In addition to syllogistic conclusions, there are also *non*— syllogistic conclusions . In these conclusions, the purpose of the conclusion is to establish between two concepts of relations according to whether a sign belongs to a concept (or property to an object) and according to whether a species belongs to a genus (or an object to a class of objects). In these conclusions, the purpose of the conclusion is to establish relations of a different kind between the objects imaginable in the premises, namely the relations between the objects *in magnitude* ("object A is larger than object B"), relations *in space* ("subject A lies higher than subject B"), the relationship between events *in time* ("event A occurred earlier than event B"), the relationship of *cause and action* ("phenomenon A is the cause of phenomenon B, and phenomenon B is the effect of the phenomenon A "), relationship of kinship (" Ivan is the brother of Peter "), etc.

Some types of non—syllogistic conclusions seem at first glance no different from syllogisms. In these conclusions, as well as in syllogisms, the relation between the subject and the predicate of conclusion is deduced from the relation established by the premises of each of these concepts to a certain third concept.

For instance:

Cervantes was a contemporary of Bacon.
Bacon was a contemporary of Shakespeare.
———————————————
Next, Cervantes was a contemporary of Shakespeare.

It might seem that this conclusion is the usual syllogism of the first figure. However, in reality, this conclusion can become a syllogism of the first figure only after we subject it to some transformation. For this, it is necessary to add one more premise to it, namely: "every two events or two persons, contemporary to some third event or person, are contemporary to each other". This premise will be the greater premise of a new—syllogistic—conclusion. Its smaller premise will be the premise obtained from combining in one of both premises of our first conclusion: "Cervantes and Shakespeare were contemporaries of Bacon." In full, the new conclusion will be as follows:

Any two events or two persons, contemporary to some third event or person, are contemporary among themselves.
Cervantes and Shakespeare are two faces, contemporary to a third party — Bacon.
————————————————————
Next, Cervantes and Shakespeare are contemporaries.

The conclusion is indeed the syllogism of the first figure. However, this inference as a form of inference is obviously not identical with the first inference from which it was obtained by transformation. His premises are other than those from which the conclusion is drawn in the first example. But in order to get this conclusion, there was no need to introduce a new premise at all. The conclusion, according to which Cervantes and Shakespeare are contemporaries, in the first example is just as correct and just as necessary logically as in the second. Therefore, this conclusion is *special* a type of inference different from the syllogism of the first figure. From the fact that this conclusion can be turned into a regular syllogism of the first figure by adding a new general premise, it does not at all follow that this conclusion is not a special and independent form of inference.

The conclusions of such a structure not only exist as a special type of inference. They are extremely common in geology, in the historical sciences about the development of life on earth, in the history of society, etc. Researchers in these sciences constantly make a number of conclusions about the simultaneity or modernity of known events, processes, and individuals. These conclusions are usually not brought to syllogistic form. The conclusions are drawn in them—without adding a new premise that turns them into syllogism—based on the relations between the concepts that are established in their premises.

These relations are no longer relations of belonging, as in syllogisms, but relations of events or persons in time, in this example—relations of the present.

In the conclusions of this type, the purpose of the conclusion is not to, based on the relationship of concepts in content, find out the relationship between them in volume. The question to which the conclusion answers is precisely the question of the relationship between concepts in content that characterizes the relationship between them as a relationship in time.

In addition to the conclusions about the present, the conclusions about the relationship in time can reveal the relationship of precedence or sequence.

For instance:

Lermontov died before Belinsky.
Belinsky died before Gogol.
———
Next, Lermontov died before Gogol.

Or more:

The discovery of the Strait of Magellan occurred after the discovery of the route to India around Africa.
The opening of the path to India around Africa occurred after the discovery of America by Columbus.
———
Next, the discovery of the Strait of Magellan occurred after the discovery of America by Columbus.

§ 3. Inferences about relationships in time — only one of the many types of non—syllogistic inferences. Another extremely common form of these conclusions is the conclusions about the relation of equality of two objects, each equal separately to a third subject.

For instance:

The diameter of ball A is equal to the diameter of ball C.
The diameter of the ball C is equal to the diameter of the ball B.
____
Next, the diameter of ball A is equal to the diameter of ball B.

And here it might seem as if we are facing the syllogism of the first figure. But here, the transformation of premises is also necessary in order to turn this conclusion into a syllogism. For this it is necessary, firstly, to add a new—bigger—premise. Such a premise here will be the judgment "two quantities equal each separately from the same third are equal to each other." Secondly, it is necessary to combine both premises of our inference into one smaller one: "the diameters of the ball A and the ball B are each separately equal to the diameter of the same third ball C". We get a new conclusion:

Two quantities, each equal to the same third, are equal to each other.
The diameters of the ball A and ball B are equal each separately to the diameter of the same third ball C.
____
Next, the diameters of ball A and ball B are equal.

The conclusion is the syllogism of the first figure. However, in order to obtain a conclusion on the equality of the diameters of the balls A and B, there was no need to complicate the conclusion so. Without adding a new package from the packages alone:

The diameter of ball A is equal to the diameter of ball C.
The diameter of the ball C is equal to the diameter of the ball B

It is necessary that the diameters of balls A and B are equal to each other. In other words, not being a syllogistic, this conclusion from the logical point of view is absolutely correct, and the connection between his premises and the conclusion is a necessary connection.

As in the conclusions about the relation in time, the relation established in the conclusion between its subject and the predicate is not the relation of belonging of an object to a class. This is not a relation between volumes of concepts. This is the relationship between them in content. The difference between this conclusion and the non—syllogistic conclusion about the relations of time consists only in the fact that the connection of concepts established by the conclusion on the content is not their connection on the side of the content that expresses the relationship of time, but on the side of the content that expresses the relationship of magnitude.

The conclusions of this kind are constantly applied in mathematics and in the mathematical sciences. A huge number of mathematical conclusions goes according to the formula:

A = B
B = C
———
Next, A = C.

These conclusions are not syllogisms. Although they can be reduced, as has just been shown, to syllogisms of the first figure, they remain a *special and completely independent* form of inference. Their truth and the logical necessity of the conclusions drawn in them do not depend on whether they are reduced or not reduced to the forms of syllogisms. The relation of concepts to which their conclusion leads is an attitude or connection in content.

In addition to the conclusions about the relation of equality, inferences of this type can also reveal relations of inequality.

For instance:

Planet Jupiter is larger than the planet Saturn.
Planet Saturn is larger than the planet Uranus.
———
Next, the planet Jupiter is larger than the planet Uranus.

Or:

The atomic weight of silver is less than the atomic weight of gold.
The atomic weight of gold is less than the atomic weight of uranium.
———
Next, the atomic weight of silver is less than the atomic weight of uranium.

§ 4. So, along with syllogistic there are non—syllogistic conclusions. Since these conclusions are extremely common in thinking—in everyday and in scientific—then logic explores them in the same way as it explores syllogisms. Logic, firstly, establishes the *types of* non—syllogistic inferences, secondly, establishes the *rules* by which logical conclusions are drawn from these inferences, thirdly, explores the relation of non—syllogistic conclusions to syllogisms.

A full examination of the theory of non—syllogistic conclusions, their types and their relationship to syllogisms cannot be the subject of a real, initial, outline of logic. Of all types of non—syllogistic inferences, this essay on logic considers only groups of so—called *inductive* inferences.

Non—Syllogistic Inductive Inferences

§ 5. Among non—syllogistic conclusions, an extremely important place belongs to the so—called inductive conclusions, or inductive conclusions. These findings, all taken together, are also called *induction*.

Inductive inferences are the conclusions of general provisions from single or private premises. Consider, for example, the premises:

On Monday last week, the weather was cloudy.
On Tuesday too.
Wednesday too.
Thursday too.
Friday too.
Saturday too.
Sunday too.

Here all the premises are single judgments.

Based on these premises and knowing that, in addition to the days listed in the premises, the week has no other days, we obviously have the right to conclude:

The weather was cloudy all the days last week.

Inference is an example of *inductive* inference. Each premise in it is singular, but the conclusion is a general judgment.

Another example of inductive inference.

Noting that *some* cats—domestic cat, lion, tiger and jaguar—have retractable claws, and when not meeting a single case when acquainting with the cat family, a cat is without retractable claws, we conclude: “all cats have retractable claws.”

And this conclusion is inductive. And in it, the premises reliably establish only a *particular* judgment: “some cats have retractable claws.” However, the conclusion states that *not only some, but all cats* have retractable claws.

Inductive conclusions *differ in a* number of features from syllogisms. But along with these differences between inductive inferences and syllogisms, there are also *common* features. As we will see, some types of inductive conclusions in their logical structure are very similar to some types of syllogism. Therefore, logic studies in inductive conclusions both how they differ from syllogisms and how they turn out to be similar to syllogisms.

§ 6. The first and most sharply conspicuous feature that distinguishes inductive reasoning from syllogisms consists in the fact that through the induction of *private* parcels can be obtained *overall* conclusions.

In syllogistic conclusions, this is impossible. Not a single syllogism — whatever its figure and whatever its mode — no general conclusion can ever be drawn from private premises. If both premises are private, then a syllogistic conclusion is completely impossible. If one of the premises of the syllogism is private, and the other is general, then the correct syllogistic conclusion can *only* be *private*. But even if both premises of the syllogism are general, the conclusion, or conclusion, will not always be a general judgment. So, in simple categorical syllogisms of the third figure in the Darapti and Felapton modes (as well as in syllogisms of the fourth figure in the Bramantip and Fesapo modes), despite the fact that both premises are general, the conclusion is only a particular one. Of all nineteen correct modes of simple categorical syllogism, only five modes yield a general conclusion in conclusion with two general premises.

These mods are: Barbara, Celarent of the *first* figure, Cesare, Camestres—of the *second* and Camenes—of the *fourth* figure.

On the contrary, in inductive inferences, as can be seen from the above examples, the particular nature of premises does not only prevent a general conclusion, but inductive inferences are precisely the conclusions in which private premises give the basis for general conclusions.

§ 7. In close connection with this feature there is another feature of inductive conclusions, distinguishing these conclusions from syllogisms. In syllogisms, reliable premises always lead to equally reliable conclusions. Syllogistic conclusions are invalid only if the premises of the syllogism are unreliable.

For example, I have the premises:

All patients with influenza are spreading influenza infections.
Mikhailov, *apparently* sick with the flu.

From these premises, an unreliable, but only likely, conclusion can be obtained:

Mikhailov, *apparently* , is a distributor of influenza infection.

However, the probable nature of the conclusion here does not depend on the fact that this conclusion is a syllogism, but only on the fact that the smaller premise of this syllogism in this case turned out to be modally not apodictic, but only a problematic proposition.

Therefore, as soon as instead of this premise we take another—reliable,—the conclusion of the syllogism immediately from the problematic will become completely reliable:

All patients with influenza are spreading influenza infections.
Mikhailov has the flu.
———————————————
Next, Mikhailov is a distributor of influenza infection.

And this is the case in all simple categorical syllogisms.

In each simple categorical syllogism, provided that only its premises are true and if the conclusion is consistent with the real relationship between the concepts of premises, the conclusion will always be reliable truth. If it is true that “all the platypuses are oviparous” and that “all the platypuses are mammals”, then the conclusion “some mammals are oviparous” will be quite reliable. This excludes any possibility of concluding otherwise, i.e., concluding, for example, that although all the platypuses are egg—laying and although they are all mammals, nonetheless, mammals are never egg—laying. Whoever admitted such an opportunity, that is, would deny the validity of the conclusion, would immediately be in conflict with the premises he himself recognized. But also in conditional syllogisms, as has already been shown, the conditional is by no means the logical connection between the premises and the conclusion, but only the assumption on which the consequence of the conditional premises depends. Even in purely conditional syllogisms, where both premises and conclusions are conditional judgments, the logical connection between premises and conclusions is absolutely necessary and reliable. Suppose that from the premises— “If A is B, then C is D” and “If C is D, then E is F”—we conclude: “If A is B, then

E is F”. In this syllogism, in spite of the fact that both premises and conclusion are conditional judgments, the logical connection between premises and conclusion is absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, on which the consequence of the conditional premise depends. Even in purely conditional syllogisms, where both premises and conclusions are conditional judgments, the logical connection between premises and conclusions is absolutely necessary and reliable. Suppose that from the premises — “If A is B, then C is D” and “If C is D, then E is F” — we conclude: “If A is B, then E is F”. In this syllogism, in spite of the fact that both premises and conclusion are conditional judgments, the logical connection between premises and conclusion is absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, on which the consequence of the conditional premise depends. Even in purely conditional syllogisms, where both premises and conclusions are conditional judgments, the logical connection between premises and conclusions is absolutely necessary and reliable. Suppose that from the premises — “If A is B, then C is D” and “If C is D, then E is F” — we conclude: “If A is B, then E is F”. In this syllogism, in spite of the fact that both premises and conclusion are conditional judgments, the logical connection between premises and conclusion is absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, the logical connection between the premises and the conclusion is absolutely necessary and reliable. Suppose that from the premises—“If A is B, then C is D” and “If C is D, then E is F” —we conclude: “If A is B, then E is F”. In this syllogism, in spite of the fact that both premises and conclusion are conditional judgments, the logical connection between premises and conclusion is absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, the logical connection between the premises and the conclusion is absolutely necessary and reliable. Suppose that from the premises—“If A is B, then C is D” and “If C is D, then E is F”—we conclude: “If A is B, then E is F”. In this syllogism, in spite of the fact that both premises and conclusion are conditional judgments, the logical connection between premises and conclusion is absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, the logical connection between premises and conclusion is a connection absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, the logical connection between premises and conclusion is a connection absolutely necessary. In this sense, the conclusion here is quite reliable. The conclusion does not claim that A is B. It is possible that A is not B. But the conclusion does not mean this. The conclusion says that provided that A is B, *E need to be the F* . In other words, if the conditions specified in the packages are made, *then E can not be the F* . Therefore, any attempt, agreeing with the premises of this syllogism, to disagree with its conclusion is impossible.

§ 8. On the contrary, in inductive conclusions even from reliable premises *far from reliable conclusions can always be obtained*.

True, in our first example of inductive inference, the general conclusion ("all days in the past week the weather was cloudy") is a completely reliable conclusion. If on each of the days of the last week individually I reliably know that the weather was cloudy on that day, and if it is reliably known that in addition to the seven days listed in the packages, the week does not contain any other days, then the general conclusion we make from these packages will be quite reliable.

However, inductive inference of this type, which gives a reliable conclusion, is only one and, moreover, as we shall see, the least valuable type of inductive inference. All other types of inductive inferences—and the most valuable types of induction for knowledge—belong to them. They give conclusions of a completely different nature.

Already in our second example of inductive inference, the conclusion was not strictly reliable. If we know that some of the cats listed in the premises have retractable claws, and if we know, besides, so far we have never seen cats anywhere without retractable claws, then, knowing this, we certainly have *some* reason assume that all other cats will also have retractable claws.

However, the conclusion is, being probable, has no credibility. This means that, although the premises make our conclusion possible and probable, they do not exclude the possibility that among cats that are still unknown to us, there may be ones that do not have retractable claws.

The conclusion, the premises of which, although they make a conclusion probable, however, they also allow the possibility of a conclusion that contradicts which one is deduced, is called *probable*. Inductive conclusions, generally speaking, are not reliable, but probable conclusions.

§ 9. Thus, a preliminary acquaintance with inductive inferences has revealed in them *two* features that distinguish these inferences from syllogisms. The first of these is that inductive conclusions give *general* conclusions from *private* premises. This is the *advantage of* inductive conclusions compared with syllogisms, in which the general conclusion is *never* cannot be obtained from private parcels.

The second feature that distinguishes induction from syllogisms is that inductive conclusions give not reliable, but *only probable knowledge* . According to this trait, inductive conclusions *are inferior to* syllogisms, in which—subject to the same reliability of the premises—the conclusion is *always reliable* , that is, it is necessary true.

§ 10. *Reliability and, accordingly, reliable knowledge do not have degrees.* If two truths are both reliable, then it cannot be said that one of them is more reliable than the other. That twice two will be four is no more and no less reliable than the fact that twice three will be six. The Pythagorean theorem is no more and no less reliable than the circle area theorem or any other Euclidean geometry theorem.

On the contrary, probability and, accordingly, probable knowledge have degrees , that is, they can be more or less probable. The probability that, for example, a meteorite will fall on a city square is many times less than the probability that it will fall in the ocean, in a field or in a forest. Under certain conditions, the degree of probability can be calculated mathematically.

Suppose I put my hand through an opening in a closed box in which, in an unknown order, ten balls of the same size, smoothness, density, weight are placed. Of these balls, seven are blue and three are red. What is the probability that I will take out a red, not a blue ball? Obviously, to solve this issue it is necessary to reason as follows.

In our task, we can determine the total number of all equally probable cases, both favourable for getting the red ball, and unfavourable for this getting. The number is ten, because there are only ten balls in the box. Of all equally probable cases favouring the delivery of the *red* ball, obviously, *three*, since there are only three red balls in the box and, taking out all ten balls in succession, more than three red balls cannot be obtained. The number of all equally probable cases that are not conducive to getting the red ball will be *seven* , since there are only seven blue balls, from which one can be pulled out instead of the red one, each time. Obviously, the degree of logically reasonable probability that the red ball will be pulled out will be expressed as a fraction of 3/10. In this fraction, the numerator (3) is the number of all cases conducive to the condition of the problem, and the denominator (10) is the total number of all equally possible cases that, in sum, exhaust all the possibilities of this test.

If all ten balls were red, then the degree of probability of the case indicated in the problem would be expressed as a fraction of 10/10, i.e., would be equal to one. In this latter case, the degree of probability would obviously be equal to *confidence*.

If all ten balls were blue, then the degree of probability of the case indicated in the problem would be expressed as a fraction of 0/10, i.e., would be equal to zero. In this latter case, the degree of probability would obviously be equal to the reliability of the non—occurrence of the event.

In general form, the degree of probability of occurrence of an event is expressed by the fraction m / n, in which m is the number of all cases favourable for the occurrence of the event, and n is the number of all equally probable cases that completely exhaust the test, i.e., representing the sum of all cases—both favourable and adverse.

The degree of probability of *non*— occurrence of the event, obviously, will be expressed by the formula 1 — m / n, i.e. (nm) / n.

No matter how small the fraction m / n is, but until this fraction has become equal to zero, there is some positive, at least insignificant, probability that this event will occur. In practice, of course, a degree of probability close to zero is not taken into account. So, although the possibility of earthquakes is not ruled out in Moscow, the probability of *destructive* earthquakes is very small here due to their weak intensity and is not taken into account in the practical calculations of builders. On the contrary, in San Francisco, where the degree of probability of devastating earthquakes, as experience shows, is incomparably greater, builders must reckon with it in their practical plans and calculations.

The conclusions of probability are of varying value for practical life and for science. Of crucial importance are the conclusions of probable *general* judgments from the judgments of *individual* and *particular* facts. Such conclusions are called *inductive,* and the whole set of techniques, or methods by which these conclusions are justified, is called *induction*.

Inductive methods, or types of induction, differ in their value for knowledge, namely: 1) in the ability to give *new* knowledge in comparison with that contained in the premises, and 2) in the *degree of probability* with which various types of induction substantiate the general conclusions.

Full Induction

§ 11. The first form of induction forms *complete induction.* This is the name of the inductive inference, in which the *general* conclusion is drawn from a series of *individual* premises that exhaust in their sum all possible cases or all possible types of a known kind.

Example of complete induction:

All types of conical sections are limited to a circle, an ellipse, a parabola and a hyperbola.
A circle cannot be crossed by a straight line at more than two points.
Ellipse too.
Parabola, too.
Hyperbole, too.
————————————————————
Next, none of the conic sections can be crossed by a straight line at more than two points.

In this conclusion, the conclusion is a general judgment about the whole genus (about all conical sections). The general conclusion is substantiated by a number of premises, each of which expresses the same predicate. This predicate is expressed not about the whole genus, but only about one of its species: about the circle, about the ellipse, about the parabola, about the hyperbole — about each separately. A special premise confirms that in addition to the listed types, there are no other types of conical sections. Since the predicate approved by each premise turned out to belong to each of the species without exception, this gives the general conclusion that this predicate belongs to the entire genus.

Another example of complete induction has already been given above—when explaining the features of inductive inferences. In this example, the general conclusion—“all days in the past week the weather was cloudy”—came from parcels that found out that the week was seven days and that each of the days of the last week, taken separately, was cloudy. Here, as in the previous example, the general conclusion is based on a complete listing of all single cases, the sum of which exhausts a known class and which are characterized by the fact that the same predicate is expressed separately for each of them. The only difference between this conclusion and the previous one is that here the general conclusion is obtained from *single* premises, while in the example with conical sections the general conclusion is the conclusion about the *genus*, the parcels only speak of *species of* this genus. But in either case — whether the premises expressing the predicate will be *individual* judgments or judgments *about species*— they will always be *private* in comparison with the conclusion.

The very course of the conclusion is the same in general cases. It consists in the fact that the predicate expressed by the premises about each individual instance of the class or about each individual form, concludes by saying about the whole class or about the whole genus, i.e. it is transferred to the whole class or genus.

§ 12. What is the logical right of such a transfer based on? It is based on the complete identity of the volumes of the concepts of a class (or genus), which is evidenced by the general conclusion, and the sum of the volumes of concepts of all instances (or all kinds of the genus), which are mentioned in particular premises. In turn, this identity of the volumes of concepts is based on the fact that both the entire class (or genus), which is mentioned in the conclusion, and each instance of the class (or each kind of genus), which are mentioned in private premises, are identical in content. This means that the signs by which a class (or gender) is thought, and the signs by which each instance of a class (or each kind of gender) is thought, are one and the same. These are precisely the signs that are thought of in the predicate of private premises.

In other words, the traits conceivable in particular objects of a certain class or in particular species of a known kind, we transfer—in the case of conclusions of complete induction—to the whole class or to the whole kind.

But we have the right to such a transfer only if we have really examined all the objects included in the class (or all species included in the genus). Only in this case, between the subject of a general judgment about the whole class (or gender) and the sum of the subjects of private judgments about individual instances of the class (or species of the genus) from which the predicate is transferred, will there be a complete logical identity, giving the right to a general conclusion.

On the contrary, in cases where private premises do not exhaust all instances of a class (or all types of a genus), there is no sufficient reason for transferring a predicate conceivable about particular objects of a class (or genus) to the entire class (or genus). In such cases, the general conclusion can easily turn out to be erroneous.

An example of such an erroneous conclusion of complete induction can be the conclusion of ancient astronomers about the direct movements of outer planets. These astronomers did not know anything about the existence of the outer planets of Uranus, Neptune, Pluto, as well as the existence of planetary moons. Unaware of their existence and knowing from observations of the three outer planets known to them that each of them, as a general rule, moves relative to stars from west to east, i.e., by so—called direct motion, these astronomers concluded that all outer planets were moving direct movement.

This conclusion turned out to be erroneous. Its fallacy was that all instances of this class included in the class of outer planets were not taken into account. In other words, the premise turned out to be erroneous, which claimed that there were no more outer planets except Mars, Jupiter and Saturn. In fact, it turned out that the class of outer planets is not exhausted by three planets known to the ancients. Moreover: it turned out that some of the satellites of the outer planets have not direct, but reverse movements. As soon as this fact was established, the basis for a general conclusion about the direct movements of all outer planets collapsed.

The error that an incomplete review of class instances or species of the genus is taken as *exhaustive* and therefore is considered as a basis for *general* conclusion about the whole class or about the whole kind, is common. In such cases, complete induction turns out to be *imaginary* complete induction, and its general conclusion is often erroneous. Confidence that a genus is exhausted by all its currently known species or class — by all specimens known hitherto, often does not have sufficient basis.

§ 13. The indicated features of complete induction determine both the *scope of its application* and *its significance for knowledge*. Full induction does not provide knowledge about other subjects, except for those that are alternately listed in private premises. Thus, the general conclusion about conical sections in our first example of complete induction does not apply to any new objects compared with those discussed in private premises. The predicate that each individual premise reiterated about the circle, about the ellipse, about the parabola and about the hyperbole, is not transferred in conclusion to any other or new curves, except for the ones listed. In this sense, that is, *with respect to the number of objects* onto which the general conclusion is transferred, complete induction does not give new knowledge in comparison with the knowledge that we had in the premises.

However, without extending to *new objects*, the general conclusion of complete induction characterizes the same objects from *a new perspective*. The subject of judgment in each particular premise was each individual object of the class (or a separate type of genus) as a separate and only separate object or type. On the contrary, in the general conclusion, the subject of the judgment is the same objects, but already considered not as separate, but as a certain class or some kind, that is, as a certain logical group. Therefore, the conclusion of complete induction is not an empty repetition in the form of a general conclusion of what was already fully conceived in private premises.

The scientific value of the conclusions of complete induction depends on whether the particular premises that substantiate the conclusion will be judgments on individual objects of the class or on species of the genus. If private premises represent judgments about individual objects of the class, then a general conclusion, according to the essence of complete induction, is possible only if all the instances of which the class is listed and considered are considered. In such conclusions, the number of instances of the class should obviously be limited, since the overview of the instances should be exhaustive. Therefore, the conclusions in this case are less valuable for knowledge.

But if the private premises justifying the conclusion, represent judgments about the *types*, then the limited number of species that make up the genus does not prevent the total amount of specimens that make up the genus is incalculably large. Although there are only four types of conical sections, but since each of them embraces countless specimens, the whole group of conical sections to which the predicate of particular premises goes in output will be a group consisting of countless specimens. Such conclusions, in which the well—known general property of a group can be attributed to each of an innumerable number of members of this group, are of greater value for knowledge than conclusions about a group consisting of a limited number of copies.

The inference from the predicate belonging to each species of a genus separately to the belonging of the same predicate to an entire genus is often used in the proofs of the mathematical sciences. Using complete induction, geometry proves the theorem according to which every angle inscribed in a circle is measured by half of the central angle based on the same arc. Geometry proves that this situation is true, firstly, for the case when the centre of the circle lies between the sides of the angle inscribed in the circle, and secondly, for the case when the centre of the circle lies on one side of the angle inscribed in the circle, and, in— thirdly, when the centre of the circle lies outside both sides of the angle inscribed in the circle (see Fig. 65).

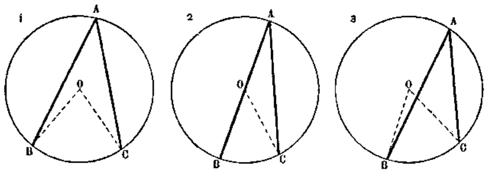


Fig. 65

Since with these three cases all possible types of the concept of an angle inscribed in a circle are exhausted, and since the proved position turns out to be valid with respect to each species of the genus separately, from this geometry concludes by the method of complete induction that this position will also be true with respect to the *whole* genus, i.e. e. with respect to *any* angle inscribed in the circle.

§ 14. In a preliminary explanation of the concept of induction, traits were *distinguished* that *distinguish* inductive inferences from syllogisms. It was also said that along with features distinguishing inductive conclusions from syllogisms, there are *common* features between them.

The above is true and relatively *complete* induction. The conclusion of complete induction, differing from syllogisms, like all inductive conclusions, by the ability to give general conclusions from particular premises, is at the same time similar to syllogisms in three respects.

Firstly, the conclusions of complete induction, as well as syllogisms, give, in contrast to other types of induction, not only probable, but quite reliable conclusions. True, the condition for the reliability of these conclusions is an exhaustive review of all specimens that make up the class, or all species that make up the genus, to which the predicate of private premises is transferred in conclusion. But even in syllogisms, the condition for the reliability of a conclusion is always the reliability of the premises substantiating the conclusion. Only the possibility, in some cases, to erroneously recognize an inaccurate premise as valid leads to an erroneous conclusion, but the general property of syllogisms does not hesitate at all — to give completely reliable conclusions, provided the premises are reliable.

Similarly, the erroneous recognition of an incomplete review of instances of a class or species of a genus for their full review leads to an erroneous generalization, but does not contradict at all that, provided a really exhaustive list of all instances of the class or all species of the genus, the conclusion of complete induction attributing the predicate to the whole class or genus will be quite reliable.

§ 15. *The second* feature, which brings together full induction with syllogistic conclusions, is that the premises and conclusions of complete induction, in contrast to the premises and conclusions of other types of induction, are usually *judgments of belonging*. So, in the example with a general conclusion about conic sections, each premise and conclusion affirm the belonging of a known property—the ability to intersect a straight line at no more than two points—not only to each individual type of conical sections, but also to their entire genus. On the contrary, in other types of induction, which are discussed above, in the conclusion of the inductive conclusion, a causal relation is usually established.

§ 16. *Thirdly*, in addition to the general similarity of complete induction with all syllogisms, there is still a special similarity between the conclusions of full induction and simple categorical syllogisms of the *third* figure.

Common between them is the very process of inference and the basis on which it is based. Indeed, in the syllogisms of the third figure, as well as in the conclusions of complete induction, we have in conclusion the transition of a predicate from a logical form to a logical genus.

For example:

All marsupials are uterine.
All marsupials are mammals.

Next, some mammals are uterine.

This syllogism is of the third figure. In it, the predicate "prenatal" is transferred in custody from "marsupials" to "mammals". But "marsupials", as can be seen from the lesser premise, make up with respect to "mammals" only one of the species of the entire genus "mammals". That is why, ascribing the predicate "prenatal" to the genus of "mammals", the conclusion ascribes this predicate not to the entire genus of "mammals", but only to that part of the volume of this genus, with which the volume of its species "marsupials" coincides.

This shows that the particular nature of the conclusions obtained on the third figure of the syllogism does not preclude the fact that the very course of the conclusion on the third figure consists in *transferring the predicate from species to genus*. From the fact that the conclusions of the third figure are always only private, it does not follow in any way that the very course of the conclusion on the third figure consists in moving from genus to species. From this it follows only that, carrying out the transfer of the predicate from species to genus, the conclusion transfers the predicate of the larger premise from the species not to the entire genus, but only to some part of it.

But the same inference takes place with full induction. And here the conclusion carries over the predicate attributed in each of the private premises to only one of the species of the genus, for the whole genus. The fact that in each particular premise it was stated about one kind of conic section — separately about a circle, about an ellipse, about a parabola, about a hyperbole— in conclusion it is stated about the whole kind of conic sections.

The difference between full induction and the third syllogism figure is *not in the process of* inference, but *as a result of* this move. The result of it in the case of the third figure will always be *only a particular*, in the case of complete induction—always a *general* judgment.

The reason for this difference is the unequal review of those species whose predicate is transferred to the genus. In the case of *complete induction, the* species survey carried out in private premises is exhaustive, that is, it covers the entire genus. Therefore, the predicate, passing in conclusion from species to genus, does not pass to a part of the genus, but to the entire genus.

On the contrary, in the case of the *third figure* the predicate expressed by the larger premise asserts only about one kind of genus. Therefore, this predicate, passing in conclusion from species to genus, does not pass to the entire genus, but only to that part of it, the volume of which coincides with the volume of the species.

But in either case, whether the predicate passes to the entire genus or only to some part of it, the predicate passes from species to genus. The general basis for the possibility of this transition is the identity between the volume of species and the volume of the genus. In the case of complete induction, this identity will be complete, since all species of the genus are taken into account, in the case of the third figure, it is only partial, since only one species is taken into account.

Incomplete Induction

§ 17. We have just seen that a pass when viewing all instances of a class or species of a genus, generally speaking, makes the general conclusion about this whole class or genus unreliable. If each of the specimens or species considered by us has the same property or trait, but if we do not know if there are still any specimens or species belonging to the same class or genus, then we cannot consider it reliable that a property or attribute, repeated in all cases so far known to us, will also be repeated in all other cases of the same class or genus. Therefore, the conclusion of complete induction, as has already been said, turns out to be unreliable as soon as it turns out that the consideration of all specimens or species that should have been carried out in private premises did not in fact exhaust the entire class or genus.

This does not mean, however, that any generalizing inductive conclusion always and without fail must be based on the study of *all, without exception,* particular cases that make up a known genus. In addition to complete induction, there is still *incomplete* induction. This is the name of the conclusion, in which the general conclusion is obtained from not all, but only a few cases or instances of this class. Moreover, each instance individually has a well—known property, which is attributed to it as a predicate in each private premise. In the conclusion, the same property is generalized, i.e., it applies not only to the considered instances for which it is installed by

private premises, but also to the entire class, including instances that have not yet been examined.
For instance:

Wheat blooms in spikelets.
Oats, too.
Rye, too.
Barley, too.
But wheat, oats, rye, barley are cereals.
____
Next, *all* cereals bloom in spikelets.

Inference is an example of *induction* , since the general conclusion about the entire genus of cereals is obtained in it from a number of premises about individual species of this genus. However, the induction here is not complete, since the general conclusion is based on a review of not all, without exception, but only some of the cereals. Wheat, oats, rye and barley do not exhaust the entire genus of cereals. In addition to these species, there are also rice, corn, bamboos and other types of cereals. Nevertheless, based on premises that do not exhaust the entire genus of cereals, we concluded that this genus is without exception.

Inference diagram of incomplete induction:

S ’	possesses the property		
S ’’	»	»	
S ’’’	»	»	
________________			
Next, and S ’’ ‘‘, in general, all S have the property R.			

§ 18. Incomplete induction differs significantly from full. That new knowledge, which gives full induction, is not knowledge of new *objects* , beyond those that were considered in the premises. Full induction does not give knowledge about new objects, but about the *new side of* those objects that were considered in the premises and which are characterized in the output no longer as separate, but as a whole class or as a logical group.

On the contrary, *incomplete induction in the conclusion gives knowledge about new objects besides those that have already been considered in the premises* . The property that these premises assert regarding a part of a class or genus transfers the conclusion of incomplete induction to the whole class or genus.

Incomplete induction of this type is called precisely because in the premises *only some part of* all cases or instances of the class is deliberately considered , while the conclusion is made regarding the *whole class* representing the total sum of all these cases or instances.

On what basis is a general conclusion possible here? What gives us the right, having considered only a few cases or objects of a known class and finding that all of them—each individually—have a well—known property, to conclude that this same property belongs to the *whole* class?

Such justification cannot be a simple enumeration of any horrible cases or consideration of any horrible instances, randomly or arbitrarily snatched from the whole class. If the general conclusion about the whole class was obtained as a result of considering only a certain part of randomly encountered instances of the class, then it is completely obvious that the position that turned out to be true in all these cases cannot be a sufficient basis for a general conclusion. If I walk along the street and if the first three passers—by whom I met on the way accidentally turned out to be old people, then this is not enough to conclude that all the other passers—by that I meet along my path will also be old people.

Incomplete Induction Through Simple Enumeration

§ 19. But, perhaps, the basis for the probability of a general conclusion is the absence of facts or cases that contradict the generalization? Perhaps the probability of a general conclusion is based not only on the fact that we know several cases or facts confirming our generalization, but also on the fact that we do not know a single case and not a single fact that would contradict this generalisation?

Of course, the absence of facts or cases that contradict the general conclusion of several particular facts confirms the probability of generalization. If we know a certain number of facts that are consistent with the generalization, but at the same time, we also know about the existence of other facts of the same kind that go against the generalization, then we cannot recognize the facts that coincide with the generalization as the basis for a probable general conclusion. The only fact that is incompatible with the content of the generalized conclusion is sufficient for this conclusion to be decisively rejected as erroneous. Indeed, the conclusion claims to be *general* , that is, it assumes that the known position is true with respect to the whole class, while the existence of facts contrary to the conclusion proves that the conclusion is actually true *only with respect to part* class, i.e., is not common.

An inductive conclusion in which a general conclusion is made only on the basis of only a part of all cases or facts that are consistent with the generalization, provided that no case or fact is known that would contradict the generalization, is called *induction through a simple enumeration* . The full name of induction of this type is *induction through a simple enumeration in which there is no contradictory case*.

§ 20. Induction through simple enumeration is the most unreliable form of incomplete induction. If the only basis for the probability of a general conclusion is ignorance of cases that contradict the generalization, then the probability of a conclusion should be recognized as poorly substantiated. In this case, the presence of probability may turn out to depend only on our ignorance. Today we do not know a single fact that contradicts my generalization from particular facts, and so far our generalisation can still be recognized by us as probable. But if I meet tomorrow with at least one fact incompatible with the generalization, then my generalization immediately becomes likely to be simply false from the likely.

But this is not enough. The disadvantage of induction through simple listing is not only the constant possibility of its refutation. Its disadvantage is that even with ignorance of facts that contradict generalization, the generalization in this case cannot be complete. If, when considering particular facts on which the conclusion is based, the selection of facts was completely random, then the generalization itself, strictly speaking, can only be valid with respect to the facts that we have considered, but not with respect to facts other than those investigated.

Given the incompleteness of the facts and the randomness of their choice, no reason can be seen which would make it possible to transfer the predicate from the cases already considered, where this predicate is set, to any cases beyond those considered.

Therefore, if incomplete induction was reduced only to that form, which consists in a simple enumeration of cases consistent with the generalization, and in the absence of cases that contradict it, then incomplete induction would be a little valuable form of inductive conclusions.

But induction through simple enumeration is only one type of incomplete induction. In addition to induction through simple enumeration, there are also types of incomplete induction in which, in addition to the absence of facts contradicting the generalization of facts, the likelihood of a conclusion is joined by the *special nature of the facts themselves, justifying the conclusion*, and *a special way of selecting facts that excludes or at least reduces randomness them for the entire class with respect to which the conclusion is drawn* . We call inductive inferences of this kind *incomplete induction through selection, excluding the randomness of generalisation*.

Incomplete Induction Through Selection, Excluding Randomization Generalisations

§ 21. In inductive conclusions of this kind, a generalization, as well as in the case of incomplete induction through a simple enumeration, is made on the basis of only a certain part of facts of a known kind.

However, these facts are selected in such a way that, as a result of their selection, generalization becomes likely. In these cases, it can be seen that the generalization is based not only on the agreement of the conclusion with the facts confirming the conclusion, and not only on the absence of facts that contradict the conclusion. In these cases, the generalization is based on signs indicating that the facts selected and examined by us are not the only ones confirming the generalization, and that all other facts of the same kind probably have the same property that was found in the facts already considered and which, in the conclusion, is transferred for the whole clan.

The main condition for the probability of inductive conclusions of this kind is the *exclusion of circumstances that make random the choice of facts on which the generalization is based* . If, having arrived in a new area and taking walks in several directions, I noticed that field hyacinths (“loves”) are often found in all these areas in the forest, I can summarize my observations and say that not only in the directions I have studied, but and throughout this forest in general, many field hyacinths grow.

This conclusion is a typical conclusion of *incomplete* induction. It generalizes (spread *to the whole* the forest of the property found in its parts) is made by considering only part of the volume of a known genus. If the number of directions we studied and, consequently, the number of parts of the forest were insignificant in comparison with the entire area of the forest, and if these directions were parallel to each other and not far from one another, then our conclusion would be poorly substantiated, and its probability would be would be insignificant.

But if walks were made in such a way that their paths crossed the forest in all possible directions, if these directions passed through all the main parts of the forest, then with the same number of walks and the same number of field hyacinths that we noticed during the walk, the probability of our The conclusion about the abundance of field hyacinths throughout the forest, over its entire area, is incomparably greater.

The reason for this difference in the degree of probability in the first and second cases of the conclusion is obvious. In the first case, the conclusions of our walks were chosen so that it remained doubtful whether it was not by chance that for the whole forest as a whole the abundance of field hyacinths that we found in the directions we travelled. Indeed, being parallel and close to each other, these directions covered a small area of the forest. Due to random circumstances, for example, due to the coincidence of the direction of my walks with a narrow swamp passing through the forest in the same direction, field hyacinths, loving wetlands and therefore abundant in this part of the forest, could be rare in all its other parts.

On the contrary, in the second case, the choice of directions for walking was such that the observation results—the presence of a large number of field hyacinths in all these directions—could not be random *for the whole forest*. If, crossing the forest from end to end *in the most diverse* directions, I have encountered many field hyacinths, then it will be very likely to conclude that *throughout* forest areas, and not just along the directions I have been following, field hyacinths grow in abundance. Moreover: with such a selection of the facts underlying the generalization, the assumption opposite to our conclusion would be unlikely. It would be extremely strange if, often getting across in all directions, crossing and crossing the forest from end to end, field hyacinths would disappear just in all areas lying between the intersecting lines of my paths.

In this case, the very choice of facts on which the general conclusion is based eliminates the moment of chance, which reduces the validity of the generalisation.

§ 22. The more varied and more numerous are the observations from which the facts underlying the generalization are drawn, the less is the danger that the properties we noticed in these facts that are distributed in the conclusion to the whole class have no basis in the properties of the whole class and depend on special and random circumstances—from the limitations of that part of the class that has been taken into account.

The exclusion of the moment of chance in the actual material increases the degree of probability of conclusions. Where the elimination of accidents is sufficiently ensured by the very conditions of choice, the conclusion of incomplete induction is quite likely. Under these conditions, incomplete induction becomes an important and widely used method of generalization in life and in science, and its conclusions become a reliable element of knowledge and practical orientation.

§ 23. From the foregoing it is not difficult to derive the logical basis of incomplete induction through the elimination of chance, and also to determine the scope of its conclusions. The basis for this type of incomplete induction is the actual identity of some objects with some part of the class. At the same time, the objects should not be horrible, but such that their properties we observed depend on the properties of the class to which they belong, which they represent and which are characterized by the same property in the general conclusion.

Inductive conclusions of this type do not differ in structure from the conclusions of complete induction and from the syllogisms of the third figure. As in the last two forms of inference, in the conclusions of incomplete induction, the conclusion is to transfer the predicate from individual instances to the whole class. The peculiarity of incomplete induction compared with full induction and comparatively with the third figure of syllogism consists in the originality of techniques by which the accidents leading to unjustified conclusions are eliminated. These techniques in various fields of knowledge are different.

In the natural sciences, there are a number of extremely important and very general provisions that encompass an infinitely large, practically inexhaustible set of facts. These provisions cannot be proved by checking them on all, without exception, facts belonging to the scope of these laws. Unavailable to verification, exhausting all cases without exception, these provisions are nevertheless justified by incomplete induction. They substantiate, since it is possible to prove that even under the most diverse conditions and in the most various parts of nature, within the limits of observable, the facts studied always have the properties that make up the predicate of these propositions.

Such general provisions of the natural sciences are, for example, the law of universal gravitation, the law of conservation of energy, and a number of other truths.

The conclusions of incomplete induction are the basis for a number of important points of the science of proper thinking itself. One of the main provisions of the logical doctrine of the so—called Bacon induction is the provision that, with a sufficient number of cases of observation, no circumstance that coincides with a known phenomenon by *chance* can occur with such constancy as its causes, all taken together. Of course, provided that this accidentally observed circumstance itself does not belong to the category of phenomena that are most common in nature.

The high probability of this situation borders on reliability. It is due to the fact that even under the most variable conditions of observation, the connection between facts that are causally dependent on each other, in a huge number of observations, is always more constant and close than the connection between facts, the coincidence of which is accidental.

But no matter how varied the conditions of observation, no matter how numerous the various cases substantiating the general conclusion, these conditions and cases, of course, can never exhaust all the possible variety of experimental conditions that can be verified. Even the most skillful and successful elimination of circumstances that make generalization random, can never certainly exclude randomness of choice. True, the elimination of chance is achieved extremely much. Where the conclusion is made on the condition that random circumstances are excluded, we can be sure that the objects whose property is characterized in the general conclusion are not separate instances to which this property belongs by chance, but in any case form a certain group.

However, even with this result, it remains unclear whether we can consider the predicate belonging to this group of objects as the predicate of the entire class without exception. Incomplete induction cannot finally clarify this question. Being probable, the conclusions of incomplete induction always leave the possibility of exceptions that undermine the universal significance of its conclusions.

So, at the beginning of the XVII century, before the discovery by Europeans of Australia, all European, Asian and African naturalists had the right to draw, based on incomplete induction, the conclusion that all swans are white. In their practice, there was not a single case that would contradict this conclusion. But at that moment when the Europeans who landed on the western coast of Australia came upon the first black swan there, this conclusion, despite the huge number of cases hitherto supporting it, was immediately refuted. Before the recent discovery of the so—called “white dwarfs”, very small, extremely dense and hot stars, the incomplete induction, according to which the stars are huge bodies with volumes close to the volume of the sun and even larger, seemed quite reasonable. The discovery of the first “white dwarf”—the satellite of Sirius, and then several other “white dwarfs” refuted this generalization. There could be many such examples. However, the possibility of such exceptions does not reduce the role of incomplete induction. Through this induction, science establishes many properties and relationships that must be recognized not only as belonging to a known class of phenomena, but in some cases even prevailing in it.

Incomplete Bacon Induction

§ 24. Among the conclusions of incomplete induction, a particularly important place belongs to the conclusions about the causal connection of phenomena. Their significance is due to the fact that knowledge of the causes and their connection with actions allows one to foresee and even bring to existence at one's own discretion a number of phenomena of value for practical life and for science.

Not a single natural phenomenon and not a single event in the life of society is committed without reason. Everything that happens for some reason, although this is far from always known to us. So, we know that the reason for the eclipse of the sun is the covering of the solar disk with the dark disk of the moon at the time of the new moon, provided that at this moment the centres of the earth, moon and sun are on the same line. For a long time, people did not know the causes of the so—called infectious, i.e., infectious, diseases. After the discovery of microorganisms invisible to the simple eye, this reason was established: it turned out that these diseases are caused by the activity of microorganisms that have penetrated the body— bacilli, bacteria, i.e., for each disease there is a special reason, its own special pathogen.

It is known, for example, that the cause of malaria is the bite of a special malaria mosquito (anopheles). This mosquito is a carrier of a malaria parasite that enters the blood of a bitten one. Knowing the cause of malaria, doctors not only establish through the microbiological examination of mosquitoes the presence or absence of malaria in the area, but also indicate a number of measures to prevent disease. These include: draining the swamps where malaria mosquitoes are found, neutralizing infected raw lowlands by pouring them with oil, etc.

However, the exact determination of the causes of various phenomena in most cases is not easy. This is explained, *firstly*, by the fact that the same phenomenon can be caused not by a single, but by a number of reasons. So, the cause of a fire in a wooden house can be a lightning strike, a short circuit of wires, and a small head falling from a neighbouring house during a fire, and an enemy incendiary bomb, etc.

Secondly, even if the phenomenon is caused by a single reason, establishing this reason is sometimes difficult due to *complexity* most phenomena. Each phenomenon occurs and is observed among the numerous conditions and circumstances that precede it or occur simultaneously with it. So, the appearance of lightning is usually preceded (and often accompanied by) hot weather, the accumulation of positive and negative electricity in the clouds, the appearance and condensation of clouds of a special structure (thunderclouds), the appearance of a pre—thunderstorm vortex, rain and hail, etc. which of all these circumstances (or which part of them) are the *direct* cause of the electric discharge, called lightning, a special study is necessary.

To explain the lightning phenomenon of each of these circumstances separately, it would not be enough. Lightning, like any other phenomenon, is possible only if there are *all the* facts and circumstances necessary for its occurrence: the formation of thunderclouds, the accumulation of positive electricity in the part of the cloud saturated with large suspended drops of water, the accumulation of negative electricity outside the region of the upward current, disintegration of drops, etc. All these facts and circumstances constitute, in their totality and interconnection, the cause of lightning, and lightning itself is their *effect* . In this case, the cause *precedes* in time, and the action *follows* in time after its cause.

§ 25. It can be seen from the foregoing that the reason is something complicated. A combination of a number of certain circumstances or conditions is required for a certain action to occur. Therefore, not every sum of facts and circumstances is the reason. The reason will be only such a combination of facts and circumstances, in which at the very moment when all these facts and circumstances are fully present, the action *must* occur.

Thirdly, the study of the causes of the phenomenon is further complicated by the fact that this study cannot be limited to *direct* observation of the relationship between various circumstances preceding the phenomenon and its attendant. *The causes of the phenomena are often inaccessible to direct observation.*. Millions of people have observed the phenomenon of the rainbow millions of times, but Newton’s genius was needed to discover the direct cause of this phenomenon, which was unnoticed directly: the decomposition of a complex white sunbeam into composite coloured rays, due to the different refractions of individual coloured rays of the solar spectrum. In order to establish the causes, one has to set up special experiments that were preliminarily thought out and organized in such a way that the cause of the phenomenon, inaccessible to direct observation, would be revealed through experience.

Five Basic Types or Methods of Bacon Induction

§ 26. In all studies of causation, that is, the relationship of cause and action, inferences and conclusions, known as *Bacon induction* , play a large role . These conclusions were first indicated in logic by the English materialist philosopher *Francis Bacon* (1561—1626). The rules of Bacon’s induction were set forth in detail and revised taking into account the facts and methods of the natural sciences that emerged at the end of the 18th and the first third of the 19th century, by English scientists John Herschel and John Stuart Mill.

There are five main types or methods of Bacon (or Mille) induction: 1) the method of *similarity* (or, as it is also called, the method of *single similarity*); 2) the method of *difference* (or the method of *single difference*); 3) the combined method of *similarity and difference* ; 4) the method of *residuals;* and 5) the *method of concomitant changes*.

1. Similarity Method

§ 27. The method of similarity is a conclusion about the cause of a phenomenon, obtained from a comparison of a number of cases selected in such a way that the phenomenon whose cause we are looking for occurs in all these cases and that these cases, which are different in everything, are similar in one common to all of them circumstance.

Suppose that in the same area, where the same gravity stress, we consider several pendulums having the same oscillation period. Suppose we wondered about the reason for this equality. The cause of this phenomenon, obviously, can be either the *composition of the substance* from which the pendulum is made, or the *length of its rod*. To solve the question which of both of these circumstances will be the cause, we will make several pendulums from various substances—rom steel, copper, iron, but we will make the pendulum rods of the same length.

Experience shows that in all these cases the oscillation period of the pendulums will be the same. From this we conclude that the reason for the equal period of oscillation of the pendulums is not the composition of the substance (which was different in all experiments), but only the same rod length in each pendulum.

Let us consider closer the course of this conclusion. In all cases taken for experience, a phenomenon whose cause was required to be established — an equal period of oscillation— invariably took place. In all these cases, no matter how different they were from one another, one circumstance was always evident: all the pendulums, regardless of the composition of the substance from which they were made, had the same rod length. Except for this unique circumstance common to all cases, all other circumstances in each case were different: chemical composition, density, weight, hardness, etc.

The reason for the equal period of oscillations could not be the differences in the chemical composition of the substance of the pendulums, since, despite the fact that the chemical composition of all the pendulums were different, all the pendulums showed the same oscillation period. Since in all the cases considered only one circumstance was always present, namely the same length of the pendulum rod, this is the only similar circumstance for all cases, obviously, is the reason, or at least part of the full reason for the phenomenon under study.

In general form, inference by the method of single similarity can be depicted by the following diagram:

ases	Observed circumstances	The action whose cause must be established
.	ABC	*a*
.	ADE	*a*
.	AFG	*a*
Conclusion: the cause of phenomenon *a* is circumstance A.		

The diagram shows that in all three cases there is some action *a* , the reason for which is the subject of research. If the cause of the occurrence of *a* were, for example, circumstance B, which was part of the first case, then in the second case *a* could not have occurred, since circumstances B were not present in the second case. So, we exclude B from among the possible causes of the phenomenon of *a* . For the same reasons, circumstances C, D, E, F, and G should be excluded. All of them, appearing in one of the considered cases, were absent in all the rest and therefore could not be the cause of *a* which occurred in all decisive cases. With the exception of B, C, D, E, F, and G, circumstance A alone remains. Since it is the only one that has occurred *in all* cases, and since all other circumstances have disappeared as possible causes of the phenomenon, it remains to conclude that it is circumstance A is the cause of the occurrence of phenomenon *a* .

In our example, the experiments were set up so that one single circumstance turns out to be similar. On this basis, the similarity method is sometimes called the *single* similarity method.

§ 28. The method of similarity is one of the forms of *incomplete* induction. As in other conclusions of incomplete induction, a general conclusion on this method is obtained from consideration *not the entire* class of cases of some kind, but only a certain part of it. So, in our example, only three pendulums were considered—from steel, copper and iron. All pendulums that can be made of other substances: brass, nickel, silver, etc., were not considered.

Being a type of *incomplete* induction, the similarity method is based, however, not on arbitrary listing or consideration of any specimens, but *on a special selection of* cases on which the conclusion is based. This is a method of *eliminating* all circumstances that cannot be the cause of the phenomenon and whose accidental presence in the cases under consideration should not affect the conclusion of the conclusion.

§ 29. The purpose and result of the conclusion by the method of similarity is to conclude on the causal relationship of phenomena, or to establish a relationship of cause and action. As an inference about causation, the similarity method is different from syllogistic conclusions, in which the goal of inference is to establish a relationship of belonging.

However, differing from the syllogism in the nature of the conclusion and in the goal, the similarity method has something in common with the syllogism in the very process of inference. Indeed, the scheme of the similarity method that we have deduced is very similar to the structure of the categorical—categorical syllogism mode tollendo ponens. A prerequisite for withdrawal of the method is the separation of similarity judgment establishes that the cause of the phenomenon *and* it can be either A, or B, or C, or D, or E, or F, or G. This premise, obviously, corresponds to the dividing premise of the categorically—dividing syllogism of the tollendo ponens modus.

The very course of the withdrawal by the method of similarity is, as we have seen, first, in the exclusion of all but one of the circumstances with respect to which it was possible, according to the premise, to suppose that they can be the cause of the observed in all cases the phenomenon *but* . This is an exception to all circumstances—B, C, D, E, F, G, except for one circumstance A, which occurred in all the cases considered, which corresponds obviously to a categorical negative premise in the separation—categorical syllogism of the tollendo ponens modus.

Secondly, the course of the conclusion by the similarity method is that circumstance A is recognized *as the* cause of phenomenon a , which one turned out to be possible, since it turned out that it alone was present in all cases, while all the others—B, C , D, E, F, G,—appearing in some cases, were absent in all others. In this case, circumstance A is recognized as the cause of the phenomenon, *and* not only for those cases that were considered, but also for all cases not considered, but similar to them, that is, for all cases in which all circumstances turned out to be different, except for a single one.

Thus, the conclusions of the similarity method are based on the conclusion that the well—known predicate is the ability of circumstance A to be the cause of phenomenon *a* established for several cases in which this circumstance turned out to be the only constant—applies or is extended *to all other cases of* the same kind.

The conclusion in which the predicate—the ability of circumstance A to be the cause of phenomenon *a*—is generalized, that is, it extends to *all* cases that satisfy the scheme of the similarity method and corresponds to the *conclusion of* separation—categorical syllogism.

§ 30. Since the same action can, generally speaking, be caused by *various* reasons, the similarity method does not give a definitively reliable, but only a probable conclusion about the cause of the phenomenon.

The degree of probability of conclusions (made by the method of similarity depends, *firstly* , on the number of cases considered. The more we take pendulums made of the most diverse materials and having similarities only in that they all have the same length of rods, the more likely to be concluded that the reason of the same period of oscillation in all these cases in a different composition of matter, and in the same length of the pendulum rod.

Second, the degree of probability of conclusions made by the method of similarity depends on how large the differences are in all other circumstances, except for the only one that turned out to be present in all cases and which turned out to be the only similar. The more diverse the composition of the substance from which the pendulum rods are made, the more likely it will be to conclude that the cause of the invariably equal period, the oscillations, are not these circumstances so different for all cases, but only that which turned out to be the same in all cases.

However, even a huge number of cases and the difference in circumstances of each case, except for one, cannot report the conclusion by the similarity method of *perfect reliability*. Due to the complexity of the reasons, it always remains possible that the cause of the phenomenon under investigation will not be the only circumstance that in all "cases turned out to be similar, but the combined effect of this circumstance with others. So, the cause of the phenomenon *a* may be in one case a combination of circumstances of the AS, in the other—EA, in the third—FG. The same action or phenomenon in each case could be caused by a special cause. The cause of phenomenon *a* could not even be circumstance A, but in one case—circumstance B, in another—circumstance C, in the third—circumstance D, etc.

Finally, even if it is true that the cause of the phenomenon is the only one similar in all cases of circumstance, the establishment of this relationship between cause A and action *and the* question has not yet *been resolved to the end*. Most circumstances are in turn difficult. At the first acquaintance, circumstance A can be considered as something whole or single, but with a more careful and deeper study of A, it can turn out to be difficult. Assume that A consists of parts α, β, γ, δ. Even circumstance A, recognized as the cause of the phenomenon *a* , could turn out to be this cause *not in its entire composition*, but only in a certain part, for example, in part α, while other parts of A, for example, β, γ, δ, could not be worthwhile in a causal connection with the phenomenon *a*.

§ 31. In addition to all the circumstances indicated here that reduce the likelihood of conclusions obtained by the similarity method, this method has another drawback. This drawback consists in the fact that in the conclusions by the method of similarity the conclusion goes from action to the cause of this action. But the researcher often already finds action, ready, already arising in nature or in public life. In relation to action, the researcher often finds himself in an observant position. He is forced to accept the action as he actually found it. And only subsequently dividing the entire action or phenomenon into its component parts or circumstances, the researcher can raise the question of which of these parts or which of these circumstances is the cause of the observed phenomenon. So, thousands of scientists have watched thousands of times the game and iridescent colours on the inner surface of mother—of—pearl shells. But in order for a hunch to arise that the cause of this phenomenon is not the chemical composition of the substance from which the shell is built, but the physical structure of its inner surface, a case was needed. This conjecture arose only after Brewster, accidentally receiving the imprint of a mother—of—pearl shell made of wax, noticed on the inner surface the same play of colours as on the inner surface of a real shell.

This case is typical of the similarity method. Guesses about causation verified by this method often require an especially favourable case for their occurrence. Where the phenomenon is repeated in nature under uniform conditions, a special, only occasionally granted, deviation from these conditions is required so that the circumstance, which remains the same under the changing other circumstances, could attract attention as a possible cause of the phenomenon. When Brewster received — by chance — the first print of a wax sink, it was no longer difficult for him to come up with a further change in all other circumstances, except for the only thing that remained unchanged — the shape of the inner surface. Having made a number of artificial shell prints—from gypsum, rubber, resin, etc., he could easily see.

2. The difference method

§ 32. The method of distinction is a conclusion about the cause of a phenomenon, obtained from comparing the case when the phenomenon occurs, with the case when it does not occur. Moreover, both cases are completely similar to each other in all circumstances except one. This circumstance is present in the first case, when a phenomenon occurs, and is absent in the second, when there is no phenomenon. Consider the example of inference by the difference method and derive its scheme.

Modern physiology knows that the light sensitivity of the eye in the dark depends on the normal formation of visual purpura in the retina. Eyes whose retina lacks the proper amount of visual purpura are hard to see in the dark. But what is the cause of the normal formation of visual purpura?

To determine the cause of this phenomenon, the physiologist sets the following experiment in his laboratory. For a number of days, the experimental rabbit is given food containing vitamin “A” among other nutrients in its composition. Then, over the same series of days, the same rabbit is given food in the same amount and of the same composition, but without vitamin “A”. At the same time, they are observing the formation of visual purpura in the rabbit’s retina and the associated sensitivity of the eye in the dark. It turns out that during the period when vitamin “A” was mixed with food, the formation of visual purpura in the rabbit’s eyes and its sensitivity to light in the dark were normal; at the same time that the rabbit was fed the same food, but without vitamin A, the formation and restoration of visual purpura in the dark and the sensitivity of the rabbit's eyes to light in the dark sharply decreased. This leads to the conclusion that the presence of vitamin "A" in food is the cause of the formation of visual purpura.

Both cases compared are similar to each other in all circumstances except one single one. In fact, in both the first and second cases, the experimental rabbit was in the same conditions, conditions, diet, quantity and types of food, etc. But in the first case, to all conditions common in the first case with the second, joins one single circumstance by which this case differs from the second—the presence of vitamin "A" in the composition of food.

Since the presence of vitamin "A" is the only circumstance by which the second case differs from the first, and since this circumstance is the cause of the phenomenon, this method is often called the method of *single* difference.

In general form, inference by the method of single difference has the following scheme:

ases	Observed circumstances	The phenomenon whose cause must be established
.	ABCDE	*a*
.	BCDE	—

Conclusion: the cause of the phenomenon *a* is circumstance A.

The conclusion will be as follows:

None of the circumstances B, C, D, E could be the cause of the phenomenon *a* . If this reason were, for example, circumstance B, then, since this circumstance was present not only in the first, but also in the second case, phenomenon *a* should have been observed in the second case as well. But since in the second case the phenomenon *a* did not arise, then B cannot be the cause of *a* . For the same reasons excluded from the possible causes of the phenomenon *and the* conditions C, D and E. There remains only one circumstance A. Since experience shows that the presence of A appears *and* on the other hand, in the absence of A does not occur and *a* , we conclude that a circumstance is A cause of the phenomenon *as well*.

§ 33. The method of difference, like the method of similarity, is a method of *exclusion* . A prerequisite for the conclusion of this method is a separation judgment, listing a number of circumstances that make up the two cases under consideration and which may be the cause of the phenomenon *a*.

The subsequent course of the conclusion consists, firstly, in the fact that all circumstances that were present in both compared cases are sequentially excluded from the circumstances indicated by the separation premise. Secondly, the only remaining not excluded A circumstance that took place in the former and absent in the second, the conclusion is recognized in the cause of the phenomenon *and* not only in the above case, but in all cases, the circuit diagram of which corresponds to the difference method.

Therefore, the difference method is also one of the methods of *incomplete* induction. The conclusion obtained with his help corresponds not only to the conclusion of the modus tollendo ponens of the categorical—categorical syllogism, but also to the generalization from several cases of the class to the whole class, which is the essence of incomplete induction.

Being the conclusion of incomplete induction, the conclusion by the method of distinction is based on a *special selection of circumstances* from which the cases under consideration are composed. Selection of this excludes the possibility that the observed relationship between us and the circumstance A phenomenon *and* was accidental connection of relevance only to the cases examined. The very choice of these cases makes them representatives of a whole class, reports the general significance of the conclusion about the causal relationship between A and *a*.

§ 34. The difference method has an important advantage over the similarity method. It consists in the fact that the conclusion by the method of difference gives a *more probable conclusion than the conclusion by the method of similarity about the reason for the phenomenon being studied* . If the introduction of the experience of the circumstances and the phenomenon *as well* , the reason we're looking for, come, and with the exception of the circumstances A—disappears, there can be no doubt that between A and B *and* have a causal relationship.

This advantage of the difference method is due to the fact that the exclusion of all circumstances, except A, from the number of possible causes of the phenomenon *a* in the case of the difference method is made by *experiment*.

In the derivation using the similarity method, the starting point of the conclusion is usually not an experiment, but an observation. Given a well—known phenomenon, it is required to establish its cause. Since the phenomenon is usually not created by the researcher himself, but only found and observed, the composition of the phenomenon is often not well known to the observer, and the phenomenon itself arises outside of his will and calculation—not in his laboratory, but in nature itself. So, the rainbow spectrum that appears on a rain cloud is not created by the observer, but arises regardless of his plans and intentions. If at the same time the cause of the observed phenomenon is unknown, then it can not always be easily caused and its action can not easily be made the subject of observation.

The situation is different in the conclusions of the difference method. Here, the appearance (or non—appearance) of an action depends *on the researcher himself* and is determined *by the very conditions of experience*. The most experience here is *artificial* experience, or *experiment*.

In the experiment, such an environment for observation is intentionally created in which the researcher, firstly, knows exactly what circumstances the observed process is made up of. Secondly, in an experiment, the researcher, at his discretion, may introduce some new circumstance into the process in order to establish what effect this circumstance will have on the course of the process. Thirdly, the experiment isolates the artificial environment of the process created by the researcher for the entire duration of the experiment. This isolation does not allow any other circumstances to penetrate the course of the process, except for those introduced by the experimenter himself in order to trace what change the new elements introduced by him will make. The last condition is extremely important, since with its presence, any change observed during the process can be caused only by those circumstances which were introduced into the same setting by the researcher himself. This ensures that the action of the new circumstances introduced into the process appears in its pure form, is not complicated by any unforeseen and unaccounted for influences. Fourth, the experiment provides the possibility of arbitrary *repeated* occurrence of the phenomenon—either under the same conditions, or under conditions intentionally and according to the plan changed by the researcher himself.

To make sure how closely the difference method is associated with experiment, let us return to our example with experience with the action of vitamin “A”. In the example of this, the conclusion was possible only because the logical conditions of its possibility and probability were created and provided by the experiment. In order to obtain two processes that are completely similar in all circumstances, except for one single one, present in one and absent in another case, an artificial environment and protection from the intervention of new circumstances that can influence the course of the process are necessary.

If during the observation of the effect of vitamin “A” on the formation of visual purpura in the rabbit’s retina, the circumstances that make up the two cases to be compared would change all the time and, apart from the researcher’s will and intentions, new circumstances would come into play, for example, all the time if the composition of food, its quantity, lighting conditions, temperature, etc. changed, then the researcher could never say with certainty which of the continuously changing circumstances is the reason for the formation or disappearance of visual purpura in the rabbit’s retina.

Only by *isolating the* observed process and thereby protecting it from intrusion of extraneous circumstances, only by achieving *complete* equality in both cases of all circumstances, except for one thing—the presence and absence of vitamin “A” in the same food composition and quantity—the researcher can be sure that the disappearance of visual purpura and the loss of sensitivity of the eye to light in the dark are caused by the absence of taken food of vitamin “A”. But this artificial setting and process isolation can only be created and provided by experiment.

§ 35. However, the difference method does not give a final solution to the issue. A new circumstance introduced by the experimenter is always a somewhat complicated circumstance. Therefore, there is always the possibility that the real cause of the phenomenon will not be the whole circumstance A as a whole, but only some of its constituent parts, for example, α. In this case, it is possible that, finding somewhere separately in the experiment α—an integral part of circumstance A, we will receive, independently of A, in its whole composition, the very phenomenon *a*, for which we considered A as a whole.

It can, for example, be established by the difference method that air (A) is a necessary condition for the life of an animal (*a*) Having placed a bird under the hood of the air pump, they then pump out air, and the bird suffocates and dies in front of the observer.

This experience, of course, proves that air is a necessary condition for the life of a bird. But air is not a simple element. Air is a complex mixture consisting of oxygen (α), nitrogen (β), water vapour (γ), carbon dioxide (δ), etc. Therefore, even by proving the need for air (A) for life, our experience leaves us the open question is what role each of the constituent elements of air plays in the process of breathing and life—α, β, γ, δ.

§ 36. But the complexity of the circumstance introduced into the experiment by the method of distinction is not the *only one* a source of insufficient reliability and accuracy of inductive conclusions that are made by this method. The second source of their lack of reliability and accuracy is the *complexity of the causal relationship itself*.

If the change in the composition of both cases, which are compared to each other in the conclusion by the method of difference, always consisted solely in the fact that only circumstances A would join the circumstances of the BCDE that make up the composition of one case, so that the composition of the second case would be composed of the same of BCDE plus A, then a comparison of the results of one and the other composition would immediately give a likely conclusion about the causal relationship between circumstance A and the new result that arose with its addition to the circumstances of the previous composition.

But this would only be possible if the circumstances that make up the composition of each case would exist simply next to the other, *without exerting any effect on each other*. In fact, this is extremely rare.

By virtue of the always existing interaction between various facts and circumstances of nature and human life, the addition of the only new circumstance A to the composition of the previous circumstances usually results not only in the emergence of a new phenomenon *a*, but, in addition, changing the circumstances of the previous composition themselves and replacing them with others. Instead of the expected composition in the second case, the circumstances of the ABCD may result, for example, the composition of the AEFD. This will be the case if the introduction of a new circumstance A results in a change, for example, circumstance B into circumstance E and circumstance C into circumstance F. Thus, by staining the plant tissue observed under a microscope, the histologist or microbiologist hopes that as a result of the addition of the observed forms of some new factor, and, moreover, the only new one, namely, the substance that stains the fabric of these forms of substance, he will again receive all the same circumstances from which the subject of his observation is composed. The only change, he believes, will consist only in the fact that the paint introduced by him will better highlight colourless ones for vision.

In fact, as is well known to scientists conducting microbiological studies, the introduction of a colouring substance rarely remains without affecting the structure itself and the course of the observed phenomena and processes. Since the colouring matter is usually a more or less strong chemical agent, then simultaneously with the absorption of this substance by the organic tissue in the composition of the tissue, changes occur in its forms and in the organic processes occurring in it, as a result of which the composition of the circumstances of the second case may turn out to be changed. This change can be so significant that as a result, instead of the previous circumstances of the aircraft, the observer will no longer have a group of circumstances ABCD (what he wanted), but some other group AEFD: not just a simple sum of all the composite factors, but *qualitatively* different composition.

It is quite obvious that with such a radical change in composition in the second case, the scheme of the difference method is not observed. Instead of the intended scheme:

.	CD	
.	BCD	

the researcher is dealing with another scheme:

.	CD	
.	EFD	

In this last scheme, there is no condition that informs the probability method of the difference method: the second case (when the phenomenon, the cause of which must be established, takes place) differs from the first *one not by only one* additional circumstance A, but, in addition, by additional circumstances E and F. True, the researcher sees that with the introduction of circumstance A, phenomenon *a* arose. Given this fact, it can, of course, to suggest that some sort of causal link between the introduction of the A and the occurrence *and* there. But he was denied the opportunity to say with certainty that the cause of *a.* And it is a fact in itself, and not those circumstances E and F, in which as a result of changed circumstances new entry A former circumstances first case B and C. In other words, A may be the cause of the phenomenon *but* not directly, but through E and F.

Thus, theoretically a very clear and simple scheme for differences rarely performed in all its clarity and simplicity in practice. Usually, the experimental conditions give only a certain approximation to that state of affairs, in which both cases compared are distinguished by only one single circumstance. The less accurate this approximation, the less reliable the conclusions obtained by the difference method.

§ 37. But even in cases where there is reason to believe that the addition of a new circumstance A will not cause any side effects in the investigated case, except for the result that the experimenter himself is waiting for, it is almost often impossible to be sure that the scheme of the difference method is precisely executed.

One of the main sources of this inaccuracy is the difficulty with which the isolation of the process performed and observed in the experiment, necessary for the success of any experiment, is connected, i.e., its protection from the influence of all extraneous additional circumstances, except those introduced by the experimenter himself according to his own plan and plan. This can be illustrated by the following example.

When Pasteur set out to prove the impossibility of spontaneous generation of living organisms, he had to set up special experiments to prove this idea. To do this, Pasteur took two flasks with a nutrient medium; He sterilized one of them, but not the other. If, Pasteur reasoned, microorganisms are brought up in a nutrient medium under ordinary conditions, then no organisms will start in a hermetically sealed bottle with a sterile (provided) medium, no matter how long it has stood. It would seem that the idea of experience is very simple and accessible to verification. The Pasteur experience is a typical difference method experience. Two cases are compared: in the first and in the second—the same nutrient medium, the same bottle shape, the same storage conditions, etc. The difference is only in that in one case the nutrient medium contains embryos of microorganisms that have penetrated into it naturally from the air, in the other this medium is sterile, i.e. all embryos that were in it are destroyed. Experience should show — by the method of difference—that in the second medium organisms will not arise.

In carrying out this experiment, however, it turned out that it was extremely difficult to achieve complete sterilization of the culture medium. The first tests showed that in an environment that was considered sterile, organisms nucleated in the same way as in an environment where their embryos were not destroyed.

What conclusion should be drawn from these experiments? Maybe the one that the organisms arose in a sterilized environment by themselves, spontaneously? Pasteur understood that such a conclusion would be premature. Before making such a conclusion, it was necessary to check whether all the conditions for the conclusion by the method of difference were really met in this case. It was necessary to make sure that in the second case the medium turned out to be really sterile, i.e. that all the embryos that were in it were really destroyed.

Verification showed that this is the main condition for the given experiment—the condition *was not* actually fulfilled. When hermetically clogged the mercury into which the corks for bottles were immersed, dust particles remained, there were unaccounted for and undetermined germ cells of microorganisms, which multiplied, gave offspring, so that the result of the experiment was the impression that these organisms arose by themselves.

3. The United Method of Similarity and Difference

§ 38. We examined the method of similarity and the method of distinction individually. But in studying the causal relationship of phenomena, these methods are sometimes used together.

The combined method of similarity and difference is as follows. A number of cases are considered in which the phenomenon occurs and in which only one circumstance is common. Then they consider a number of cases in which the same phenomenon does not occur and which have nothing in common with each other, except for the absence of exactly the same circumstance. Then the circumstance, by the presence or absence of which only the two series of cases differ, is either a consequence, or a cause, or part of the cause of the phenomenon.

So, having noticed that the mushroom aspen is always found in that part of the forest where aspens grow, and not finding it in any other parts of the forest where aspens do not grow, we can conclude from this that it is the presence of aspen that favours the growth of aspen mushrooms. This reasoning is an example of the combined conclusion of similarity and difference. First, by the method of sole similarity, the probability is established that it is the presence of aspen that favours the reproduction of boletus. Then, by the method of the only difference, it is established that the absence of aspen in a certain part of the forest excludes the possibility of the growth of aspen trees in this part. Since both of these series of cases differ only in the presence or absence of aspen, i.e., one single circumstance, the probability of a conclusion obtained already from the first row.

§ 39. Scheme of the combined method of similarity and difference

			Circumstances of each case	The phenomenon whose cause must be established
I row	1st	case	ABC	*and*
cases	2nd	»	ADE	*and*
II row	1st	case	BC	—
cases	2nd	»	OF	—
Conclusion: circumstance A is the cause of phenomenon *a* .				

In practice, compliance with all the conditions of the indicated scheme of the combined method of similarity and difference is difficult to achieve. It is difficult to exclude all circumstances, except for a single one, from the cases of both series we are considering, both in which the phenomenon under study is present and in which it is absent.

Sometimes the complication of a simple similarity method is achieved by one increase in the number of cases considered. So, when studying the cause of the rainbow play of colours on the inner surface of the pearl shell, the probability of output was greater, the greater the number of prints that differed in all circumstances except one — the shape of the inner surface.

4. The method of residues

§ 40. The method of residuals is a conclusion about the cause of the phenomenon, obtained from the study of a complex phenomenon, which, in addition to the already known circumstances that produce a known action, also includes some, as yet unknown, reason that produces a homogeneous, but additional action. The conclusion by the method of residuals is the conclusion that the cause of this homogeneous incremental action must be the circumstance that remains as a result of subtraction of the circumstances already recognized as belonging to the number of reasons for the observed action from the total amount of homogeneous circumstances that could be the causes of the same action.

Consider the example of inference by the residual method and derive its scheme.

It is known that the moon *on the full moon* when in relation to the Sun it is *on the other* side of the Earth on a straight line connecting the centres of the Moon, the Earth and the Sun, it produces by its attraction a tide phenomenon, the magnitude of which can be accurately measured in each given area. The same Moon at the *new moon,* when in relation to the Sun it is *between the* Earth and the Sun on the same straight line, produces by its attraction the same tide phenomenon, but stronger. What is the cause of this added tidal force?

To answer this question, it is necessary to subtract the smaller value obtained in the first case from the larger value of the phenomenon that occurs in the second case. This determines the remainder corresponding to the difference between the circumstances of the phenomenon in the first and second cases. Now it is necessary to determine what consists and in what this difference is expressed in the circumstances. And at the time of the full moon and at the time of the new moon, the attraction of the Moon acted, if we neglect the small difference in the distance from the Moon to the Earth in these phases, with the same force. But the Earth is affected not only by the attraction of the moon, but also by the attraction of the sun. This solar attraction will not produce the same result depending on the position of the moon relative to the earth. At the time of the new moon, when the Moon is between the Earth and the Sun, the Earth's attraction by the Sun and the Moon acts in the same direction. Therefore, the result of this attraction is added up. At the time of the full moon, the Earth's attraction by the Moon is weakened by the opposite attraction of the Sun. Therefore, in the case of a full moon, from the value that measures the Earth's gravity by the Moon, one has to subtract the value by which the Earth's gravity is measured by the Sun. This remainder, obviously, will explain the observed and measured difference in the height of the tide in both cases (see Fig. 66).

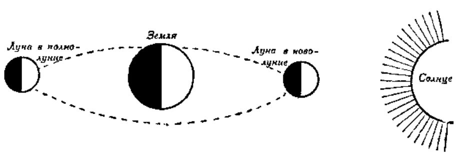


Fig. 66

In general form, the inference by the residual method can be represented by the following diagram:

The circumstances of ABC are the only ones that can cause the complex phenomenon of *abc* .										
ut	it is known	hat	circumstance	T	there is	reason	arts		phenomena	*bc*
	»		»		»	»			»	*bc*
Conclusion: circumstance A is either the cause of part *a of the* phenomenon *abc*, or at least it is causally connected with *a* .										

§ 41. It can be seen from the example and from the diagram that the method of residues is an inference from a certain totality to its elements or parts. The conclusion here is that, having considered the set of circumstances ABC, which is characterized by the fact that it alone can be the cause of the complex phenomenon *abc*, and comparing it with the elements of this set B and C, which is already known that B is the reason for component *b* and C—the reason part *with*, the complex phenomenon of *abc* , we conclude that the cause of the last part of *a* complex phenomenon *abc* have the circumstance a, which is obtained as a result of subtracting from the totality of its parts ABC B and C.

The conclusion by the method of residuals, as well as the conclusions by the method of similarity and difference, is the conclusion of an *exception*: from all circumstances ABC, which together constitute the cause of the complex compound phenomenon *abc*, all those circumstances (B and C), which, being each the cause of the corresponding parts *b* and *c of the* whole phenomenon *abc* cannot be the cause of part *a of* this phenomenon.

§ 42. The residual method is widely used in science in the study of causal relationships. In many cases, science reveals a causal relationship by examining the part of a phenomenon that is obtained after subtracting the rest of it, which has already been known before and reduced to known causes. Thus, they learned about the existence of a number of new chemical elements by establishing that in the spectra of some complex substances, in addition to the spectral lines caused by the presence of elements known to science in these substances, there are other spectral lines. These lines do not coincide with the lines of known elements and therefore prove the presence of some new, previously unknown elements in the composition of the studied complex substance.

The type of inference underlying the residual method is not limited in its application to inductive conclusions about causation alone. In essence, the same type of inference is widely used in non—inductive non—syllogistic conclusions, for example, in some conclusions of the mathematical sciences.

In these sciences, conclusions are constantly being made like the following: "If A + B + C + D = a + b + c + d and if B + C + D = b + c + d, then it follows that A = a." This conclusion and countless conclusions similar to it are based on the axiom: "If equal values are taken away from equal values, then the residuals will be equal." But this axiom, like the residual method inference scheme, logically represents only various cases of applying the same form of inference—from the totality to its part.

The logical feature of all conclusions of this type is that in them the conclusion is based not on the consideration of relations between the genus and species or logical group and objects of this group, but on the consideration of the relationship between a certain aggregate representing a known whole and elements or parts, this aggregate. When in the conclusion by the method of residuals in one of the premises of the conclusion it is stated that the circumstances ABC are the only ones that can cause the complex phenomenon of *abc*, this definition does not apply to A separately, and not to B, and not to C, but only to their totality, considered as a whole. Separately taken A is the cause of only *a* , and not the whole phenomenon of *abc*. There is only reason *b*and C is the reason only *with* . Only the totality of all the circumstances of ABC is the cause of the whole complex phenomenon of *abc*. It is from the aggregate ABC to its part A as the cause of *a that* we conclude.

This conclusion by the method of residues is significantly different, for example, from the syllogism of the first figure. True, in the syllogism of the first figure, the conclusion is, as in the conclusion, by the method of residuals, also from the general to the particular. But this common in the case of syllogism is not the totality, but the genus; knowing that the predicate P belongs to the whole genus M and that S belongs to the genus M as its species, we conclude that the predicate P must also belong to the whole S as one of the species of the genus M.

It is easy to see that here the predicate P or the definition of the genus M is such a definition that is applied not only to the set of all objects that make up M, but also to each of the items that make up this set individually. This means that the predicate P does not belong to the aggregate combination of all objects that make up the genus M, but to each of the objects of the genus M separately. The very conclusion is that, due to the logical identity of any item of type S with any item of genus M, the predicate P of each item of genus M must be recognized at the same time as the predicate of every item of type S, i.e., be transferred from the genus M in appearance S.

5. The method of concomitant changes

§ 43. A modification of the methods of similarity and difference is represented by the *method of concomitant* *changes* . This method is a conclusion about the cause of the phenomenon, obtained from a comparison of cases in each of which one observes, as in the conclusions by the similarity method, the same phenomenon, but not to the same degree. Moreover, all the circumstances in each case, as in the conclusions by the method of difference, are completely similar, with the exception of one. This last circumstance is also present in all cases, but it is observed in each of them to varying degrees. The conclusion consists in the conclusion that the cause of the phenomenon, the intensity of which varies in each case, is a circumstance that alone turned out to be changing, that is, having a different degree in each case.

Consider an example of output using the method of concomitant changes and derive its scheme.

Through the method of concomitant changes, the physicist proves, for example, that friction is the cause of the always—observed deceleration.

According to the well—known law of inertia, the rectilinear motion imparted to the body will continue rectilinearly at the same speed until the shock imparted by another body changes the speed and direction of this motion.

Let a ball roll on a horizontal surface. In its movement, this ball must certainly experience friction—no matter how smoothly polished the surface of the ball and the surface of the board on which it rolls. If the friction of the ball on a point on the surface were completely eliminated, i.e. reduced to zero, then the law of inertia could be proved by the method of a single difference. To do this, it would be enough to compare two cases, in one of which the movement would have occurred without friction, and in the other under the same circumstances, but in the presence of friction, i.e., the resistance of the particles of the surface along which the ball moves. In the case of the truth of the law of inertia, this experiment should show that in the absence of friction the motion of the balls will be uniform and rectilinear, and in the presence of friction the movement should slow down until the ball stops completely.

However, in reality, such an experiment can never be produced. It is impossible to reduce friction to zero. The only difference that in this example should be the absence of friction present in another case cannot be made.

However, this fact does not mean that the law of inertia should simply be taken on faith. This law is confirmed *by the method of concomitant changes.*. Although friction cannot be completely eradicated, it can still be greatly weakened. It is possible to set up a series of experiments with the movement of the same ball on a horizontal surface made of various materials, giving more or less friction. Moreover, all the circumstances of the experiment will be the same in all cases, and the difference between one experiment and another will only be that the inevitable and unavoidable friction will be greater in some cases, less in others. A comparison of a number of such experiments shows that the greater the friction, the greater the deceleration, and, conversely, the less friction, the less the deceleration.

Or another example.

If the pendulum is suspended without taking special measures to reduce friction at the point of gain, then, being out of equilibrium, the pendulum will stop after several swings. But if you build the pendulum so that with the help of special devices the friction at the point of gain will be greatly weakened, and the air resistance is eliminated, then, being removed from the equilibrium position, the pendulum can be pumped, without any additional push, for several tens of hours. This comparison gives grounds to conclude that if friction had been reduced to zero, motion would have continued without deceleration.

In both examples—with the movement of the ball and with the swing of the pendulum—the conclusion is made by the method of *concomitant changes*. The conclusion here is based on the idea that *every phenomenon that changes in a certain way in a certain way, while another phenomenon also changes in a certain way, is connected with this last phenomenon by the connection of cause and action.*

In general form, the course of inference by the method of concomitant changes can be represented by the following diagram:

Circumstances	ABC	—	the only ones	previous	phenomenon	*and*
»	A_1BC	»	»	»	»	a_1
Conclusion: circumstance A is causally related to phenomenon *a* .						

§ 44. From our example and from the diagram it is clear that the conclusion by the method of concomitant changes assumes in the form of a premise a judgment according to which the phenomenon *a* can have as its circumstances only circumstances AA_1BC. But about each phenomenon, which is preceded solely by the circumstances of AA_1BC, we have the right to assert that the cause of this phenomenon should be among these circumstances. This statement is, from a logical point of view, there is a judgment about the circumstances of the group AA_1BC, to which, as a group of possible reasons include the phenomenon *and*.

So, the first premise of the conclusion by the method of concomitant changes is the judgment of a certain group, which has a number of predicates. Each of these predicates—separately or in conjunction with another predicate of the same group—indicates one of the possible causes of the phenomenon *a*. Such a reason can be either A, or A_1, or B, or C, or AB, or $A_1 B_1$, or AC, or A_1 C, or BC, etc.

Considering the group of possibilities expressed by our premise, we the right to divide the whole group of these opportunities into two subgroups. The first will include all predicates indicating circumstance A (with all its changes, for example A_1), as the cause of the phenomenon *a* or at least as part of this reason (A, A_1 AB, A_1 B, AC, A_1 C). The second will include all predicates pointing to other circumstances, as possible causes or as part of the causes of the phenomenon *a* (B, BC, C). As a result of this separation, our judgment on the group of circumstances AA_1 BC will take the following form: the cause, or at least part of the cause of phenomenon *a*, may be either circumstance A, or all other circumstances.

The second premise of the conclusion by the method of concomitant changes is a judgment certifying that the phenomenon *a* in each case changes: in the first it appears as *a* , in the second as a_1.

But about the changing phenomenon, we have the right to assert that circumstances cannot *completely* be its cause, which themselves remained unchanged during its change. Therefore, knowing that the phenomenon *a* in the second case has changed to a_1 , and knowing that circumstances B and C in the second case have remained unchanged, we have the right to exclude circumstances B and C from the possible causes of the phenomenon *a* indicated by our first premise—a judgment on group AA_1 BC.

Thus, from both subgroups between which all its predicates were distributed in the first premise, we must exclude the entire subgroup in which, as possible causes of the phenomenon *a* and its changes (a_1) circumstances B, C, BC are indicated. It remains to conclude that the cause of the phenomenon *a* with all its changes (a_1) should be seen either in the changing circumstance A (A_1), taken separately, or in its combination with any of the other circumstances (A, AB, AC, A_1, A_1B, A_1C).

In other words, the entire course of the conclusion by the method of concomitant changes consists in the inference from the judgment on the group, indicating the whole set of circumstances that may be the causes of the phenomenon *a*—through the exclusion of those that in both cases remain unchanged—to the fact that turned out to be changing in this case.

An exclusion condition, by means of which a conclusion is reached on a causal relationship between a changing circumstance A and a changing phenomenon *a*, is the *assignment of a phenomenon to a logical group* twice repeated in the conclusion. For the first time, phenomenon *a* belongs to a group of phenomena, the only previous circumstances of which are AA_1BC. This judgment of the group already limits the entire area within which we could look for the cause of the phenomenon of *a* to two subgroups—a subgroup characterized by the presence of a changing circumstance A (A_1), and a subgroup characterized by the presence of all other circumstances (B, C, BC).

The second time, the phenomenon *a* belongs to the *group of changing* phenomena. Thus, from the whole group of previous circumstances, among which one could look for the cause of phenomenon *a* , the entire subgroup, characterized by unchanging circumstances, is excluded. The result of this successive exception is the conclusion. It consists in the fact that the phenomenon *and* tolerated as it causes a subgroup of changing circumstances, remaining after excluding other subgroups.

From this it can be seen that the method of concomitant changes, like the other methods of back—century induction, is a method of *elimination*. The sought reason is obtained in it *by* *eliminating* all circumstances which, as it turns out during the conclusion, cannot contain the causes of the phenomenon *a*.

§ 45. Of all the other inductive methods closest to the method of accompanying changes worth *similarity method* . In both of these methods, the conclusion goes from the *general to the particular* , *from the group to a separate subject*.

Indeed, the prerequisite for the conclusion by the method of similarity is, as we have seen, the *judgment of the group* relating the phenomenon *a* to the group of circumstances ABC, among which may be the cause of the phenomenon *a*. In turn, the ABC group, to which we attributed the phenomenon, can be divided into two subgroups. One of them is characterized by the fact that in it a circumstance, which may be the cause or at least part of the cause of the phenomenon *a*, is circumstance A, taken separately or in conjunction with other circumstances. Another subgroup is characterized by the fact that in it, as a possible cause of phenomenon *a* , all other circumstances are indicated (B, C).

A comparison of the case of the circumstances of ABC and the case of the circumstances of A_1BC immediately shows that the assumption that the cause of phenomenon *a* can be found among the circumstances of the *second* subgroups (B, C) are extremely unlikely. If this assumption were true, then it would turn out that circumstance A, which, according to this assumption, is not the cause of the phenomenon *a* , at the same time occurs in conjunction with this phenomenon as often as all real facts in conjunction with it his reasons put together.

But the rejection of the assumption that the cause of phenomenon *a* may be among the circumstances of the second subgroup means that this reason should be sought among the circumstances of the first subgroup, i.e., that the cause *a* is A.

Thus, in the conclusion by the method of similarity and in the conclusion by the method of concomitant changes, the course of inference is essentially the same. Both methods are inferences from the group to a separate subject of the group. Both methods differ from syllogistic conclusions in that the condition for the conclusion is the exclusion from the judgment on the composition of the group, to which the phenomenon belongs, all the predicates that cannot belong to this subject.

§ 46. On the contrary, there is an important difference between the conclusion by the method of residuals and the conclusion by the method of concomitant changes.

In the conclusions of the residual method, the cause of the phenomenon under study has a complex composition. It consists of part of the known and already studied, part of the unknown before the experience of the reasons. Moreover, the existence and nature of unknown causes are established, as we have seen, by examining the remainder between the full and incomplete action of the cause (*abc* — *bc*). The remainder in this case is due to differences in the circumstances of the phenomenon (ABC in the first case, BC in the second).

As for the circumstances themselves, or the reasons for the combination of which causes the phenomenon, they—in the case of the residual method—do not change significantly. So, the action *The moon* on water particles in the Earth's oceans remains (if we neglect the small difference due to the unequal distance of the Moon from the Earth to the new moon and the full moon) the same in both phases. In exactly the same way, the effect of the Sun on these particles remains (if we neglect the small difference in the distance of the Earth from the Sun, due to the motion of the Earth around the Sun in an elliptical orbit) the same both in the full moon and the new moon. The remainder, characterizing the difference between the first and second cases, shows here that, in addition to the previously taken into account and known reason (the action of the moon), the intensity of the phenomenon is affected by some additional, not yet taken into account or even completely unknown reason (the action of the sun). The manifestation of this reason is dependent on various circumstances of the phenomenon and gives a certain balance when subtracting one action from another.

On the contrary, in the conclusions by the method of concomitant changes , *the very cause of* the phenomenon under investigation *changes* , and, depending on its change, the strength of its action changes.

§ 47. Like all inductive methods, the method of concomitant changes gives a probable conclusion about the causal relationship of phenomena. At the same time, however, this method leaves unclear the question of what is the causal relationship in each given case. Where ABC circumstances *precede the* phenomenon *as well*, and the circumstances of A_1 BC—phenomenon *and* $_1$ *bc*, when inference according to the method of accompanying changes in the ability to exercise and what circumstance And is *full* cause of the phenomenon *as well*, and that it is *only a part of* all reason *a* . But where changes are ABC and *abc* are strictly parallel, so that any change occurring with A corresponds to a certain definite and simultaneous change occurring with *a* , it remains unknown whether circumstance A is generally the cause of *a* . Here it is possible, firstly, that A and *a* are both the actions of some common cause for them. Secondly, it is also possible that A is an action and *a* is the cause.

Therefore, the method of concomitant changes is usually used in the first stage of research, when the task is to establish the fact of causality, and not to clarify its nature. At this stage, the method of concomitant changes is of great importance, since with its help a fact of a still unknown causal dependence can be discovered. Thus, a weather study of the number of spots and groups of spots on the Sun, the order of their appearance, the determination of the belts within which these spots develop, as well as the magnetic properties of the spots themselves, led to the establishment of an amazing relationship between the phase of this periodically repeating process and the sign of the spots themselves, which after an eleven—year period changes from positive to negative and vice versa.

But an explanation of the essence and nature of the dependence itself cannot be achieved by the method of concomitant changes alone and requires special studies in each case. Where changes are simultaneous parallel and where it is impossible to establish that what precedes—A precedes *and* vice versa,—a method of accompanying changes in reserves even unclear which of the two, respectively, changing phenomena is the cause and what—the action. This method only detects that each specific change in a certain circumstance corresponds to a certain change in the phenomenon.

Logical errors possible in inductive outputs

§ 48. When using all considered inductive methods, logical errors are possible, as in all actions of thinking. As in all other types of inferences, in inductive inferences all the premises on which the conclusion is based must be true, that is, correspond to reality.

One of the premises of the inductive conclusion is usually a judgment on a certain group of circumstances, among which there should be a circumstance related to a causal relationship with the phenomenon *a* . Therefore, the first condition for the correctness of the inductive conclusion should be the truth of the premise expressing a judgment about the group.

The error of this premise may consist, *firstly,* in the fact that among the circumstances preceding the occurrence of the phenomenon *a*, not all the circumstances that may be the cause of this phenomenon will be noted. Due to the complexity of all facts and phenomena, there is always the possibility that, among the circumstances included in the group of possible causes of the phenomenon, the circumstance that constitutes its true cause will be unaccounted for and missed in the judgment of the group. So, in Pasteur's experiment on the issue of spontaneous nucleation, the possibility of the appearance and reproduction of organisms was not initially taken into account due to insufficient sterilization of plugs immersed in mercury, which clogged flasks with nutrient broth.

Secondly, the fallacy of a premise expressing a judgment on a group of circumstances that could be the cause of the phenomenon *a*, may consist in the fact that, correctly indicating these circumstances, the package *does not take into account the complexity of their composition* . Underestimation this can lead to the fact that factor A, remaining on the exclusion of all other circumstances, it may be the cause of the phenomenon *but* does not entirely, not throughout its structure, i.e. E. Not as a circumstance A, but only to some of its parts *α*, which can occur in experience and cause the appearance of, *and* even in the absence of A in its entirety. Thus, air is not a necessary condition for breathing in its entirety, as could be concluded from an experiment with a bird placed under the bell of an air pump, but only to the extent that oxygen is included in the air.

The possibility of this error cannot be overestimated. In all branches of knowledge, at every stage of the development of science, in countless cases, complexity is revealed where simplicity was previously assumed. What could be simpler than the idea of ancient physicists about atoms as continuous homogeneous whole and unchanging lumps of matter? However, this idea had to be abandoned, since the assumed simplicity of the structure of the atom turned out to be incompatible with the huge number of phenomena observed by physics and chemistry.

The transition constantly taking place in science from the concept of a simple composition of a phenomenon to the concept of its complexity cannot remain without a trace for all conclusions about a causal relationship. A researcher using inductive methods should always be ready to review the current conclusion about the causal relationship between A and *a* — as soon as it turns out that A itself contains a number of circumstances: α, β, γ, δ, etc..

In all such cases it is necessary to raise the question whether there is a cause of the phenomenon and *as* in all its composition, as *the totality of* the circumstances of α, β, γ, δ, or a cause is any of these circumstances, *taken separately*.

§ 49. But even the full truth of the premise that characterizes the group of circumstances, among which we should look for the cause *and* does not provide more inductive inference is correct. Second, after the truth of the premises, the condition for the correctness of this conclusion consists in the correctness of the inductive inference itself.

Since inductive inferences are used in studies of the causal relationship between phenomena, the first source of logical errors encountered in these inferences is the mixing of causation with a simple sequence in time.

Every connection of cause and effect proceeds in time. If a physicist wants to detonate a mixture of detonating gas in a flask, he must *first* bring a lit match, and only *then* an explosion will follow. A thunderclap does not precede a flash of lightning, but vice versa: *first* lightning flashes, and only *then* a thunderclap is heard.

But from the fact that a cause precedes its action, it does not at all follow that every phenomenon that follows in time after another phenomenon is an action, and that which precedes is a cause. A simple sequence of two phenomena in time alone does not give any reason to believe that the previous phenomenon is the cause, and the next is the action.

A logically undisciplined mind, in particular the mind of a person devoid of scientific concepts of the world, tends to fall into the mistake that the sequence of two events or phenomena in time is taken as a causal connection, as if existing between them. This error is called in logic the conclusion error by the formula post hoc ergo propter hoc ("after that, therefore, because of this"). According to this formula, people who are at a low level of cultural development, prone to superstition, believing in signs, and so on, have reasoned and are now reasoning.

Who, on the basis of the many times observed change of dawn and sunrise, would make the impression that dawn is the cause of sunrise, and sunrise is the effect of this reason, he would be reasoning by the formula post hoc ergo propter hoc. When superstitious people proclaimed the great comet that appeared this year and preceded the outbreak of war, the cause of the 1812 war, they also reasoned by the formula post hoc ergo propter hoc.

Arguments of this kind, of course, have no basis and therefore no evidence. Although all phenomena are interconnected and do not proceed independently of each other, this does not mean that any phenomenon that precedes this is its cause. To make sure that the preceding phenomenon is the true cause, and the subsequent is the true action, it is not necessary to simply observe the sequence in time, but a real proof. Induction methods play a prominent part in this evidence. When a physicist introduces a new circumstance, using the method of difference, into a new circumstance, notices the appearance of a new action, and then, excluding this new circumstance, observes the disappearance of this new action, he no longer simply establishes the sequence of two phenomena in time: he proves, that there really is a causal connection between the two phenomena. This conclusion is no longer based on post hoc ergo propter hoc. The conclusion here is based on the observation not of a random connection between the two phenomena in time, but on experience, which proves that each time a certain circumstance, which is supposed to be a reason, once introduced into our experience, really causes a certain action, and being excluded, leads to disappearance actions.

The second important source of logical errors in inductive conclusions is the mixing of the *probability of* inductive conclusions with *reliability*.

Whatever the perfection of the inductive methods associated with the experiment and with all the advantages that the experiment informs the conclusion, these conclusions always have only a greater or lesser probability, but not unconditional reliability.

Even an extremely large number of individual cases confirming the general situation that is deduced from them by induction, *taken by itself, without other justifications,* cannot turn an inductive conclusion into an unconditionally reliable proposition. On the other hand, as we have seen, one single case that contradicts the conclusion is sufficient—and the generalization, no matter how large the number of cases confirming it, is refuted.

Very often encouraged by the multiplicity of cases, apparently confirming the generalization or assumption, the researcher is inclined to ignore facts that contradict the generalization.

This mistake, psychologically very understandable, is extremely common. The scientist observed many individual cases, developed an assumption about the relationship between them, summarized his observations and established, as he believes, a certain regularity. Until now, experience has not refuted, but, on the contrary, as if confirming its generalization. Such a scientist is often extremely unpleasant to make sure that the generalization obtained by him with such difficulty, verified in so many cases, is still erroneous. Such a scientist is inclined not to notice facts and cases incompatible with his generalization, theory or hypothesis. When Galileo discovered with the help of the invented pipe of the moons of Jupiter, there were scientists who did not even want to look into the tube and make sure the actual existence of the satellites. These scientists understood that if they saw satellites, about the discovery of which Galileo announced, then this fact alone would be enough to refute the old ideas about the number and nature of celestial bodies. These scientists preferred the denial of the obvious fact to perception, which should, according to the laws of logic, make them recognize the falsity of their generalizations and theories.

On the contrary, an important quality of a real scientist lies in his ability and aspiration, developing a well—known generalization and finding a number of facts confirming this generalization, to actively look for facts that are incompatible with his generalization. Knowing that, even substantiated by a large number of separate confirming cases, many hypotheses turned out to be refuted as experience was increased and contradictory cases were discovered, a true scientist does not turn his assumptions and generalizations into dogma into prejudice, fettering his mind and making his blind and insensitive to the perception of new data. Very many seemingly brilliant and promising generalizations had to be abandoned as soon as facts were found that were incompatible with these generalizations. A true scientist not only knows how to make a generalization on the basis of the studied particular facts, he also knows how to temporarily and without regret refuse any generalization as soon as it turns out that there are facts that contradict this generalization. Such a scientist, for example, was the great Russian physiologist I. Pavlov. He possessed both of these features to a high degree: the ability to generalize the great number of observed particular cases and facts, as well as the ability to irrevocably and ruthlessly discard even the assumption or generalization, which seemed to be firmly established by successful explanations of particular facts, which turned out to be incompatible with new facts. Possessing this very valuable quality for a scientist, I. Pavlov developed this quality in his students as well. that there are facts that contradict this generalization. Such a scientist, for example, was the great Russian physiologist I. Pavlov. He possessed both of these features to a high degree: the ability to generalize the great number of observed particular cases and facts, as well as the ability to irrevocably and ruthlessly discard even the assumption or generalization, which seemed to be firmly established by successful explanations of particular facts, which turned out to be incompatible with new facts. Possessing this very valuable quality for a scientist, I. Pavlov developed this quality in his students as well. that there are facts that contradict this generalization. Such a scientist, for example, was the great Russian physiologist I. Pavlov. He possessed both of these features to a high degree: the ability to generalize the great number of observed particular cases and facts, as well as the ability to irrevocably and ruthlessly discard even the assumption or generalization, which seemed to be firmly established by successful explanations of particular facts, which turned out to be incompatible with new facts. Possessing this very valuable quality for a scientist, I. Pavlov developed this quality in his students as well. as well as the ability to irrevocably and ruthlessly reject even an assumption or generalization that seemed to have firmly established itself with successful explanations of particular facts, which turned out to be incompatible with new facts. Possessing this very valuable quality for a scientist, I. Pavlov developed this quality in his students as well. as well as the ability to irrevocably and ruthlessly reject even an assumption or generalization that seemed to have firmly established itself with successful explanations of particular facts, which turned out to be incompatible with new facts. Possessing this very valuable quality for a scientist, I. Pavlov developed this quality in his students as well.

Tasks

Having considered the following inductive inferences, determine the type of induction used in them, determine whether the conclusion is correct and, if it is erroneous, what the error made during the conclusion consists of:

1) "The development of heat is constantly observed on the axis of moving wheels of all kinds; therefore, the cause of developing heat is the transfer of motor energy into heat. "

2) "Various salts of radium, enclosed inside a thick lead shell, all the time give off heat, which is 135 calories per hour per gram of radium. Radium remains warmer all the time around. Since the observed heat must cause some kind of change, and since no chemical process occurs in the radium compounds, it should be assumed that the reason for the constant release of heat by the radium salts is a change in the radium atom itself."

3) "If a layer of yellow sand is scattered evenly across the red floor, and if this sand is sufficient so that the thickness of the layer is equal to at least the thickness of one grain, then the whole floor will appear yellow. But if there is half as much sand, then the red color of the floor will inevitably shine through; experience shows that in this case it is impossible to scatter sand with an even layer with a thickness of half grain. From this we conclude that a sudden change in the properties of the sand layer is caused by the granular structure of sand."

4) "Almost all absorption lines found in the spectrum of the solar atmosphere can be attributed to atoms known on earth; the same is true for the spectra of stellar atmospheres of all stars in the sky. Therefore, the entire universe is built only of those types of atoms that are found on earth."

5) "Short waves in the ocean are dangerous for small ships, longer ones for large ones; therefore, a wave with a very long wave does little harm to both."

6) "Some of the visitors to the dining room were poisoned; the study found that all the poisoned people ordered various dishes, but among these poisoned ones, ice cream was ordered; the same study found that not a single visitor to the canteen who ordered ice cream

was poisoned; hence, the investigators concluded that the cause of the poisoning was the poor quality of the ice cream.”

7) “There are only twelve constellations of the Zodiac: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces. Aries constellation is located in the ecliptic belt, Taurus constellation is also, Gemini constellation is also, Cancer constellation is also, Leo constellation is also, Libra constellation is also, Scorpius constellation is also, Sagittarius constellation is also, constellation Sagittarius is also, constellation is K too, the constellation of Aquarius — too, the constellation of Pisces — too. Consequently, all the constellations of the Zodiac are located in the ecliptic belt.”

8) “To determine the speed with which excitation propagates in the motor nerves, experiments are carried out on the frog. These experiments consist in the fact that the frog muscle is irritated by an electric current passed through the nerve. In one series of experiments, a nerve site close to a muscle is subjected to irritation, and in another, a distant place. Experiments find that the time interval between any of the stages of muscle contraction and the moment of irritation will be greater in the case when the nerve site being irritated is more distant from the muscle. Obviously, the effect of irritation on the intramuscular branching of a nerve occurs later when a distant place is exposed to irritation; however, it proceeds in exactly the same way as in the case of irritation of the near end.

9) “The muscle of an adult chicken under examination under a microscope turns out to be composed of bundles, bundles of filaments, filaments of very thin fibres, distinguishable only at high magnification. The younger the chicken, the thicker these elementary muscular fibres, while in the embryo at the age of the middle of incubation their diameter is even larger. These observations, which are true for muscles, are also true for other tissues and parts of the chicken body. It follows that all parts of the chicken’s body are rougher and less formed, the younger they are.”

CHAPTER XII. INDUCTION AND DEDUCTION

The Logical Basis and the Logical Formula of Conclusions about Probability

§ 1. The forms of inductive inferences considered in the previous chapter in some respects form groups of conclusions other than syllogistic conclusions.

This difference concerns, *firstly*, the problem of conclusions itself. The task of syllogistic conclusions is to establish the relationship or belonging of a property to an object or object to the class of objects. The task of inductive conclusions is usually to establish a causal relationship between the phenomenon and the circumstances preceding the phenomenon.

Secondly, the difference between inductive conclusions and syllogistic conclusions concerns the relationship between the generality of premises and the generality of conclusion. In syllogistic conclusions, the result of the conclusion is the receipt of a general or private judgment from other general or private judgments. Knowing, for example, that all objects of a known class have some property and that a given species belongs to this class, we conclude with certainty, according to the rule of the first syllogism figure, that all objects of this type will have the same property as all objects of this class. Here the conclusion goes from the general to the general subordinate to it, i.e. to the particular.

In inductive conclusions, the result of the conclusion is, on the contrary, the establishment, by examining individual cases, specially selected according to the rules of inductive methods, of a certain position that applies not only to the cases considered, but to the entire class of objects. In this sense, the inductive conclusion comes from particular cases to the general.

Thirdly, the difference between inductive conclusions and syllogistic conclusions consists *in the different reliability of* inductive and syllogistic conclusions. In syllogisms, provided that the premises are true and if the logical conclusion is correct, the conclusion will be *reliable* truth.

In inductive conclusions, on the contrary, the truth of the premises and the correct logical course of inferences cannot provide the conclusion with complete reliability. Here the conclusion is *just plausible*. And although in many cases this probability is so great that it almost approaches reliability, the fundamental difference between probability and reliability still remains and cannot be completely eliminated.

§ 2. The differences between inductive conclusions from syllogistic conclusions indicated here and, first of all, the difference between the course of the conclusion from particular to general and from general to particular form the basis for combining the entire group of syllogistic conclusions into a group of conclusions of the so—called *deduction* and for distinguishing *deductive* conclusions from *inductive*, or from *induction*.

In a broad sense, the word *deduction* means in logic any conclusion of some propositions from others, including the conclusion of only probable judgments. In a narrower sense, deduction logic calls all kinds of conclusions of reliable judgments from other reliable and, moreover, more general judgments.

On the contrary, *induction* refers to any conclusions of probable judgments from other less general reliable or probable judgments.

§ 3. The difference between induction and deduction remains valid where the conclusions are considered: 1) from the point of view of their *problem* , 2) from the point of view of *generality of the* conclusion of the conclusion compared with its premises and 3) from the point of view of the *reliability of the* conclusion.

But if conclusions are considered *from the point of view of their logical basis*, i.e., according to the logical type of inference that determines the transition from premises to conclusion, then the difference between deductive and inductive conclusions is far from unconditional.

Indeed, some types of syllogisms and some types of inductive inferences regarding the course of inferences are extremely close to each other. Such, for example, are the third figure of a simple categorical syllogism, the conclusions of complete induction and the conclusions of incomplete induction.

At first glance, it might seem that between the third figure, on the one hand, and the inductive conclusions of full and incomplete induction, on the other, there is all the difference that exists between the deductive conclusion, going from the general to the particular, and the inductive, coming from the particular to the general.

In fact, *according to the third figure,* conclusions are obtained, as you know, only private. This property of the third figure in Darapti and Felapton modes, where particular conclusions are obtained from *both general premises, is* especially striking . But in the other modes of the third figure, particular conclusions are obtained from premises, one of which is certainly common. On the contrary, in the conclusions of *complete* induction, single or partial premises justify the *general* conclusion. The same is true for the conclusions of *incomplete* induction: the conclusion established in them as a probable conclusion is a general conclusion: it applies not only to the cases considered and confirming the conclusion, but also to all cases not yet considered of the same kind.

If, however, we take a closer look at the logical basis of the conclusion in all these three forms of inference, then it turns out that the difference between them from the course of thought from the general to the particular and from the particular to the general does not reveal the essence of that train of thought, which in all these forms leads to the conclusion.

§ 4. Let's start with the third figure. The predicate in the conclusion of conclusions made by the third figure does not really apply to everyone, but only to a part of the objects of the class to which the subject of the conclusion belongs. In this sense, we have the right to say, as was said above, that the conclusions of the third figure correspond to the interest of cognition directed to the particular.

However, the matter is not limited to this. The conclusions of the third figure acquire their full meaning only on condition that we clarify the meaning of the question for which these conclusions are applied. As you know, these conclusions are often used to refute false judgments about a whole gender or class. Suppose a student claims that all arthropods are insects. To refute this statement, it is enough to prove the truth of the statement contradicting it. Such a contradictory statement would be the partial negative judgment "some arthropods are not insects." It can be proved by the third figure of the syllogism (Felapton modus):

Not a single spider is an insect.
All spiders are arthropods.
———
Some arthropods are not insects.

Of course, the conclusion was *private* . But the whole point of this conclusion, as was already shown above, is not at all just that it limits the subject of judgment to certain instances. The full meaning of this proposition is that it characterizes the *entire* class of arthropods as such a class, regarding which it is incorrect to say that all its representatives are insects. In other words, *by means of a conclusion that is private in terms of the number of conclusions, the conclusion on the third figure expresses a judgment not about a part of a group, but about a whole group of objects.*

It can be seen from this that the opposition of the third figure as a deductive inference to inductive conclusions is the only apparent opposition. What seems to be the opposite here — the train of thought in the case of the third figure from the general to the particular and the train of thought in the case of complete and incomplete induction from the particular to the general—is the opposite, which does not affect the *logical* the basics of the conclusion in both cases. The basis of this is not the ratio of the number of parcels to the number of conclusions, but the possibility of a transition from a judgment on certain subjects of a group to a judgment on an entire group of subjects. This possibility undoubtedly exists in the conclusions of complete and incomplete induction, differing only in the completeness of the cases on which the conclusion is based, and therefore in the degree of probability of the conclusion itself. But this possibility also exists in the conclusions of the third figure. In the syllogism:

All platypuses are mammals.
All platypuses are egg—laying.
Some ovipositors are mammals.

a conclusion is a statement or judgment about the *whole*

genus of mammals as such, regarding which it is incorrect to think that there are no oviparous in its volume. Here essentially the same transition of thought from species to class or genus is found in the conclusions of complete and incomplete induction. Platypuses are just one type of ovipositor. But since the entire volume of the platypus species is included as part of the volume of the mammalian genus, we have the right to transfer the predicate (belonging of the platypus to the egg—laying) to the mammalian genus (noting, of course, that this ability characterizes only part of the genus). But even being an incomplete, limited *part of the* representatives of the genus, the definition remains the definition of the *genus*, not a species. In other words, the conclusion here is to transfer the predicate from some species or even from one species to a genus.
It can be seen from this that deductive and inductive conclusions, being, to a certain extent, different and even opposite, are not opposite in the sense that is of the greatest importance for characterizing the logical originality of conclusions: in relation to the course of inference. Some types of deductive conclusions, such as, for example, the third figure of syllogism, are much closer to inductive conclusions (to conclusions of complete and incomplete induction) than to other forms of deductive (syllogistic) conclusions.

§ 5. This is the case when comparing deductive and inductive conclusions from the *point of view of a logical process* , or *the rationale for the* conclusion.

But the situation is not different if the question of the difference between deduction and induction is approached from the point of view of the relative probability (or reliability) of the conclusions obtained by induction and deduction.

It has already been shown that, provided that the premises are true and the conclusions are *correct* , deductive conclusions give *reliable*, and inductive—only *probable* knowledge.

This difference — where it takes place — should be recognized as significant. In its place, it was already explained that the reliability has no degrees, while the degree of probability can vary from a value close to complete unbelievability to a value approaching full reliability.

And yet, no matter how great the difference between probable and reliable knowledge, it cannot be the basis for the unconditional opposition of the induction of deduction.

§ 6. *Firstly*, there are forms of inductive inferences, through which not only probable, but also completely reliable conclusions are obtained. These are the conclusions of the *complete* induction. Given the truth of its private premises and subject to exhaustive consideration of all the specimens (or species) that make up the class with respect to which the generalization is made, the conclusion of complete induction is quite reliable. Since the conical sections are exhausted by a circle, an ellipse, a parabola, and a hyperbola, and since it is reliably known for each of them that it cannot be crossed by a straight line at more than two points, it will be no less reliable to conclude that all conical sections are not can intersect in a straight line at more than two points. And no matter how small the *novelty of the* conclusions obtained by complete induction, in terms of *reliability*, these conclusions are not inferior to the reliability of deductive conclusions.

§ 7. *Secondly*, even among those forms of induction by which unreliable, but only probable, conclusions can be obtained, there are forms whose conclusions regarding the degree of probability can infinitely approach reliability.

With the exception of induction through a simple enumeration, which gives conclusions based partly on cases confirming the conclusion, partly on the absence of cases contradicting it, other types of incomplete induction—

induction through the exclusion of random circumstances and Bacon induction—provide knowledge whose probability can increase to values close to reliability. Therefore, with regard to the likelihood (as well as the reliability) of conclusions, the opposite between induction and deduction is not unconditional. There is a type of induction by which reliable knowledge is obtained, as well as with the help of deduction. There are types of induction by which, due to the characteristics of the methods used, the probability of inference can be very high.

§ 8. Finally, and in the *third* respect, in respect of *goals* or *objectives of* inference—the opposite between induction and deduction also cannot be recognized as unconditional.

The same complete induction, which, as we already know, in terms of the reliability of its conclusions, should be put on a par with deductive conclusions, does not differ from them in the nature of its conclusions. Just like in syllogisms, the conclusions of complete induction usually represent judgments about whether a property belongs to an object or about whether an object belongs to a class.

§ 9. Until now, speaking of the absence of an absolute contradiction between deduction and induction, we have relied on those forms of induction that, in the course of inference, in terms of its probability and in its task should, like complete induction, be placed next to syllogistic, or deductive, conclusions.

But the same absence of the absolute opposite between deduction and induction can be proved in another way — by analysing those forms of inductive conclusions that, like Bacon’s induction, undoubtedly differ from syllogistic conclusions and in the *degree of probability of* conclusions that never reaches full certainty, and in their *the goal of* causation.

Indeed, the general outline of all Bacon's inductive methods is, as we have seen, the separation—categorical syllogism of the modus tollendo ponens.

Regardless of the particular inference method for each method, each Bacon induction method consists, from a logical point of view, in that, taking into account the totality of circumstances incompatible with each other, regarding which it is possible to think that each of them can be the cause of the phenomenon under study, they consistently exclude all of them that, as it turns out from the analysis, cannot be such a cause in this case. As a result, only one single circumstance turns out to be possible, which is the reason (or part of the reason) of the phenomenon. In the case of the *similarity* method, the circumstance that remains the same in all the cases under consideration remains unanswered, while all other circumstances turn out to be different in each case. With the *difference* method the circumstance by which this case differs from all other cases when the phenomenon occurs remains unexclaimed. With the method of *residues,* the circumstance that cannot be the cause of any component of a complex phenomenon, except for the one whose reason must be established, *remains* unexcluded. Finally, in the case of the method of *concomitant changes,* the circumstance that one changes in degree remains unexcluded, while all the others in all the cases studied are unchanged.

So, with all the undoubted difference that exists between deduction and induction, this difference is by no means the absolute opposite of mutually exclusive types of inference.

§ 10. But this is not enough. The absence of the absolute opposite between deduction and induction consists not only in the fact that in a series of deductive and inductive conclusions the course of inference, with the apparent difference, turns out to be essentially the same. The absence of the absolute opposite between deduction and induction is also reflected in the fact that, even if different, induction and deduction *supplement each other and suggest each other* in many types of scientific research.

Usually, scientific research is a difficult task, the solution of which can only be achieved by the combined use of deduction and induction. Even with conclusions that often seem to be inductive, thinking always relies on deduction as well. So, in order to begin to study the causes of the phenomenon using one of the methods of Bacon—age induction, it is necessary to assume that this phenomenon is a special case or a particular manifestation of the universal law of causation. But this proposition is the conclusion of a deductive—syllogistic—conclusion.

§ 11. Even in cases where the inductive conclusion precedes the deductive proof, the final reliability of the conclusion is achieved not by induction, but by deduction. It is known from the history of sciences that even in the proofs of mathematical theorems induction was used. Some and, moreover, very important theorems of number theory, for example, Fermat’s little theorem [1], were first found by induction. By induction, the area of the parabola was found by Archimedes: Archimedes took sheets of tin of the same thickness, cut pieces of parabolic shape from them, and then weighed them. And only after the formula for the parabola area was found by induction, it was possible to deduce the same formula in a deductive way.

However, these theorems did not acquire the significance of universal truths, not on the basis of the initial inductions by which they were found, but on the basis of *deductive proof*. Only it turned out to be able to raise these positions from the stage of probable or fair for only some cases provisions to the level of truths, quite reliable and strictly proved.

§ 12. On the contrary, in cases where the mathematical generalization does not go beyond incomplete induction, it can always be, like any conclusion of incomplete induction, refuted by the first fact that contradicts the generalization. The same Fermat expressed, on the basis of induction, the assumption that all numbers of the form (2 ^ 2 ^ n) +1 are prime numbers, that is, numbers that are divisible only by themselves and by one. At the same time, he relied on a sequential series of four cases, or examples, which all yielded a result, generalized by Fermat in his formula. And indeed: $2^{2}+1 = 5$; $2^{4}+1 = 17$; $2^{8}+1 = 257$; $2^{16}+1 = 65\,537$, i.e., all four cases considered and forming a sequential series result in prime numbers and, therefore, confirm the formula. But as soon as Euler calculated the result for the next fifth case ($2^{32}+1$) and showed that this number—4 294 967 297—is divisible by 641, Fermat's assumption, found by incomplete induction, was refuted, since the case was discovered contrary to generalization.

§ 13. But also deductive research cannot do without induction. Induction not only leads to initial conjectures about the general rules and laws, which are subsequently justified by deduction. Induction leads to the formation of those *concepts* and *definitions* which form the basis and starting point of the deductive sciences and their deductive conclusions. True, in their current form, these concepts, definitions, axioms or postulates may seem completely independent of any experience or of any induction. The concept of a geometry about a point, about a straight line, about a plane, about parallel ones, etc., may seem to exist only in the geometrical thought, but not in reality itself. In fact, every straight line has not only length, but also width and height. In the thought of a geometric line is only the length. In fact, every point is a very small *body* , that is, just like a straight line, it has both length, width, and height. In the mind of a geometer, a point has neither length, nor width, nor height, etc.

And yet, no matter how significantly the concepts and definitions of mathematics differ from real objects and the relations of these objects in the real world, these concepts and definitions once arose on the basis of experience and generalizations derived from experience. Of course, the concept of a geometry about a straight line is not only the concept of the limit to which the straight line drawn on ink draws as its width and height become smaller and smaller in the hands of a skilled draftsman. There is a difference between the “thinnest” and “lowest” straight lines drawn in the drawing and a straight line, conceivable by a geometer, that is, having *only one length*, which will not be filled by any possible applications and transitions in the experiment. Here, the thought makes a transition, as a result of which something new appears, which cannot be deduced from any induction.

But if the geometer did not rely on numerous observations that show that it is possible, without changing the length of the drawn line, to change, namely reduce, its thickness and height, if, in addition, he would not have to ask a number of questions regarding the line to solve of which neither height nor width is important, but only its length alone, then the geometer would never be able to form in his mind and with the help of his imagination the concept of a straight line as a line having only one length. Induction cannot, without the aid of deduction, prove a single proposition as an absolutely *reliable* proposition, but the very concepts underlying all the judgments of the deductive sciences are formed from experience and through inductive generalizations.

§ 14. The interrelation of induction and deduction clearly appears in complex scientific research. These studies rarely begin with the precise formulation of the law. Usually, the exact formulation of the general law is preceded by an approximate, often crude and very inaccurate test of such a formulation, based on very still imperfect inductions, or conclusions from particular cases. But even at this stage, anticipation of the general formula and deductive conclusions from it play a large role, which point the way for further research. Taking his approximate generalizations as truth, the researcher extracts by *deduction* conclusions about how "the general laws he proposed should appear in other cases, beyond what is already known from experience. Having obtained these conclusions, the researcher again turns to experiment to check to what extent the consequences, deduced by him deductively from the assumptions made by induction, are consistent with real facts.

Before Galileo, for example, physicists, noting that water rises in a pump, explained this phenomenon by the fact that nature is supposedly afraid of emptiness: as air is pumped out by a pump, water becomes a place of air.

Galileo already knew from experience with the glass tube of the suction pump that no matter how long the water was pumped and how long the pump tube was, the water raised by the pump never rose above 32 feet. This fact established by observation inspired Galileo with a hunch that the "fear of emptiness" is not unlimited, but has a limit. Galileo's student Torricelli completely abandoned the assumption that nature is afraid of emptiness. According to him, the limit of water rise in the pump is due to the fact that the earth's atmosphere, having a limited height above the ground and therefore limited severity, presses the water in the pump tube. The weight of water raised to a height of 32 feet is exactly the weight of an atmospheric column above the surface of the water in the vessel from which water is pumped into the pump. From this conjecture, Torricelli made a *deductive* conclusion. If the weight of the liquid raised in the pump must exactly equal the weight of the atmospheric column above the surface of the liquid, then the height by which the liquid rises in each individual case will obviously depend on the specific gravity of the liquid taken for testing. So, for example, mercury, which is almost 14 times heavier than water, will obviously rise not 32 feet, but only 1/14 of that height, i.e. 30 inches, since a column of mercury 30 inches tall weighs so much how many pillar, 32 feet water. Indeed, the experiments made by Torricelli showed that the deductive conclusion made by him from Galileo's assumption was fully justified: mercury rose no higher than 30 inches.

Nevertheless, this coincidence of the results of the experiment with the deductively derived consequence of the theory was not conclusively convincing in the eyes of many. This coincidence could be accidental, and the rise of water and mercury in the pump to an unequal height could be explained by the action of a specific cause in each of both cases.

To eliminate all doubts about the truth of Torricelli's guesses, Descartes came up with, and Pascal and his son—in—law Perrier carried out a new experiment. From the guesswork of Torricelli, Descartes drew a deductive conclusion, the verification of which was to bring a real solution to the question. It was necessary, Descartes reasoned, to set up such an experiment that would leave no doubt that it is the pressure of the atmospheric column above the liquid level in the vessel that determines the limit to which the liquid in the pump can be raised. If it were possible to show that with a change in the weight of the air column above the liquid level the column height of the same liquid raised in the pump would also change, then Torricelli's conjecture would thereby be proved. But the weight of the air column, Descartes continued to argue, depends on the height of a given area above sea level. *part of* this pillar. Therefore, at the top of a high mountain, the level of liquid raised in the pump will be lower than the level of the same liquid in the same pump at the bottom of the mountain: the weight of the air column at the top of the mountain will be balanced by the smaller liquid column in the pump.

All of Descartes's arguments presented a series of deductive conclusions from Torricelli's hunch. It was necessary to check how real facts are consistent with these conclusions. This check was made by Perrier.

The presented history of the development of the theory of the barometer is an excellent example of the mutual connection of induction and deduction. From the generalizations found by induction, usually still imperfect and inaccurate—through the consequences of these generalizations, deduced by deduction—to checking these consequences through new experiments and new inductions, this is the usual way of scientific research.

Estimation of the Probability of Inductive Inferences

§ 15. From a comparison of inductive conclusions with deductive ones, it was deduced that, in addition to complete induction, which gives *reliable* conclusions, all other types of induction give *probable* conclusions.

This difference, taken by itself, does not, however, solve the question of the comparative scientific value of deductive and inductive conclusions. True, reliability always remains above probability. However, the probability may have varying degrees. Under certain conditions, the degree of probability can increase so much that in practice the probability can infinitely approach reliability.

Since inductive conclusions give, generally speaking, probable knowledge, the scientific significance of these conclusions will obviously be determined by the degree of probability attainable for them in each individual case and in each type of induction.

It follows that in assessing the scientific value of induction, it is necessary to get acquainted, *firstly*, with the method by which the degree of probability can be determined, *and secondly* , with special methods by which the degree of probability is determined in the case of *inductive conclusions* .

§ 16. Above, we have already examined the basic method of calculating the probability and improbability of an event. But since the mathematical calculus of probability, the reception of which is indicated, should obviously have a *logical* basis and be based on a *logical* formula, the mathematical formulas being applied to a particular field, this logical basis and this logical formula must also be established. The latter is also necessary because in some cases the probability cannot be accurately calculated mathematically, but nevertheless it can be characterized with a certainty sufficient to weigh the comparative value of one or another possibility, between which the solution to the question is distributed.

§ 17. From a logical point of view, the conclusion about probability has the premise of a judgment about a certain group of objects. And indeed, this conclusion should contain a complete indication of all possible cases between which the trial is distributed. If there are eight red and four blue balls mixed with each other in a closed box, and if the question is what color the ball will be, which we will take out of the box, then it is obvious, firstly, that only red or blue ball can be taken out . Therefore, the first approximation to the solution of the question will be the judgment: “A ball drawn out can be either red or blue.” Judgment is a separation judgment, listing all mutually exclusive possibilities between which choice is distributed.

However, to confine ourselves to this judgment alone in this case, when we know not only what colours can be found among the balls placed in the box, but we know, besides, how many red and how many blue balls are in the box, it would mean not to bring research up to the certainty possible under the given conditions.

It is true, of course, that in order to answer the question posed, we must form a *separative*, rather than any other opinion. If the proposition expressing the degree of our knowledge of which ball will be pulled out were not dividing, then our conclusion would not indicate that the whole group of objects has not the same, but different predicates, i.e., that it has some set of predicates between which all possible cases are distributed.

But, on the other hand, one dividing proposition, establishing that a ball drawn out may turn out to be either red or blue, will, of course, not be enough. This proposition *precisely lists* possible predicates in this case, that is, existing in the group. However, it still says nothing about *what meaning* has each of the predicates compared with others in the same group. In order to highlight this side of the issue, it is necessary to transform our separative judgment about the group so that it would be possible not only to transfer the predicate indicated by each member of the separative judgment to the subject in question (i.e., to the ball that should be pulled out), but, in addition, so that the separation judgment itself accurately expresses our knowledge of the comparative significance of each predicate for the whole group.

Taking this requirement into account, we will now mentally divide the entire number of balls in the box into groups of four balls in each, and in such a way that in each of the groups resulting from the division, the balls of the same color appear. You will get two groups of red balls and one group of blue. We call one of the four red balls “the first group” of red balls, the other — the “second”. Then, obviously, we have the right to make a judgment: “Any ball that can be taken out of the total number of balls in the box must belong either to the first group of red balls, or to the second group of red balls, or to the group of blue balls.”

This proposition, like the previous one (“A drawn ball can be either red or blue”), is a separative proposition about a group of objects. There are *three in it* predicates, which completely exhaust all our knowledge of the group and therefore are equal in rights.

Having formed this judgment, we can now transfer the definition of the whole group, expressed by the transformed separation premise, to the ball that should be taken out. And in the transformed form, as well as before the transformation, our dividing judgment expresses that the ball drawn out will turn out to be either red or blue. Both first groups, or fours (red balls), express the first possibility, the third group, or four (blue balls), expresses the second. The statement that the ball turns out to be red will be justified if each of the first two members of the dividing judgment that we converted is realized when the ball is delivered. In other words, this statement expresses the chances of the third member of our separation judgment. And since the rights of each case represented by four balls of the same color are equal, the probability that the first proposition turns out to be true (“the red ball will be drawn out”) refers to the probable truth of the second judgment (“the blue ball will be taken out”) as two refers to one.

Now it is easy to characterize the logical course of the considered conclusion about probability. This conclusion—

from the point of view of its *logical* type or character—is nothing more than an *inference from a group of objects to a separate object* . Moreover, the judgment about the group, justifying the transfer of the predicate to a separate subject, is a complex dividing judgment about the composition of the group. This proposition not only exhausts all the predicates existing in it, but also characterizes the comparative significance of each of them in the group.

The logical formula of the mathematical conclusions about probability described here is a formula that covers only the simplest conclusions of mathematical probability. When the conditions for determining probability are complicated, the logical formula for the conclusions of probability, without changing in essence, undergoes a corresponding complication.

§ 18. However, there are also conclusions about probability in which the course of the conclusion coincides with the course of the conclusions of incomplete induction. Let us imagine, for example, the case when, getting balls of different colours put into it from a closed box, we do not know in advance either what color the balls are in the box, nor how many balls of each color are in the box. Imagine that the question is no longer about what color the drawn ball will turn out to be, but about what color is dominant in the entire given group of balls and how does the number of balls of one color relate to the number of balls of all other colours.

The task set in this way is clearly different from the previous one. In the previous one, we knew *in* advance, *firstly, the* total number of balls in the box, *and secondly*, it was known how many of this total number of balls there are red balls and how many blue ones. The question was to determine the degree of probability of both the fact that the first ball drawn out would turn out to be red, and that it would turn out to be blue.

On the contrary, the second task is *inverse* to the first. Here, neither the total number of balls in the box, nor the distribution of this number between groups by color is known. It is required to determine what color of balls will be the most in the group and in what respect the number of these balls will be among the balls of all other colours.

The first task was solved, as we saw, by calculating probability, based on a separation judgment, accurately expressing all our knowledge about a group, and on transferring the definition of a group expressed by a separation judgment to a separate subject.

In the second task, we obviously cannot *immediately* formulate, as it was in the previous case, a separation judgment that would accurately express our knowledge of a group of objects. However, in this case, an *approximation* to such knowledge is possible . To do this, let us take out the balls one after the other from the box so that the conditions of each individual access are as diverse as possible, that is, each time we take the ball out of different parts of the box.

If the conditions for getting balls are varied enough, then by arranging the balls into groups so that each group includes balls of the same color, and determining both the total number of balls already drawn and the number of balls of each color, we can to answer not only the question of what color of balls is most in the box, but also the question of in what relation the number of balls of each color is among the balls of all other colours.

As soon as the number of balls drawn in such a way from the box and distributed by colours is large enough, we get the right to inference, which, if we consider its *logical* basis, turns out to be the *conclusion of incomplete induction by eliminating random circumstances*.

In fact, under the indicated conditions, we are dealing, as in the conclusions of incomplete induction, with a certain group of objects (namely, balls in a box), the number of which, although it is not known, but quite definitely, which are concentrated in a strictly defined and accessible to experience field and which can be removed, generally speaking, in conditions that exclude random circumstances.

When all these requirements are met *and with a sufficient number of seizures (in relation to the total number of balls),* we get the right to look at the balls pulled out and distributed in color no longer as random specimens of the group.

We get the right to see objects in them, the ratio of the number of which in each group of one color to the number of them in groups of other colours is indicative not only for that part of the balls that turned out to be covered by the test.

We have the right to believe that the same ratio expresses the comparative numbers of all groups within the total number of balls in the box and not yet fully covered by the test.

§ 19. This conclusion is a conclusion that gives only probable, but not unconditionally reliable knowledge. Its probability depends, *firstly,* from the thoroughness with which random circumstances were eliminated, *and secondly,* from the ratio of the number of already completed deliveries to the total number of balls in the box. The probability of withdrawal, which is insignificant with a small number of deliveries, approaches the reliability as the number of balls remaining in the box and not yet covered by the test decreases.

As in all the conclusions of incomplete induction, the conclusion is that the properties and relations of a certain part of the group, established by the experiments performed, which obviously do not exhaust all the objects of the group, are transferred to the whole group. The reason for the transfer here is, as in the other conclusions of incomplete induction, the exclusion of random circumstances affecting the conclusion. As a result of this exception, the right arises to consider the examined part of the group not as composed of random specimens, but as such a part, the properties and relations of which characterize the properties and relations of the whole group.

§ 20. The inference course used here essentially does not change when the task changes. Suppose that the total number of balls in a box has become known to us; Suppose we also know what colours the balls in the box can be. Under these conditions, the very question regarding the possibilities offered here should change. It will not be a question of what color of balls is most in a box, but a question of how many balls of each color are available. Until we knew the total number of all balls and the total number of colours, only the question of what predicates a given group of objects should be characterized and what is the significance of each of them in a group could be solved. This question, by its very meaning, is vague.

On the contrary, now that the total number of balls and the number of colours in which they are painted, it is known, another question can be solved — the relative probability of several, this time already quite definite, assumptions. Indeed, since the total number of balls in the box, as well as the number of colours, we know, we can form a few assumptions regarding the number of balls of each color. The logical form by which these assumptions are expressed will be a *dividing judgment on a group of objects* , indicating several possible, under given conditions, solutions to the question posed.

This difference in the conditions of the problem leads not only to a change in the question that is to be investigated. It also leads to a change in the role that the process of getting balls from the box plays in the entire test.

While the number of balls in the box and the number of colours were unknown, the sequential retrieval of balls from the box was a means for inductively deducing which predicates belong to this group and what significance each of them in the group has. In this case, the very conclusion is to transfer the predicate from individual group objects to the entire group.

On the contrary, as soon as the number of balls in the box and the number of colours in which the balls are painted becomes known, along with the change in the formulation of the question, the value of the process of getting the balls changes. From the means for establishing the inductive derivation, the process of getting the balls becomes the basis for transferring the differently coloured balls, which follows from the assumption regarding their distribution, to the result that is observed when the balls are delivered.

In this case, however, the very process of inference does not lose the character of *inductive* inferences. Indeed, making a number of assumptions regarding the possible conditions for the distribution of balls in the box is necessary only where the number of deliveries was too small relative to the total number of balls in the box and where, therefore, possible randomness of getting could remain unresolved.

But if there is reason to believe that the conditions for taking out the balls were quite diverse, so that the accidents were eliminated, if the number of deliveries was sufficiently large with respect to the total number of balls, then the inference process remains the same as in the problem with the previous conditions. This move consists in transferring the comparative value of the predicates (in this case, various colours) established for the observed balls, i.e., only for part of their total number, to the *whole* set of balls in a box. This transfer is possible without drawing up special assumptions about possible cases of distribution of balls in a box and without determining the probability of these cases. It is possible, since the multiplicity of the cases of getting and their conditions, eliminating the influence of randomness on the conclusion of the conclusion, constitute a sufficient basis for convincing that, for example, the prevalence of some particular color among the extracted balls is not due to accidents that favoured the delivery of the balls of this particular color, but only to the character the group itself or the value that this color has in the group.

Thus, when deciding on the comparative number of subjects of each group and which group prevails, as well as when deciding on the predicate of the subject to be taken out, the course of the conclusion, despite all the changes resulting from changes in conditions tasks, it remains one and the same. This is an inductive inference through the exclusion of random circumstances.

§ 21. We became acquainted with the logical structure and with the logical basis of conclusions about probability. We did not find in them any forms of inference that would give the basis to single out conclusions about the probability from the group of conclusions of incomplete induction already known to us. Regarding the conclusions of *mathematical* probabilities, then they turned out to be conclusions that fit the signs of the conclusions already known to us, consisting in transferring the complex definition of a group to a separate subject.

Now we can get to the question of how the probability of conclusions of Bacon induction is determined. It is easy to verify that the methods for this determination, generally speaking, will not differ from the methods for determining the probability of inductive inferences of other types.

Indeed, in determining the scientific evidence of Bacon induction methods, two questions must be solved, as in the case of other inductive conclusions: 1) how much the applied method in its very logical form contributes to the elimination of randomness and 2) is it possible to repeat the experiment so often in the conditions of this study and numerous, so that this frequency, combined with a variety of conditions excluding randomness, increases the probability of a conclusion.

§ 22. We consider from this point of view the *similarity* method. We already know that according to the scheme of this method, cases are compared, characterized by the fact that 1) in all these cases the phenomenon, the cause of which must be established, always occurs; 2) all the circumstances preceding the onset of the phenomenon are different in each case, except for one unique one, which in all cases remains the same.

It is quite obvious that the probability of a conclusion obtained by this method depends, firstly, on how diverse and numerous circumstances are, different in all cases. Compare two examples of applying the similarity method:

Cases		Circumstances preceding the phenomenon	The phenomenon whose cause must be established
Scheme 1	1st	ABC	*a*
example	2nd	ADE	*a*
Conclusion: circumstance A is the cause (or part of the reason) of the phenomenon *a* .			

Cases		Circumstances preceding the phenomenon	The phenomenon whose cause must be established
Scheme 2	1st	ABC	*a*
example	2nd	ADE	*a*
	3rd	AFG	*a*

	4th	THERE	*a*
	5th	AKL	*a*
	6th	AMN	*a*
Conclusion: circumstance A is the cause (or part of the reason) of the phenomenon *a* .			

It is easy to verify that the probability of a conclusion in the second example is higher than in the first. In the first example, as well as in the second, the conclusion consists in the exclusion of all circumstances that cannot be recognized as a possible cause of the phenomenon *a* . But the basis for such an exception in the second example is more compelling.

In fact, in the first example, the conclusion was made based on an analysis of only *two* cases. As a result of this limited number of cases, the variety of circumstances in which the first case differs from the second (B, C, D, E) is much less in the first example than in the second, where the conclusion is drawn from an analysis of six cases and where the circumstances by which each case differs from all others, much more (B, C, D, E, F, G, H, I, K, L, M, N).

And in the first and second examples, the possibility that the cause of the phenomenon *a* will not be circumstance A, the only similar in all cases, but in each of these cases any other circumstance , is not excluded . However, to assume that in each case the cause of the phenomenon *a* is not circumstance A, but some other circumstance, in the second example is much more difficult than in the first.

Already in the first example, the assumption that the cause of *a* is in the first case, B, and in the second D, is much less likely than the assumption that such a cause is A. If A is not the cause of *a* , then in the first and second case A preceded by *a* by chance. But to assume this means to admit, as if accidentally preceding the occurrence of *a* circumstance, And just as often precedes him, as often his real reasons—B and D, combined together.

It is unlikely in the first example, where there are only two cases, and four different circumstances for each case, this assumption seems even less likely in the second example, where there are already six cases, and twelve different circumstances. Assume under these conditions, though in all six cases, the circumstance A precedes the phenomenon *and* quite by accident and, moreover, as often as often precede it all possible reasons, taken together, then obviously go in defiance of probability.

The increase in the number of cases under consideration raises the possibility of withdrawal, not only because it makes improbable haphazard appearance And every time there is a *well*. The probability of a conclusion pointing to A as the cause of the phenomenon *a*, increases also because with an increase in the number of cases, as well as with an increase in the variety of previous circumstances, an explanation of the appearance of *a* from a plurality of reasons becomes less and less likely . While there were only two cases, the supposition that the cause of the phenomenon *as* in the first case there is a circumstance B, and D in the second, taken by itself, it does not contain anything surprising or impossible Nogo. But if there are six cases, as in our second example, and if all the circumstances of each case are completely different; except for A, in which only a randomly preceding circumstance is assumed, then under these conditions it is assumed that in each of the six cases the phenomenon *a* each time it is caused by some new, different from all other reason, it is possible only with a big stretch. The larger the number of cases and the more varied the previous circumstances, the less likely such an assumption, the more evidence in favour of the idea that the cause of the phenomenon *a* should not be seen in numerous, from case to case, changing circumstances B, C, D, E, F, G and so on. d., and in the circumstance and that one thing was evident in all cases when there is a *well*.

§ 23. The significance which, in order to substantiate the conclusion, has the variety of numerous circumstances of each case with the constancy and similarity of one single circumstance, repeated in all cases of the occurrence of a phenomenon, is clearly outlined in the conclusions *by the method of concomitant changes*. The more varied in each case the circumstances that remain unchanged, the higher the probability of a conclusion according to which the cause of the changes in the intensity of the phenomenon is not the circumstances that in all cases remained unchanged, but the circumstance A, which in each case turned out to be changed. The more diverse the pendulums having the same rod length are made in each individual case, the more probable is the conclusion that the reason for the equality of the oscillation period observed in all cases is not in the substance of the pendulums, but only in the same length of their rods.

§ 24. The findings of the *sole distinction* method the method scheme itself reduces the possible influence of randomness on the conclusion of the conclusion. In the conclusions of this method, the possibility of a conclusion based on a possible multiplicity of reasons is reduced. Since in the absence of A the phenomenon *a was* also absent, and with the introduction of A, on the contrary, it immediately appeared, and since all other circumstances were the same in the case when *a* occurred, and when it did not, then I suppose one of these similar circumstances, the cause *and* is obviously impossible. Here, in an extreme case, it is only possible to assume that A is not the whole cause of the phenomenon of *a*, but only one of the conditions for the full cause of this phenomenon:

Cases	Circumstances prior to phenomenon *a*	The phenomenon whose cause must be established
1st	BC	*a*
2nd	ABC	*a*
Conclusion: circumstance A is the cause (or part of the reason) of the phenomenon *a* .		

Indeed, it is possible to assume that the cause of phenomenon *a* is not only circumstance A, but connection A, for example, with B. And with this assumption it is clear why in the first case *a* did not occur: that part A of cause AB was absent, without which the totality of conditions cannot be complete. The possibility of a complex composition of the causes of the phenomenon under study is constantly available in science. Usually, an action occurs as a result of not just one, but a whole sum of circumstances, since only the presence of all these circumstances makes the beginning of the action possible.

§ 25. There may be cases when the cause also includes such a circumstance or such an element of an event that, without *directly* causing no changes in the investigated phenomenon, nevertheless, must be present in order for these changes to occur.

In this sense, for example, the so—called *enzymes* belong to the composition of cricine. This name denotes substances that themselves do not directly participate in important processes and reactions for the body, but without which these processes cannot take place. So, mustard seeds could not be the cause of pungent smell and taste if they did not have the enzyme *myrosin* . This enzyme, with the assistance of water, decomposes the salt of myrononic acid located in these seeds and releases acute volatile mustard oil from it.

§ 26. Since in the conclusions by the method of the only difference the phenomenon occurs only in one of the two cases compared, namely, when, in addition to all other circumstances, the case also includes circumstance A, the exclusion of all other circumstances as incapable of being the cause *and* in these conclusions much more reasonable than in the conclusions of the similarity method. With the similarity method, the possibility of a multiplicity of reasons is so great that where the number of cases to be compared is small, one always has to reckon with it.

In contrast, with the difference method, all other circumstances, except A, immediately disappear at the very beginning of the study. All things being equal, each of these circumstances (B, C, D, E) may, in extreme cases, be *incomplete* cause (as is always possible with the similarity method), but *only part of the* full reason. *In any* case, its other part will always be A.

Uncertainty in the scientific value of the conclusions obtained by the method of distinction consists *in the inconclusiveness of the answer achieved through it to the question of causation.* That A should be at least part of the cause *a* — this method of distinction confirms us with complete certainty. But this method leaves *two* questions open . The first of them, as we have just seen, is the question of whether A is *only a part of the* full reason *a*. The second question, which remains open at conclusions on the method of differences, there is the question of whether A (if the cause can not be recognized by any one of the other circumstances) cause *as a whole* , in *all* its structure, or a reason to be any *part* or *any parts of A is* recognized *:* α, β, γ, δ, etc.

If it turned out that the cause *a* is not the *entire* composition of A, but only any *parts of* this composition, then the initial conclusion, It was held in recognition of the cause *and* circumstances A may be only preliminary. In this case, the conclusion, however, outlines the field of facts and circumstances, among which we must look for the cause *a* , but does not give an exact answer to the question for which it was intended. This answer can only give further research. In the course of this study, even the possibility that *a*it turns out that not only α, or β, or γ, but in one case one of them, in the other — the other, in the third — the third, etc. In other words, in the case of the difference method, the exclusion of the multiplicity of reasons that distinguishes the difference method from the method of similarity, is not yet unconditional. The area within which a plurality of causes may manifest itself, in the case of the difference method, narrows significantly. It is limited to those circumstances from which the complex composition of circumstance A is composed. But also limited, the multiplicity of reasons remains possible in this case as well.

This makes it clear why, despite the higher probability of conclusions by the method of difference compared to conclusions by the method of similarity, the method of difference still provides only probable, but not unconditionally reliable knowledge.

§ 27. The degree of probability of inference is further enhanced when the similarity method is combined with the difference method. Already in a separate application, the similarity method gives a probable conclusion that the cause of the phenomenon *a* is circumstance A. However, it is not excluded, as we know, the possibility that A is only present in all cases of the occurrence of *a* and that the cause *a* — B , or C, or D, or E.

But if, having shown by the method of similarity the probability that the cause of phenomenon *a* is circumstance A, then we will show — by the method of difference, that in the absence of A, the phenomenon *a* does not come, we will obviously make our conclusion even more likely. Indeed, with such a combination of both methods, we not only see that phenomenon *a always* occurs *in all* cases when circumstance A exists, but at the same time we see that phenomenon *a* does not occur *in any* case when A is absent.

Previous circumstances		The phenomenon whose cause must be established
ABC		*a*
ADE		*a*
BC		
OF		

With this connection, both methods the probability that the cause *and* would not A but, for example, B, or C, or D, or E— much smaller than in the derivation of only one method of similarity. Here the possibility is already excluded that the full cause of *a* can be B, C, D, E. Since in the presence of BC (as well as in the presence of DE) the phenomenon *a* did not occur, then B, C, D, and E can, in extreme cases, be each only part of the composite reason, the other part of which in any case will be necessary A.

Finally, the likelihood of deriving similarity and difference by the combined method becomes even higher when the number of cases of applying the similarity method and cases of applying the difference method combined together into one composite method.

CHAPTER XIII. HYPOTHETICAL CONCLUSIONS, OR HYPOTHESES. INFERENCES BY ANALOGY

Constructing Hypotheses and Turning Them Into Reliable Truth

§ 1. Considering the logical basis of the conclusions about probability, we found that one of the possible examples of the application of these conclusions is the case when the question of the relative probability of several assumptions is resolved. If from the conditions of the problem the total number of balls in the box and the number of colours in which they are painted are known, and if the question of the comparative number of balls of each color in the box is to be decided, then the logical type of inference will depend on how large the number is deliveries in relation to the total amount of balls. If this number is so large that the influence of randomness in all cases of getting can be considered insignificant, then the conclusion, as we already know, has essentially the same structure as the conclusions of incomplete induction.

But if the number of deliveries was too small in relation to the total number of balls, and therefore the randomness of getting could remain unresolved, then to solve the problem it is necessary to make several certain assumptions, and then establish or calculate the probability of each of them individually.

So there are conclusions of a special structure, called *hypothetical* . When applied to the special tasks of individual sciences, these conclusions are called *hypotheses*.

§ 2. In common use, the term "hypothesis" has several meanings. "Hypothesis" is called: 1) a simple guess; 2) an assumption about the cause of a known set of phenomena that is currently unavailable to detection, but unavailable only due to random circumstances, so this reason can be detected at any time and can become an object of observation; 3) the assumption of the existence, at present or in the past, of such a regular order or reason that, given a given state of science or due to their cessation in the past, cannot be directly observed, but which, once we assume their existence, explain a certain the totality of phenomena observed in reality or well known from history.

In science, the last meaning is accepted for the word "hypothesis", and therefore in logic only assumptions of this third kind are considered. We will call them further *hypotheses*, or *hypothetical conclusions*.

In the natural and historical sciences, as well as in other sciences, the hypothesis is often featured prominently, with the hypothesis put forward not only in those parts of the natural sciences, which study the current state of nature, but also in those, the subject of study which is the *development* : the development of space life the development of our planet, the development of organic life on it, as well as social life.

Thus, the general correspondence of the coastal outlines and the similarity of the geological structure of the continents, which are now divided by wide oceans, inspired some geologists (Wegener, Keppen, etc.) to think that these continents once made up a single, closed mass. Separated from each other, they gradually took their present position, so that the oceans separating them show the distance that the torn parts of the once much more vast masses of land swam.

This theory is a hypothesis. Her proposed explanation of the geological similarity of the currently separated oceans of the continents, as well as the correspondence of their external outlines of the proposed line of discontinuity, cannot be directly observed—both by remoteness in time preceding the appearance of a person on earth, and by the inability to directly verify the existence of those movements continents, which were, according to this theory, to divide once solid land masses.

This theory is hypothesized by the complete impossibility of directly checking—In the *current* state of science— the *main* assumption of Wegener's theory. The question of whether continents that seem to be motionless, floating islands—like icebergs floating in the Arctic Ocean—cannot be resolved until science has a number of geodetic observations that are accurate enough to eliminate possible observation errors. Wegener's theory is hypothesized by its ability to explain the established, but still unexplained anomaly in the distribution of plants on the Earth: the modern distribution of plants on the surface of the Earth apparently requires communication in the distant past between those areas of the land that are currently separated by thousands kilometres of the ocean.

In the historical sciences, a number of issues also have to be hypothesized. For example, modern linguistics, which studies the Indo—European group of languages, has established that only a part of the eight cases of the ancient Indo—European declension has survived in various languages of this group; all forms of cases of a specific meaning—instructive, local, depositional—disappeared and only *grammatical* forms were preserved cases—nominative, vocal, accusative, genitive and dative. Moreover, in different languages these losses turned out to be different: while neither Homer nor any of the dialects of the ancient Greek language preserved the sixth case, the Armenian, Lithuanian and Slavic languages and now have a rich form of declension. In these languages, cases of a specific meaning are well preserved: for example, Lithuanian, Polish, Ukrainian, and also modern East

Armenian languages distinguish seven cases out of eight, known in the ancient Indo—European language, and in East Armenian the cases are still used now and now are the deposits, local and instrumental, which Greek does not know already in the most ancient period of its history.

These facts, well studied and established, require explanation in the history of the Indo—European language. But such an explanation can only be hypothetical. None of modern scholars can directly observe the reason that in the remote times of the existence of the Indo—European language could produce this unevenness in the loss of ancient forms of declension. Unavailable to direct observation, this reason is indicated hypothetically by linguistics. Namely: this unevenness is explained by the influence of the population, with whom the immigrants who spoke the Indo—European language mixed, settled on Greek soil. This assumption is supported by facts, according to which in all cases when the declension met conditions favourable for conservation, it turned out to be represented by a large number of cases.

§ 3. In the previous paragraph, we examined examples clarifying the *function of a* hypothesis in scientific thinking. But logic cannot be satisfied with a single *description of the* hypothesis. Logic should find out the *logical* nature of scientific constructions called hypotheses.

In many manuals of logic, the question of a hypothesis is posed in the section on research *methods* . The basis for this premise of the hypothesis in the section on the method is the function of the hypothesis, as well as the complexity of its logical structure.

But if the question of a hypothesis is approached from the point of view of the *logical type* to which the forms of thinking called hypotheses belong, then the hypothesis, like induction, should be attributed to *inferences*.

Namely: a hypothesis is an inference, or conclusion, that a well—known set of phenomena, the thought of which forms a predicate of judgment, can be explained as a result of some directly observable regular order. The thought of this regular order should become the *subject of a* judgment formulating the main hypothesis of the hypothesis.

The general scheme of hypothetical inference will be as follows:

We have a predicate R. This predicate represents a certain set of phenomena, the cause of which, or the regular order that determines it, is yet to be explained. The thought of this regular order, or of this reason, will constitute the subject of judgment. Since this subject has not yet been found, we denote it by X. We have: X — P. Comparing the predicate P with the predicate P_1 of the proposition S — P_1, we establish that these predicates are identical in a certain part, that is, that the studied set of phenomena, the reason which we are looking for, in some part is identical to another set of phenomena known to us, the cause of which has already been established. Based on the partial identity of the predicates P and P_1 we conclude that the subjects representing the idea of a reason, or the regular order that determines the identical sets of phenomena, must also be identical, that is, we find that X is S.

Thus, the hypothesis, no matter how complex the predicates that are compared in it, are complex, there is nothing more than an inference from the identity of predicates to the identity of subjects, namely: the desired subject with the subject of judgment, the predicate of which turned out to be identical to the predicate of the studied proposition.

§ 4. A hypothetical conclusion, or hypothetical inference, differs from most of the types of inferences we have examined so far. With the exception of the syllogisms of the second figure, all the conclusions we have studied so far are based on a comparative examination of the *subjects* in judgments, which play the role of premises of the conclusion. Thus, finding from a comparison of the subjects of two propositions that these entities are identical, and knowing, in addition, that one of these entities has a certain predicate, we obviously have the right to ascribe this predicate to the other entity in another proposition. The basis for this transfer of the predicate from one proposition to another will be the identity of the subjects in both propositions.

The difference between the form of the conclusion based on the identity of the subject in the premises and the subject in the conclusion, and the form of the conclusion about belonging, the subject of one judgment of a predicate belonging to the subject of another judgment, depends on whether the conclusion 1) goes from separate objects to separate objects or 2) from individual objects to a group of objects, or, finally, 3) from a group of objects to individual objects. AT *in the first* case, when the conclusion goes from individual objects to separate objects, numerous conclusions arise about the relations of the identity of objects, the identities of parts of their contents, about the relations of simultaneity, etc. In the *second* case, when the conclusion goes from individual objects to a group of objects, conclusions of complete and incomplete induction, the conclusions of the third figure of the syllogism, and the conclusions, consisting in the application of inductive reasoning and conclusions of the third figure to a number of judgments—conditioned, judgments about the composition of objects, etc. The.. *third* in the case when the conclusion goes from a group of objects to individual objects, conclusions are obtained on the first figure of a simple categorical syllogism, on the ponens mode of conditional syllogism, conclusions of a separation syllogism and conclusions of probability.

§ 5. In contrast to all these forms of inference, a *hypothetical* conclusion, as well as a conclusion on the second figure of a simple categorical syllogism, *proceeds from a comparison not of subjects, but of predicates of premises*.

Consider an example of a hypothetical inference. When the task of explaining the mechanism of light propagation was set in physics, the following hypothesis arose among other assumptions to answer this question. It was suggested that the propagation of light is similar to the movement of waves on the surface of a reservoir, going in circles from a stone thrown into a reservoir.

What is the logical course of the conclusion that led to this hypothesis? The first stage in the formation of a hypothesis is the study of a collection of phenomena accessible to observation, the cause of which must be found. The thought of this totality will constitute a predicate of judgment, the subject of which must still be indicated. In this case, the subject will obviously be the thought of a regular order explaining the phenomena of light propagation known from experience and observation. The study consisted in the fact that the totality of these phenomena was expanding, and therefore the alleged subject, representing their cause, had to correspond to all the facts observed during the propagation of light: he had to explain the directness of the propagation of light, and the phenomena of reflection of light, and the phenomena of its refraction, deviations, interference, polarization, etc.

Each such group of phenomena, the entire amount of which was to be explained, *firstly* , made it possible to include the desired mechanism of light propagation in a number of other mechanisms that determine the same features that this group has. *Secondly* , each such group of phenomena made obvious the need to exclude the desired mechanism, or reason, from the circle of all those mechanisms that could not be due to the features characterizing this group. So, whatever the unknown cause of the propagation of light, this reason must be capable of producing the phenomenon of *reflection* Sveta. Consequently, this reason should be sought among all those processes and mechanisms of nature that are capable of producing reflection phenomena observed in experiment. And, on the contrary, it should not be sought among those processes of nature that cannot give reflection phenomena.

But the unknown reason for the propagation of light produces, in addition to the phenomena of reflection of light, also the phenomena of its *refraction*. This is a new definition of the sought cause. Like the previous one, it simultaneously shows that the cause of the propagation of light should be sought among the mechanisms, or the reasons that can give the phenomenon of refraction, and that it should not be sought among the processes lacking the ability to cause these phenomena.

Given this second definition, we *narrow* the area of mechanisms, or causes, in the circle of which the desired cause can be found. Indeed, the reason unknown so far should not only belong to the mechanisms, or reasons, causing the facts of reflection. It should no *longer* be sought *in the entire* area of these causes, but only in that part of this area to which the mechanisms or processes that can cause both reflection and refraction phenomena belong.

In a further study, a whole series of new definitions of the desired cause should be taken into account. The hitherto unknown cause of the propagation of light should, in addition to the phenomena of reflection and refraction, also explain the phenomena of interference, polarization, etc. Each of these new definitions of the desired cause further narrowed down the area of mechanisms, or processes, of nature, within which a mechanism capable of cause all these phenomena.

Finally, physics has come to the assumption that such a mechanism, or process, may be a *wave—like* process movement. Now the hypothesis has already been formulated. To all the features that the whole sum of phenomena of the propagation of light that must be characterized must be characterized, a reason is found, or a regular order, not directly given in the experiment, only assumed, but capable, once it is supposed to exist, to explain all those phenomena that in total make up the phenomena of light propagation known from experience. This reason is the *wave — like* transmission of light. At the same time, the thought of this reason is the alleged subject for all those predicates that have been consistently found and which represent each group of phenomena known to us from experience that are discovered during the propagation of light.

§ 6. But can we consider it *reliable* that the reason we have suggested is *indeed the* basis for the subject of all these predicates?

To answer this question, it is necessary to consider what is the basis of our assumption. Thus the basis is the belief that the assumptions we have as the cause of a natural order, or the mechanism *better than other* known way to explain *all* the totality of facts and phenomena of the propagation of light established on the basis of experience. But this preference, which is shown to the subject supposed by us as the reason, can be caused only by those facts that we know from experience. It is possible that with the further expansion of experience in the phenomena under study new facts will be discovered, for the explanation of which the previously assumed cause will be insufficient or even completely unsuitable, that is, incompatible with these facts.

Therefore, in the state of knowledge and experience that occurs at the time the hypothesis arises, the hypothesized reason or regular order cannot yet be recognized as the basis for *reliably* an established subject of complex judgment, the predicate of which represents the entire sum of the facts of the distribution of light known to us.

At the same time, the alleged cause is a *possible* basis for the subject of all known predicates. Moreover. In this state of knowledge, it seems to the author of the hypothesis the most acceptable as the basis for the subject of all the predicates known to us, representing the entire sum of known phenomena.

Thus, with *logical* from the point of view of the hypothesis, there really is an inference, consisting in the fact that the predicate of a certain judgment is attributed to a subject taken from another judgment. The concept of wave—like motion, which was the subject of judgment, whose predicate is the concept of phenomena observed when waves propagate on the surface of the water, is transferred as a subject to another proposition, the predicate of which represents all the features and phenomena of light propagation known from experience.

In this case, the subject transferred from one judgment to another is not assigned the immutable, but only the relative right to be the subject of a new judgment.

The basis for the transfer of the subject from one judgment to another is the identity of their predicates. Since Huygens and other physicists who created the wave theory of light, it seemed that the process of wave propagation on the surface of the water gives rise to phenomena that are identical to the phenomena observed during the propagation of light, they suggested that the hitherto unknown reason for the propagation of light is the same wave—like motion, which has already turned out to be the cause of the identical phenomena of wave propagation.

Since the same predicate, generally speaking, can belong not to one single subject, but to several subjects, the identity of the features characterizing a certain area of phenomena, the cause of which is already known, with the features characterizing the area of the studied phenomena, the cause of which is still only it must be established, it cannot be taken on its own as a sufficient basis for transferring the subject of one judgment to another.

Therefore, at the beginning of the study of the issue, usually not one, but several hypotheses arise, in a certain part similar, in other parts different from each other. This happens when experience knows about the existence of not one, but two or more objects or processes that can cause phenomena that are identical with those observed in the study area, and when data are still unknown that would make it possible to admit that in reality only one of these objects or processes can fully satisfactorily explain all the phenomena whose cause is the subject of research, while other objects are able to explain them only partially.

When the ancient Greek physicists posed in the 5th century BC the question of the causes of changes in things—the reasons for their occurrence, increase, decrease and destruction—these physicists put forward several hypotheses, each of which seemed to its author able to explain the facts observed in nature birth, growth, decline and death. So, *Empedocles* put forward a hypothesis for the explanation of all these facts about the existence of four physical elements — fire, air, water and earth—periodically connecting, sometimes disconnecting by two driving forces of "love" and "enmity." *Anaxagoras* tried to explain the same facts through the hypothesis of the existence of an infinitely large number of very small and infinitely divisible particles, embodying the embryos of all the qualities of things and put in order by a special driving force—the "mind". To explain the same facts, *Democritus* developed the hypothesis of the existence of an infinitely large number of very small particles, incapable of further fission, differing only in shape, order and position and moving flows in empty space.

§ 7. At a higher stage in the development of science, it is possible to find reasons sufficient to exclude all hypotheses that simultaneously exist on this issue, with the exception of one of them, and also open up the possibility of testing the only hypothesis that remains unverified.

Since the choice of the reason on which the scientist's thought stops mainly as the subject's base in judgment, where the predicate represents all the features of the studied area of phenomena, depends on the level of knowledge in this area, with the expansion of this knowledge the probability of an assumption constituting the content of the hypothesis may undergo verification.

The wider, richer and more accurate knowledge becomes in the area to which the hypothesis belongs, the more it becomes possible to discover such facts or phenomena in the light of which the rights of several alleged reasons for the role of the subject, previously recognized as equal, are already unequal. At this new, stage of development of science, it is possible to find reasons sufficient to admit that only one of all these causes is capable of giving all the phenomena observed in the field under study.

§ 8. This shows that hypotheses can differ one from another in the degree of probability, or validity, of the assumptions put forward in them. If the transfer of a subject from one proposition to another, namely, to conclude a hypothetical conclusion, is based solely on the fact that the reason presented by the transferred subject is more than the others capable of explaining all the known phenomena of the studied area, represented by a predicate of a hypothetical conclusion, then such a hypothesis still cannot be considered the most reasonable.

In order for the hypothesis to be considered the most justified, it is necessary to convince that of all the reasons known to us from experience, the basis for the predicate in the conclusion of a hypothetical conclusion can be only one, namely the concept of which is transferred to this conclusion as a subject. The improvement of the hypothesis is that the range of reasons, to which the whole set of phenomena can be attributed, is narrowed down, so, finally, there remains only one, which is transferred to the final proposition, all the other reasons, which were also considered earlier as grounds for possible subjects of a hypothetical conclusion, they are recognized as incapable of claiming this value.

Such an improvement in the hypothesis is possible only if the deepening of knowledge related to the studied area is sufficient to add new ones to the previously established predicates characterizing the observed phenomena. So far, only the phenomena of rectilinear propagation of light, reflection, refraction, polarization, the assumption that the cause of all these phenomena is the *wave— like movement of* light, have as many recognition rights as the assumption that the reason for this is the *expiration* light. But with the discovery of a number of new light phenomena—diffraction, light interference, etc. — the rights of both hypotheses to recognition turned out to be unequal. Joining all the previous ones, each new characteristic of known phenomena reduced the number of reasons that could be attributed to all these phenomena.

§ 9. The exclusion of a hypothesis from among the assumptions that can explain the observed course and order of phenomena is necessary when at least one fact is discovered that contradicts the *basic assumption of the* hypothesis. For example, the hypothesis of medieval physicists who attributed the rise of water in a pump to the fact that nature seemed to "fear the void" turned out to be refuted as soon as it was established that the water in the pump did not rise above 32 feet.

To test the hypothesis that remains after eliminating all previously "competing" hypotheses that turned out to be insolvent, it is necessary to derive as many consequences as possible from its main assumption.

After these consequences are deduced, it is necessary to compare the findings with the data of observation and experience. If at the same time it turns out that the data of observation and experiment are in real conflict with at least one single consequence that we deduced from the hypothesis, then this hypothesis should be immediately rejected, as it is undoubtedly false. If it turns out that not a single corollary derived from the hypothesis contradicts any phenomena known to us, then the hypothesis should be considered probable. Moreover, the probability of a hypothesis turns out to be the greater, the greater the number of consequences that have been deduced from it and the more diverse these consequences themselves.

§ 10. However, even a very large number of tested in practice and free from contradictions with it does not yet give the right to consider the hypothesis finally proved as *reliable* truth. A large number and — even more importantly — a variety of consequences consistent with observation and experience, significantly increase the likelihood of a hypothesis, but can never remove the trait that separates *probable* knowledge from *reliable* knowledge. And in this case, as in others, the absence of a refutation of this judgment should not be taken as sufficient proof of its truth.

§ 11. The hypothesis is considered proven and passes from the category of probable and unrebutted hitherto assumptions to the category of reliable truths in two cases. The first of these is the case when the reason suggested by the hypothesis, previously inaccessible to direct perception, becomes due to successes in the development of science and technology available to direct observation.

Some time after William Herschel discovered the planet Uranus, it turned out that the actually observed positions of this planet on the vault of heaven, representing the projection of its actual movements in space in orbit around the sun, deviate from those that should be expected, according to Newton's law of universal gravitation, even if taking into account all the influences that all other bodies of the solar system should have on the movement of Uranus. To explain the observed accelerations in the motion of Uranus, two hypotheses could be put forward: either suppose that the motion of Uranus does not obey the law of universal gravitation, or suppose that the acceleration in the motion of Uranus is caused by the existence of another, until now unknown planet, which produces its own attraction, in full accordance with Newton's law.

The first assumption was too unlikely and too contradictory to all the data of physics and all data about the motion of other planets in order to seriously stop at it. There remained a second assumption — the existence beyond the orbit of Uranus of some unknown planet, causing acceleration in the motion of Uranus incomprehensible outside this assumption. The decisive means of verifying this assumption would, of course, be the discovery of the proposed planet by direct observation, but where, in which place in the vault of heaven to look for it? Almost simultaneously, the English mathematician Adams and the French mathematician Leverrier took up the task. Both relied *in* their study, *firstly*, on the established data on the actual discrepancy between the observed provisions of Uranus and the provisions calculated on the basis of the law of universal gravitation. *Secondly*, these scientists made a number of consequences arising from it. The conclusion of these consequences greatly facilitated the verification of the hypothesis itself. If it is true, Adams and

Leverrier argued that deviations in the movement of Uranus are caused by the action of some unknown planet whose orbit lies outside the orbit of Uranus, then the belt in the heavens within which this planet should be searched should obviously coincide with that belt in both sides of the ecliptic, in the boundaries of which all the outer planets move. To more accurately determine the location of the proposed planet inside the Leverrier ecliptic belt, taking into account all the data on the mass of Uranus, the shape of its orbit, the position of the orbit in space, the magnitude of the observed accelerations in its motion, I also made assumptions about the mass of the planet we are looking for, its average distance from the Sun and etc. Based on all these data and assumptions, Leverrier made extensive and extremely complex calculations, as a result of which he determined the approximate place where the planet should be sought. The planet was indeed discovered within the specified zone and named Neptune.

At that moment, when, following the instructions of Leverrier, the astronomer Halle found with a telescope a planet whose existence was assumed by Leverrier and whose location was determined by him from this assumption and from the data on Uranus, the hypothesis of the existence of a new planet turned into a reliably established truth . The story of the discovery of Neptune is a classic case where a hypothesis becomes true, proved through direct observation.

§ 12. The second case of the transformation of a hypothesis into reliable truth is a case where the position constituting the content of a hypothesis is deduced as a result from reliable premises.

A weighty argument in favour of the hypothesis is the discovery, through experimental verification and observation, of a fact that was not known at all before the creation of the hypothesis and whose existence was deduced as a consequence, which follows from this hypothesis.

According to the correct observation of Fresnel, one of the main creators of the wave hypothesis, the correct hypothesis should lead to the discovery of numerical relations connecting very dissimilar phenomena. On the contrary, an incorrect hypothesis can accurately represent only those phenomena for which it was invented, just as the empirical formula generalizes the measurements made in itself only to the extent for which it was calculated. So, for example, Bio, trying to find the laws that govern the effects of staining, discovered by Arago in crystalline plates, found that the colours obtained in these plates follow the same laws with respect to their thicknesses as coloured rings, namely, that the thicknesses two homogeneous crystalline plates, painted in any two colours, are in the same ratio, as the thicknesses of the air layers reflecting the same colours in the color rings, respectively. So, using the principle of interference, which is a direct consequence of the wave hypothesis, Jung discovered another, much closer relationship between these two different phenomena, namely: he discovered that the difference in the paths of rays refracted in a crystalline plate in an ordinary way and rays, refracted unusual, just equal to the difference in the paths travelled by the rays reflected from the first and second surfaces of the air layer, which gives the same color as the crystal plate.

§ 13. Science resorts to the construction of hypotheses not only to explain the directly not perceived connection of facts. Science turns to hypothesis construction to explain observed *deviations* from the course of phenomena that is required by a hypothesis that already exists and is generally accepted. So, with the improvement of measuring instruments, it turned out that the apparent motion of the planets deviates from those movements that would have to be observed if

Ptolemy’s hypothesis were true about the central position of the Earth in the universe and its motionlessness. But, when this fact became clear, astronomers did not immediately recognize the old hypothesis as false. They did not want and did not dare to immediately abandon the usual, consistent with direct perception and supported by the clergy teaching on the immobility of the Earth and on the movements of all the stars around the Earth as the fixed centre of the universe. Therefore, these scientists have repeatedly altered the Ptolemy hypothesis so that, without departing from the main thesis for it about the central position of the Earth and its immobility.

For this purpose, some additional assumptions were introduced into the Ptolemy hypothesis. It was suggested that the outer planets move around the Earth not just in circles, but in such a way that each planet moves around the circumference of a small circle, the centre of which moves around the circumference of a large circle near a stationary Earth. Large circles were called deferents, small circles — epicycles. The hypothesis of the existence of epicycles and deferents was *auxiliary* in relation to the main hypothesis of Ptolemy. Thanks to this auxiliary hypothesis, it was possible—for a while, until a new improvement in measurement methods—to achieve a satisfactory agreement between these observations and the pattern of movements resulting from the complicated hypothesis of Ptolemy. Indeed, the movement of the planet along the epicycle, the centre of which moves along the deferent, could generate — in projection onto the vault of heaven — a picture of either direct or reverse motions, which cannot be directly derived without this auxiliary hypothesis from the Ptolemy hypothesis.

§ 14. The more artificial and complex the auxiliary hypotheses become, the more doubts arise about the truth of not only these additional assumptions themselves, but also and especially of the truth of the hypothesis, which is their basis. If in order to preserve the old hypothesis, one must admit the existence of an extremely complex and, moreover, completely artificial mechanism, specially invented with the sole purpose of "saving" the old hypothesis, then this state of affairs is usually a strong argument against the truth of this hypothesis.

So it was with the hypothesis of epicycles. When the measurements of the angular distances between the stars reached greater accuracy, it turned out that the assumption of epicycles and deferents cannot lead to agreement of the observational data with the motions provided by the auxiliary hypothesis. I had to introduce into the Ptolemy hypothesis new and even more complicating the general picture of the world of assumptions. I had to admit that the planet moves along the epicycle around a point that moves around the circumference of another epicycle, and only the centre of this last moves around the circumference of the deferent around the Earth.

But it was precisely the extreme artificiality of this building that betrayed that all these auxiliary hypotheses were not an explanation of real movements, but only a means of supporting, contrary to new observation data, the doctrine of the central position and immobility of the Earth, which came into clear contradiction with these observations. As is known, further successes of astronomy consisted in the fact that Copernicus refused to build new auxiliary hypotheses, boldly declared false the very basis of the Ptolemaic theory—the doctrine of the Earth's immobility and its central position—and explained the observed inequality in planetary motion as a visible result not equally fast the motion of the earth and other planets around the sun.

The Main Logical Types of Hypotheses

§ 15. Since the logical course of the conclusion in all hypotheses consists in transferring the subject from one judgment to another, the predicate of which represents the sum of the phenomena known from experience and to be explained, the variety of possible *logical* types of hypothesis will obviously depend, *firstly* , from the logical structure of that subject, which is introduced in conclusion of a hypothetical conclusion, *and secondly*, from the logical structure of that complex predicate for which the subject is searched in a hypothesis.

Let us consider from this point of view some of the most important varieties of hypothetical conclusions. The first of these is the conclusion that the subject known to us in another connection from other propositions is connected with the established predicate on the basis that the predicates of these propositions are identical and that only one of the known subjects can be assigned the established predicate. Examples of hypotheses of this kind may be the so—called *conjuncture*, i.e., corrections of corrupted places in manuscripts, proposed by philologists working to establish the exact text of ancient authors. The scientist sees that some word or expression is clearly distorted by an ignorant scribe, resulting in complete nonsense. The question arises: what word or expression was in the original manuscript of the author before its distortion? To solve this issue, a scholar—philologist offers a hypothesis. In this case, the predicate to which the subject is sought, obviously, will be the thought of the whole context, which includes the distorted or substituted word. The subject that is transferred to the final judgment, obviously, will be the thought of another word proposed by the philologist instead of the spoiled one. The basis for the assumption that initially it was precisely this word proposed by the philologist that stood, for example.

In the example we have examined, the hypothesis is sent from a judgment *on a specific subject*. The philologist found in other places by the same author the very word that he suggests was substituted for in the phrase he was studying, and, moreover, found it in the very context of the context of the phrase being studied. He transfers this word into a damaged context, since, according to his knowledge of the author under study, only this word alone can fit this context.

§ 16. Another type of hypothesis is a hypothetical conclusion in which the subject transferred from one judgment to another is not an idea of a specific object, as in the previous example, but an idea of an object considered as a representative of a known *logical group*.

In turn, this group can be either a collection of *objects* considered from the side of some property belonging to all of them, or a collection of *relations* characterized by some features belonging to all of them.

An example of a hypothesis in which a transferable subject is the thought of an object considered as a representative of a logical group can be taken by us as an example of a hypothetical conclusion, the physics hypothesis on the mechanism of light propagation. When Huygens suggested that all the facts and phenomena known in his time observed during the propagation of light can be explained under the assumption that the light propagates like the waves diverging from a stone thrown into a pond, he put forward the hypothesis of just that kind of. In this case, all the facts of light distribution known to Huygens constituted a definition for which it was required to find the only suitable subject for him. The subject transferred to conclude a hypothetical conclusion was the concept of a process, or of the mechanism of wave propagation. Wherein, *a* *specific* case or the fact of the propagation of waves — in a pond, in a river or in the sea. The propagation of waves on the surface of the pond was an object of Huygens's thought only as such an object that represents a whole logical group of homogeneous objects. These objects are a totality of relations, which are all characterized by the same properties, identical both in all cases of wave motion and in all cases of light propagation.

§ 17. But the logical variety of the hypothesis depends not only on the logical nature of the *subject* , which is sought for the predicate established and identified with the predicates of other propositions. The logical variety of the hypothesis also depends on the logical nature of that *predicate.*, the subject to which is sought in the hypothesis.

So, a special kind of hypothesis is obtained when the predicate to which the subject is sought is a predicate representing an object or phenomenon that has a complex composition and is composed of different parts. Such a predicate itself is complex and consists of parts. Under this condition, the subject, connected in the conclusion of a hypothetical conclusion with an established predicate, does not have to be a concept about the whole subject. If from experience we know separately all the partial subjects to which respectively all the partial predicates belong, which together constitute a complete complex definition of the region under study, then, in conclusion of a hypothetical conclusion, all partial subjects are transferred as a composite subject connected to the same composite predicate.

Hypotheses of this kind are very common in the *natural* sciences. This includes, for example, all hypotheses in which a certain sum of partial causes is assumed as the cause of a complex phenomenon, the actions of which, taken separately, are already known from experience.

18. Hypotheses in which a predicate represents a complex *composition of an object* have a special variety. This variety arises under the condition when a complex predicate is composed not of *distinct* private predicates, but of private predicates, which, from a logical point of view, should be recognized as *identical*. In this case, instead of many partial subjects representing the reasons known to us from experience and capable by their combined action of causing the entire complex set of phenomena that make up the complex predicate under study, we find *only one* partial subject known to us from the experience that connects to a part of a complex predicate. We mentally increase this partial subject and accordingly increase the part of the predicate connected to it. The task of this increase is to achieve the identity of the enlarged part of a simple predicate with that complex predicate, which is composed of concepts about all the phenomena of the studied area that are known to us and for which we are looking for a subject corresponding to it.

Hypotheses of this type are found in *geology* and in *cosmology*. One of the most important tasks, for example, cosmology, is to explain the reasons why the Moon currently rotates around its axis during its daily rotation during the same period of time in which it revolves around the Earth.

From the study of the Earth's ebb and flow, it is known that the tidal wave produces an effect that slows the daily rotation. Knowing this, the cosmologist considers the modern slow diurnal rotation of the Moon as a result of continuously accumulating, over the course of a huge period of time, enormous number of very small decelerations. These decelerations were produced by a tidal wave that arose in the lunar crust due to the strong attraction of the Earth.

In the hypothesis of this subject — the thought of the insignificant in magnitude action of the tidal wave, which inhibits the daily rotation of the Earth. This subject is mentally increasing. Accordingly, the predicate with which it connects increases—the thought of the result of the accumulated over a huge period of time and the summed up braking. Having reduced the predicate increase to the size at which the hypothetically increased predicate represents the *currently* observed slow diurnal rotation of the Moon, the cosmologist transfers to the judgment of this predicate a subject formed by adding up a huge number of logically identical partial subjects. These partial subjects are notions of insignificant in the force of the inhibitory actions of the tidal wave caused in the lunar crust by the Earth's gravity.

For the possibility of such a transfer of the subject into judgment, it is necessary to believe that, in the current state of science, we cannot find other subjects capable of presenting reasons whose actions would be able to explain the currently significant result of the inhibition of the daily rotation of the Moon.

§ 19. The increase in the subject and its predicate, carried out in hypotheses of this kind, can be done *in two* ways. *The first* consists in the assumption that the reason assumed by the subject, the effect of which is currently very small under the conditions known to us, had a different value under hypothetically assumed conditions, so large that the effect of this reason could coincide in magnitude with the size of the phenomenon represented by the predicate for whom we are looking for the subject.

So, cosmology suggests that in a very remote era from us, the tidal wave on the surface of the Moon, produced by the gravity of the Earth, was much more powerful than at present, since at that time the distance between the Earth and the Moon, formed from one celestial body, was much smaller than now.

The disadvantage of this type of hypothesis is that they are based on the assumption that with the strengthening of the cause we do not get anything new except the corresponding strengthening of the action. It is assumed that an enlarged or strengthened subject is only a certain amount of logically identical unexpanded or un reinforced subjects.

However, the result or amplification of the cause presented by the subject is rarely the mere increase or amplification of the action. Usually, with the strengthening of the cause, the very nature of the action it produces changes. Therefore, assuming a subject representing an intensified cause, we cannot be sure that the reason conceivable in this new quantity is only the sum of the identical unenhanced causes known to us from experience. And from this it follows that in hypotheses of this type not only the conclusion is hypothetical, that is, the assignment of the subject to the predicate representing the phenomenon, but, in addition, the premise on which the conclusion is based is deprived of unconditional reliability.

§ 20. *Second* the method of mental enlargement of the subject, possible in the hypotheses of the type under consideration, is based on the fact that the action of the cause is presented to increase not because of the increase in the cause itself, but because the huge number of reasons or conditions that remain unchanged are summed up due to the addition of each such reason, or each such conditions to the reason preceding it in time. With this construction of a hypothesis, the whole series of following one after another causes or conditions is only the sum of their values: adding to its previous one, each new condition only repeats it, remains identical to it, and therefore the nature of the action of all these repeating conditions does not change. In such hypotheses, only a conclusion is hypothetical, that is, the transfer of the subject into judgment with the predicate to which the subject is sought. On the contrary the premise on which the conclusion is based is quite reliable. So, the geologist rightly considers the rise of the coast above sea level in some places as the sum of the very small, in its insignificance, identical elevations that have formed over the centuries.

§ 21. A special type of hypothesis arises when the investigated predicate itself, to which the subject must be found, represents a phenomenon that appears in our experience in a slightly modified form. Such a modification can take place where the subject of the predicate consists of two parts: one that represents the cause known to us in its action, and the other that is unknown. In such cases, the predicate corresponding to the whole subject may turn out to be not just a thought about the sum of two actions: one, caused by the part of the reason known to us, and the other, related to its unknown part. The phenomenon represented by the predicate may turn out to be changed in comparison with the sum of the action of both parts of its cause, which these parts each have individually.

Under these conditions, the task of the hypothesis is no longer to find the subject to the existing complex predicate. The question is to find a *part of the* reason, but a part whose action, combined with the action of another part of the same reason known to us, could cause a phenomenon that has changed compared to the phenomenon represented by the predicate relating to a known part of the desired reason.

Hypotheses of this type are extremely widespread where science encounters some change in the actions well—known to it, caused by reasons just as well—known to it. Thus, the observed forms of comet tails, and therefore the directions along which the luminous particles of gases constituting the comets tails are located in space, cannot be explained by the action of the law of universal gravitation alone and the laws of planetary motion. To explain the observed types of comet tails, astrophysicists have to introduce into their hypotheses, in addition to all these reasons, the effect of light pressure discovered by Lebedev.

Analogy

§ 22. We examined hypothetical conclusions, or hypotheses. We have seen that, from a logical point of view, they are all based on a comparison of the predicates of two propositions. The very conclusion in them is that, having convinced ourselves of the identity of the predicates of both propositions, we transfer the object from one proposition to another.

At the same time, however, it remained not completely clarified how the idea might arise that the sought—after, but hitherto unknown subject of the studied judgment is most likely the already known subject of the judgment with a predicate identical to the studied one. Indeed, the mere identity of the juxtaposed predicates, taken in and of themselves, does not provide a *sufficient* basis for transference: when the predicates are identical, the subjects of judgments may not be identical.

Therefore, the emergence of a hypothesis that carries out the transfer of a subject from one proposition to another proposition with the same predicate is often preceded by a special conjecture consisting in identifying, at least partially, the subjects of two propositions that have identical predicates. This conjecture forms the so—called *conclusion by analogy*.

Consider an example of such a conclusion. Imagine the following case. To the teacher’s question: “Where—on the first or second syllable—should be emphasized in the word “thinking?”—the student replied: “On the second.” When the teacher asked: “Why do you think so?”—the student explained his answer with the following reasoning:

“The word thinking,” said the student, “is similar to the word crash.” Both of these words are verbal nouns, both are derived from the verbs “it”: “think”, “destroy”. Since the word “crash” stresses on the second syllable, the word “thinking”, which is similar to the word “crash” in terms of word formation, stress should also be on the second syllable.”

The reasoning of the student is an example of a conclusion by analogy. Consider the logical course of this conclusion, as well as its logical validity.

At first glance, it might seem that the considered conclusion is based on a comparison of only two objects.

The very conclusion, apparently, is the inference from the property that one of the objects found in combination with a number of other properties to the existence of the same property in the second object, since this object has the same other properties.

In fact, *not only is the* subject compared with the subject. The student compared the word “thinking” with the word “wreck” only because the word “wreck” represents in his mind a *whole group of words*, such as “decision”, “wearing”, “reckoning”, etc. All these words, being derived from the verbs on “it,” have an accent on the second syllable. The very conclusion is that, since the word “thinking” is also a derivative of the verb “it,” it too, like the words “wreck,” “decision,” “wear,” “calculus,” etc., in which the origin of the verbs “it” is associated with the stress on the second syllable, will also have stress on the second syllable.

Thus, the analogy is the conclusion, consisting in the conjecture that a property belonging to objects of a known group and occurring in them together with some combination of other properties will belong to these objects, besides these objects, another object that is similar to objects of the group, since it has the same combination properties.

This shows that the analogy is not a conclusion from the properties of one object to the property of another, but a conclusion *from a group to a separate object* . But since at the same time the group is characterized by only one of the objects included in it ("crash"), at first glance it seems as if the conclusion does not come from the group to the object, but from one separate object ("crash") to another separate object ("thinking").

From this example it can be seen, further, that the analogy is the conclusion from the already clarified *partial* similarities between the objects of the group and an individual object to *a fuller* and *deeper* the similarities between them. Indeed, a property found in group objects beyond those properties that are common to them with the properties of an object compared with a group is supposed to belong not only to the group, but also to the object compared to the group. Thus, an item is included in the group to which a specific representative or member of the same group, similar to this item in known features, belongs.

The conclusion by analogy has no probative value: its value lies in the ability to speculate on the still unreliable features of an object or phenomenon.

With respect to evidence, an analogy should be reckoned with the conclusions of probability, but not reliability. In fact, the basis of the analogy is the assumption that the relationship found in one of the group members between a certain system of its properties and another property of it is not a random connection, and that therefore, *any* object that contains the same system of properties should also have the property with which this system exists in the group representative.

But it is clear that this assumption is only a guess, and not a reliable truth. Since the connection between the system of properties and the additional property of the representative of the group is only a connection of coexistence, it is possible that this connection is random and that it will not occur in other representatives of the group.

So, in our example, the student concluded by analogy that in the word “thinking” the stress should be where it stands in the word “crash”. But the basis for this conclusion was only the similarity between “thinking” and “collapse” in terms of word formation, as well as the fact that the word “collapse” emphasizes the second syllable.

To recognize the conclusion as reliable, the basis is clearly insufficient. Without special research, it is not clear that the connection between the method of word formation and the place of stress was a *necessary* connection. It is not clear why words that have the same mode of origin from verbs with the same infinitive ending could not have stress on *different* syllables.

§ 23. The evidentiary power of analogy is negligible. The mere comparison of the similar features of a certain object with the subject of the group — no matter how large their number — does not in itself give reason to believe that these objects will *necessarily* be similar in other ways, except for those whose similarities have already been established. It is possible that this similarity will take place, but it is also possible that beyond the limits of the similarity proved in certain features in all other features, these objects will turn out to be completely dissimilar. In other words, conclusions by analogy give not only probable conclusions, but, in contrast to inductive conclusions, the likelihood of conclusions by analogy is incomparably lower.

In assessing these findings, it is not so much the number of similarities that matters, but their interconnection. In cases where the number of similar traits is clearly greater than the number of different traits, the analogy often seems more justified. However, here the question of the soundness of the analogy is not solved by mechanical counting of signs. In addition, the number of similarities is often exaggerated. If a series of similar features represents the action of one and the same reason, then, strictly speaking, all these features should be taken into account as one single similar property, and not many similar properties.

§ 24. If the subject of which a conclusion is drawn by analogy reveals the presence of a property that is incompatible with the property attributed to it by the conclusion of analogy, then the similarity of the compared objects in other features loses all significance, and the analogy turns out to be completely unfounded. If, for example, it is considered established that the existence of organic life, such as that known on earth, requires air, water and the presence of temperature fluctuations that do not exceed known limits, then the existence on other planets of conditions incompatible with these requirements makes any a conclusion by analogy regarding the presence on these planets of organic life, similar to that which exists on earth. So, the Moon has many signs common to it with the Earth: the same average distance from the Sun, close to spherical shape, solid crust, change of day and night, annual movement with the Earth around the Sun, etc. Is it possible, based on the presence of all these common features, to conclude that, since, in addition, it is known that on Organic life exists on the Earth, then the same life should probably exist on the Moon? Obviously not. In fact: it is known that on the Moon, unlike the Earth, there is neither water nor air. It is further known that temperature fluctuations at the same point on the lunar surface, depending on the change of day and night, are huge and far exceed the limits within which life on Earth is possible. Since the Moon is not protected, like the Earth, by a thick cover of the atmosphere, softening the sharpness of temperature fluctuations, with the onset of the day, the temperature of the lunar surface rises to 100 ° above zero within a few minutes.

These conditions are obviously so incompatible with the conditions of life existing on the Earth that there is no sufficient reason *to conclude* that there is an organic life on the Moon *similar to life on Earth* , despite all the many similarities between the Earth and the Moon in other respects.

Moreover, if an object has a property incompatible with the one whose existence is concluded by analogy, many other similar features argue *against* analogies. Indeed, if the Earth and the Moon are similar to each other in so many ways, it is natural to expect that the conditions under which life is possible on them should also be similar. If on the Moon, where the conditions of life were to be extremely close to earthly, in reality, these conditions sharply contradict the conditions of life known on Earth, then the probability that life on the Moon will be similar to earthly should be recognized as very low.

This is the meaning of analogy in terms of its evidentiary power. Analogy is not proof. Conclusions by analogy do not have credibility, but only probability.

§ 25. However, this does not solve the question of the meaning of analogy in thinking and in science. In addition to the question of the right of analogy to be a means of proof, there is the question of the role, which analogy plays *when speculation arises about the similarity between phenomena and objects of nature*.

In developing these conjectures, analogy is often an *extremely fruitful* form of thinking. Unable to tell the conclusion the reliability or at least the probability that is inherent in inductive conclusions, the analogy often *leads to guesses* , the correctness of which is clarified by further investigation and further verification.

Of course, these conjectures are no longer justified by analogy, but by means of genuine evidence, but for the first time they are put forward and often found precisely by analogy.

Such a—fruitful—analogy was the analogy between sound and light phenomena. Comparison of the phenomena of sound and light proved that these phenomena contain a number of similar properties: both sound and light obey the laws of linear propagation, reflection, refraction, deviation and interference. With regard to sound, it was also proved, through experiments with a siren and a monochord, that sound is caused by periodic movements. From this we concluded to the likelihood that light is caused by similar movements. It was this analogy, noticed by the Dutch physicist and mathematician Huygens, that led him to the concept *of a light wave*. The analogy noted by Ohm between the distribution of heat and the distribution of electricity in conductors made it possible to transfer to the domain of the phenomena of electricity the equations developed by Fourier for the phenomena of heat. The analogy between magnets and electrical insulators has played a prominent role in the development of physical studies on magnetism and dielectric polarisation.

These examples are not single and not random. A physicist, chemist, biologist strive not only for the accumulation of facts and materials, but also for the unification of the studied field of phenomena in the theory covering this entire field. In this case, the researcher is often guided by the analogy that he finds between the phenomena studied and the phenomena observed in another field. In some cases, the analogies found in this way are erroneous, and the researcher subsequently has to discard them as unsuitable. But in many cases, the conjecture arising by analogy is verified by more rigorous methods of proof, and by verification it turns out to be true.

§ 26. Why, in some cases, is the analogy true, and in others false?

The possibility of true analogies is explained by the mutual relationship between phenomena and between the constituent parts or different sides of the phenomena. If there is really a similarity between some object and the object of the group, then it is not surprising that this similarity will be found not only in those features that are already known to be similar in both objects, but also in that line, which besides obviously similar, it is present in one of them, but with respect to which it is still unknown whether it exists in another. In this case, the truth of the analogy is based on the similarity of the object with the objects of the group, the very discovery or discretion of the analogy depends on the insight with which the researcher predicts the necessary connection existing between properties similar in both objects and that additional property.

But if the similarity between the compared objects does not extend far, then it can easily happen that, in addition to the features already common to both objects, there will be no other similar properties between them.

In the first case, the analogy will be true, fruitful, expanding knowledge, in the second—false, barren, incapable of moving knowledge forward.

But in the first case, the analogy is only an anticipation of the truth, but not a proof of the truth itself. Therefore, in all disputes over issues of importance to knowledge, one should never consider analogy as a means of proof. The truth found for the first time by analogy and subsequently proved to cease to be just a “guess by analogy” as soon as genuine evidence is established. Such truth is included in the number of knowledge, the basis of which is not rooted in a simple analogy, but in the knowledge of the necessary connections between phenomena.

Tasks

Investigate which of the following conclusions are *hypothetical* and which *conclusions are by analogy;* in the case of hypothetical conclusions, determine the logical type of hypothesis:

1) “If two celestial bodies collide in space, then most of them undoubtedly melt. But it seems equally reliable that in many cases, a mass of fragments scatters in all directions, among which many are not more damaged than fragments of rocks during a collapse or when the rocks explode with gunpowder. If our earth, in its present state, with its vegetation cover, collided with a celestial body equal to its size, then many fragments carrying seeds, living plants and animals would have scattered in space, no doubt. Since, no doubt, from time immemorial there have been star worlds that are carriers of life, we must consider it highly probable that there are infinitely many meteorites that wander in space, bearing seeds on them.

2) “Man is called the ancients by the small world, — and there is no dispute that this name is appropriate, because as a man is composed of earth, water, air and fire, so is the body of the earth. If a person has bones that serve as his support, and covers of meat — in the world there are rocks, supports of the earth; if a person has a blood lake, where the lung grows and decreases when breathing, the body of the earth has its own ocean, which also grows and decreases every 6 hours, when the world breathes; if veins originate from the named blood lake, which, branching, diverge over the human body, then the ocean fills the earth’s body with endless water veins in the same way. There are no tendons in the earth’s body, which are not because tendons are created for the sake of movement, and since the world is in constant balance, there is no movement here, and since there is no movement, then tendons are not needed.[1].

3) One layer of liquid cannot slide on another layer without the formation of waves on the surface separating these layers. We are very familiar with this phenomenon when waves form on the surface of the water in the wind. The mathematician can easily prove that the period of oscillation of the pendulum varies in proportion to the square root of its length. The proof does not depend at all on the complete solution of the problem of the period of oscillation of a pendulum of any particular length, so if we did not know this solution at all, we would still be sure of the correct relationship between the length and time of oscillations of various pendulums. If a given pendulum completes its oscillation in a certain period of time, then we probably know that a similar pendulum, four times longer, requires twice as much time for its oscillation.

The wave—like motion on a surface separating two liquids of different densities is a task of exactly the same type, and if the results are known for one pair of liquids, they can be reliably predicted for another pair. Namely, the ocean waves formed by the wind can be considered studied and well known.

Small "lamb", which we often see in the sky, prove the applicability of the theory of sea waves to air currents. At the same time, atmospheric moisture thickens in the clouds on the crests of the air waves, and in the troughs of the waves the moisture again turns into steam. Thus, a motley change of narrow clouds, which are called lamb, is obtained. These lambs cannot be seen in the sky in stormy weather, as their presence proves that one layer of air glides over the other only at a relatively moderate speed. The distance between the crests of successive waves, expressed in a linear measure, should be very significant, but nevertheless we must consider these lambs to be simple ripples formed depending on the low relative speed of both layers. By making a plausible assumption about the densities of both layers of air and their relative velocity, we can show that sea waves ten yards long correspond to air waves with a length of more than twenty miles. A wave of this length should cover the entire horizon and may have a period equal to half an hour. It is clear that the sheep disappear in stormy weather, since we are then too close to the crests of the waves to observe their correct change and to see the separation of cloud forms[1].

4) To explain the process of formation of organic forms, Darwin turned to observations on the process of changes in these forms under the influence of the conscious will of man. "This comparison was so bold that for many it seemed incomprehensible to many ... Between the fall of a body on the earth's surface and the movement of a planet in its orbit, the difference, of course, was not so deep as the difference between a process led by a person's rational will and a process that is the fatal result of physical factors that determine the existence of the organic world. And on the other hand, where was there to look for a key to explanation, if not in the only examples of the conversion of organic forms that we reliably know? It was necessary first to find out how a person acted in such cases in which he was, so to speak, the creator of new forms.

Going through all the means by which a person exerts his influence on organic forms, we can bring them into three general categories. These categories are: 1) direct impact through the influence of external factors, 2) crossbreeding, and 3) selection. Of these three paths, only the first two exclusively attracted the attention of thinkers and scientists who were trying to find a natural explanation for the origin of organic forms in their natural state. It seemed all the more obvious that only these processes are the same both with and without human participation. But it was they who did not give the sought—after explanation, did not explain the most mysterious side of the phenomenon, striking any, even superficial observer of nature, its expediency, passing through the whole and particularities of the organization of every living creature. Third way.

Summing up the results achieved by man in the direction of improving artificial breeds of animals and plants, Darwin recognized the selection of the most outstanding role on the basis of the following considerations. By direct exposure to external factors and by interbreeding, a person, of course, can cause shape changes, but these changes are not deep, limited, not durable, they are little subject to his will, in the sense of anticipating the result, and in reality did not play such a role in the formation of known breeds which belongs to the selection. Only through selection did a person move in a certain desired direction, and changes evolved gradually, and not by random sharp leaps — in a word, only through selection did the works marked by clear traces of a person’s ideas and requirements bear that imprint of expediency, which, in a different direction, it also strikes us in the works of nature ... Man, as it were, sculpts, line by line, the desired form, but not himself, but only uses the inherent spontaneous plasticity, so to speak. Nature gives him rich finished material; man only takes from this ready—made material that which corresponds to his goals, eliminating that which does not correspond to them, and in such an indirect, indirect way imposes on the body the stamp of his thought, his will. Consequently, the result is not achieved immediately, but in two steps, by two completely independent processes. Darwin will seek the same in nature ... Nature gives him rich finished material; man only takes from this ready—made material that which corresponds to his goals, eliminating that which does not correspond to them, and in such an indirect, indirect way imposes on the body the stamp of his thought, his will. Consequently, the result is not achieved immediately, but in two steps, by two completely independent processes. Darwin will seek the same in nature ... Nature gives him rich finished material; man only takes from this ready—made material that which corresponds to his goals, eliminating that which does not correspond to them, and in such an indirect, indirect way imposes on the body the stamp of his thought, his will. Consequently, the result is not achieved immediately, but in two steps, by two completely independent processes. Darwin will seek the same in nature ...

But what can nature introduce to us analogous to a complex selection process? The first half of the process — the delivery of material — and in the selection process belongs to nature, is carried out without human intervention; therefore, in their first stage, both processes are identical. The whole question is: what will we put in place of the human influence improving this material? What will impose on this, and here and there, indifferent material the seal of expediency?

.... Firstly ... the process of selection, long before its application in its modern conscious form, man carried out completely irresponsibly and, therefore, in relation to the result was the same unconscious figure as other factors of nature. But, having allowed a conscious and unconscious selection in the human activities, we are forced to admit the possibility of the same unconscious selection, on an even wider scale, and in an unconscious nature. Secondly, we note that the results carried out by artificial selection bear the imprint of usefulness only from the point of view of a person, while the results of a similar natural process bear the imprint of exceptional utility for an organism possessing this feature. Finally, thirdly, let us pay attention to the fact that, in its very broad form, the selection process is reduced not so much to the allocation and protection of indivisibles with a chosen feature, but to the extermination of indivisibles that do not have it. Substituting all these three conditions into the general concept of selection, we get an idea of the process, which can fully correspond to it in nature. This will be a process in which in a fatal, mechanical way, all organisms that do not have features that are useful to them or possess them to a lesser extent than others are doomed to extermination. Such a process, by its results, should be recognized as completely similar to the selection " which may well correspond to it in nature. This will be a process in which in a fatal, mechanical way, all organisms that do not have features that are useful to them or possess them to a lesser extent than others are doomed to extermination. Such a process, by its results, should be recognized as completely similar to the selection " which may well correspond to it in nature. This will be a process in which in a fatal, mechanical way, all organisms that do not have features that are useful to them or possess them to a lesser extent than others are doomed to extermination. Such a process, by its results, should be recognized as completely similar to the selection "[1] .

5) "When we observe that one body acts on another at a distance, before accepting that it is a direct and immediate action, we usually examine whether there is any material connection between the bodies; and if we find that the bodies are connected by threads, rods or some mechanism that can give us an account of the observed actions of one body on another, we prefer to explain the actions using these intermediate links rather than admit the concept of direct action at a distance.

So, when we pull the bell to ring, the consecutive parts of the wire are first pulled and then set in motion until finally the bell rings at a distance through a process in which all the intermediate particles of the wire took part one after another. We can make the bell ring from a distance and otherwise, for example, by pumping air into a long tube, at the other end of which there is a cylinder with a piston, the movement of which is transmitted to the bell. We can also use the wire, but instead of pulling it, we can connect it at one end with an electric battery, and at the other with an electromagnet, and thus make the bell ring through electricity.

Here we have indicated three different ways to set the bell in motion. But in all these methods there is one thing in common, that there is a continuous connecting line between the caller and the bell and that at each point of this line some physical process takes place, through which the action is transferred from one end of the line to the other. The transfer process is not instantaneous, but gradual; so that after an impulse is given at one end of the connecting line, a certain period of time elapses during which this impulse makes its way until it reaches the other end.

It is clear, therefore, that in some cases the action between bodies at a distance can be explained by the fact that in a row of bodies occupying an intermediate space, a series of actions are performed between each two adjacent bodies of the row; and proponents of the medium's action ask: is it not more reasonable in cases where we don't notice any mediating agents — will it be more reasonable, they say, to allow the existence of an environment in these cases, which we cannot indicate so far as to state that the body can act where it is not.

To whom the properties of air are unfamiliar, the transfer of force through this invisible medium will seem as incomprehensible as any other example of action at a distance, but in this case, however, we can explain the whole process and determine the speed with which the action is transmitted from one part of the medium to of another.

Why, then, cannot we assume that the familiar way of communicating motion by pushing and pulling with our hands is a type and a vivid example of any action between bodies, even in cases where we cannot notice anything between the bodies that apparently would take part in this action "[1] .

CHAPTER XIV. THE PROOF AND ITS STRUCTURE. TYPES OF EVIDENCE

Evidence

§ 1. Any truth is not only a *true* judgment, that is, a judgment corresponding to facts. All truth is, in addition, a *justified* judgment , that is, a judgment whose statement is accompanied by an indication of the grounds by which it is true and must be recognized as true.

There are judgments, the truth of which is verified by simple perception. Such judgments are called *directly obvious* and do not need proof. Examples of directly obvious judgments: “I see something white,” “this line is broken.”

Some of the directly obvious judgments form the basis of a whole series of truths, including those that do not have evidence, but belong to the same field of knowledge. Such judgments are called *axioms* . An example of an axiom: “the whole is larger than its part.”

Judgments whose truth does not have immediate evidence are *proved* , that is, they are brought to evidence by indicating the grounds by virtue of which they are true.

Evidence, as we already know, is one of the most important conditions for scientific knowledge. The vast majority of scientific truths are not given directly to our perception. Moreover. Direct perception often misleads us, as it often shows us phenomena that are not what they really are. For example, for direct perception, the just ascended Moon seems larger than the same Moon when it rises high above the earth. In fact, the angle at which the diameter of the moon is visible is the same at the moment when the moon rises, and at that when it stands high above the horizon. Since the evidence of perception can be deceiving, in the most exact sciences, like mathematics, they do not trust the immediate evidence of perception and try to prove, if possible, all the truths, with the exception of a very small number of axioms. But the axioms were also revised from time to time in mathematics with the goal of establishing whether their number could not be reduced by proving those that, in comparison with the others, are not so obvious.

§ 2 . In the broad sense of the word, proof is any way of elucidating the grounds on which a certain judgment is considered true. In this broad sense of the word, *conclusions*, or *conclusions*, also belong to evidence.

In the conclusion, the basis for the conclusion is not direct perception, but the truth of other judgments, recognizing which as true we cannot help but recognize the conclusion as true. In the conclusion, the truth of the judgment is not only affirmed, but proved. However, the proof here consists only in discerning the necessary connection between the premises and the conclusion; the premises themselves are accepted as true without investigation and without verification of their truth.

§ 3. In a narrower and special sense, proof is not all conclusions, but a *special* kind of conclusion or a special form of substantiation of truth. In this—special—sense, proof is the study of the truth (or falsity) of *judgments* . Namely: proof is such an inference by which the truth (or falsity) of a given judgment is verified.

From this point of view, we compare the following two conclusions:

First . ”Since all cereals bloom in spikelets and since all bamboos are cereals, all bamboos also bloom in spikelets.”

Second. ”If it is true that all cereals bloom in spikelets, and also that all bamboos are cereals, and if the conclusion is correct, then it is also true that all bamboos bloom in spikelets. But the claim that all cereals are spikelets and that all bamboos are cereals are true. The course of the conclusion itself is also correct. Therefore, the conclusion that all bamboos bloom in spikelets is true.”

In a *broad* sense, both of these conclusions are *evidence* . In the *special* sense of the concept, the proof will be only the *second* conclusion, the first will be an ordinary inference. *The first* conclusion is the discretion of the necessary connection between the premises and the conclusion. *Second* the conclusion is proof that the conclusion, that is, the judgment “all bamboos bloom in spikelets,” is true. *The first* conclusion consists only of a comparison of the premises and of the discretion of the conclusion arising from them. *The second* is more complex and represents inference about inference. Namely: inference, which is the subject of another inference, is *conditional* inference: “If the propositions” all cereals bloom in spikelets “and” all bamboos are cereals “are true and if the conclusion itself is correct, then the conclusion” all bamboos bloom in spikelets “is true.” The second conclusion confirms the truth of the first: “Since it is true that all cereals bloom in spikelets and that all bamboos are cereals, and since the conclusion turned out to be correct, the conclusion” all bamboos bloom in spikelets is true.”

§ 4. One would have thought that the difference between a conclusion, or inference, from a proof is that in the conclusion the thought goes from the premises to the conclusion, and in the proof, on the contrary, from the position being proved to the premises, or the grounds from which it is displayed.

In fact, both in the conclusion and in the proof, the train of thought can be both. In some cases, the conclusion is that the premises are given and a conclusion must be drawn from them. For example, the premises were given: “potassium is a metal”, “potassium does not sink in water”; the question must be answered: what conclusion follows from these premises? Answer: “some metals do not drown in water.” Here the thought goes from premise to conclusion.

Another example. The judgment is given: “some metals do not sink in water”; it is required to answer the question: by what premises can this statement be substantiated as a conclusion to a conclusion? Answer: such premises can be, for example, the premises: “potassium — metal” and “potassium does not sink in water.” Here the thought goes from the conclusion to the premises justifying this conclusion.

But the situation is not different with the proof. And in the proof there are possible, as we will see below, two ways of establishing the truth of the statement being proved: one is that from established or recognized statements the argument goes through a series of consequences derived from these statements to the statement being proved; the other is that, having considered the proved proposition, they show that, provided that this proposition is accepted as true, a number of provisions follow from it, the truth of which has already been established and which have been proved in other ways.

Thus, the difference between the proof and the conclusion, or inference, is not at all that the thought goes from the premises to the conclusion in the conclusion, but vice versa in the proof. Both in conclusion and in proof, both of these lines of thought are equally possible.

The main difference between the proof and the conclusion is that the conclusion is the discretion of the necessary connection between the concepts that form the final judgment, the proof is not only the discretion of the connection between the *concepts*, but also the discretion of the truth of the *judgment*. It is clear that where the truth of the judgment is justified, as in the example considered above, by inference, the proof takes the form of inference of inference.

§ 5. This distinction between conclusion and proof determines the *structure of the proof*.

From a logical point of view, proof is not the process of proof itself. The proof is a *special logical form*, expressing the logical result of an already established process of proof, that is, the justification (or refutation) of the position being proved (or refuted).

In every proof there is, *firstly*, *a provable statement* proving that the well—known thesis is true (or, conversely, false). So, in the example considered by us, the proved position is the position: "the conclusion that all bamboos bloom in spikelets is true." This shows that the *position* to be *proved* must be distinguishable from the *thesis* . A thesis is a proposition whose truth or falsity is proved. In our example, the thesis is the proposition: "all bamboos bloom in spikelets." The proved position is a *proposition about the thesis*, or a judgment in which the thesis is verified as true or false. In our example, the provable position is the following: "the conclusion that all bamboos bloom in spikelets is true." The proved position contains the thesis as its part.

The difference between the position being proved and the thesis clearly appears in the proofs whose task is to *refute* , i.e., to prove the *falsity of* the thesis in question. When refuting a proven position, it is always formulated so that it is clear not only what the thesis is in question, but also that this thesis is false. Here, both the thesis and the characterization of this thesis as false are given separately.

On the contrary, in evidence whose task is to *justify*, i.e., the proof of the *truth of* the thesis under consideration, the position to be proved is very often formulated so that only the thesis itself is expressed, while the characteristic of the truth of the thesis is omitted. In our example, the proven position instead of the full form ("the proposition" all bamboos are cereals is true “) could be expressed in an abbreviated form: “all bamboos are cereals”.

However, regardless of whether the position being proved consists of only one thesis or a thesis accompanied by a separate characteristic of its truth (or falsehood), the main task of any proof is *precisely the characteristic of the truth (or falsehood) of the thesis*. Where the position to be proved consists of only one thesis, the characteristic merges into one with the statement of the thesis, but does not lose its significance from this.

The second component of any evidence is the *basis* , i.e., the judgments, the truth of which is either already established, or at least is supposed to be undoubted, and which therefore can serve as premises for inferences, through which the proposition about the truth (or falsehood) of the thesis is proved.

The third component of any evidence is *reasoning (argumentation, demonstration)*, i.e., a number of conclusions proving the truth (or falsity) of the thesis. The reasoning compares the grounds, which serve as premises of conclusions, with the conclusions that follow from these grounds. The terms “argument” and “argument” are sometimes used to indicate parts of evidence in general. Sometimes the “argument” (or “argument”) refers to the *whole proof as a whole* , that is, the thesis, grounds and reasoning. Sometimes these terms denote the basis of evidence.

The Main Types of Evidence

§ 6. All evidence can be divided into two large groups— depending on whether they examine the *truth of the* content and the correctness of the logical connection between the grounds and the thesis or investigate the *origin of the* judgments included in the proof, the *source* from which these judgments are drawn, *the conditions* under which they came to us or are passed on to us, etc.

The evidence in which the content of the grounds is examined, as well as the logical connection between the bases and thesis, are called *substantive evidence* . In this evidence, nothing is required to prove the truth (or falsity) of a thesis except to examine the grounds for *the essence of their content* and in addition to considering the logical connection between the grounds and thesis. In substantive evidence, to characterize a thesis as true or false, it suffices to make sure that there is a necessary logical connection *between the content of true judgments* , which play the role of grounds, and the *content of the thesis* .

The evidence that examines the *origin of the* judgments included in the evidence, as well as the conditions under which these judgments have come down to us, are called *evidence based on the source of the* judgments, or *genetic* (from the Greek word “genesis” meaning origin).

§ 7. If every judgment that we adopt as true was accepted by us only on the basis of substantive evidence, then the volume and variety of our knowledge would be much smaller than what they really are.

There is a number of knowledge in which the belief in their truth arises as a result of our belief that the sources from which this knowledge is drawn cannot lead us astray. Such in the vast majority of cases, all the knowledge we acquire at school related to the field of geology, geography, astronomy, etc. Not only the student, but also the teacher, relying on the textbook on which the subject is taught, cannot prove each of their statements essence, that is, by considering only the *content of* judgments and the *logical connection* between these contents.

None of us could visit *all* corners of the globe, in order to verify by substantive evidence all those truths that are communicated, for example, by a geology textbook on the composition and structure of the earth's crust in various places and on the nature of the processes of formation and disappearance of mountains, seas, continents, etc. Of course, many of these truths that we learn from the textbook by proving by the source of their origin could be verified by us also by proving the essence of their content, if we could only visit all the places studied by geologists and make sure by referring to observations and to experience, in the truth of their statements.

However, the need to be satisfied with evidence from the source of our knowledge is determined not only by the limited nature of our personal experience and our inability, within the boundaries of this experience, to verify all the unimaginably great multitude of truths established by science through substantive evidence.

There are a number of sciences and branches of knowledge in which, by the very nature of these sciences and these branches of knowledge, many proofs are always forced to remain merely *genetic* , that is, proofs of the source of the origin of judgments. In all *historical*. There is no other way for the sciences to prove the great number of truths established in them, except to ensure that the sources from which we derived these truths are trustworthy, cannot deceive us or mislead us. None of us witnessed, for example, the battle of Borodino, but we know with absolute certainty that this battle took place on August 26, 1812, that the Russian army was commanded by Kutuzov, and that of the French army by Napoleon, that the Russian troops repelled all French attacks and inflicted on the army invasion, a fatal blow for her, etc., etc. All of these and countless similar truths are justified by *proving by the source of our judgments*—through the study, comparison, critical examination and comparative assessment of official documents, messages, reports, orders, reports, diplomatic correspondence, diaries and eyewitness accounts, memoirs of participants in these events, journalistic literature, etc.

Therefore, *genetic* evidence is special a group of evidence that cannot be reduced solely to *substantive evidence* . Moreover: *genetic* evidence in its importance to knowledge is *not only not inferior* to substantive evidence, but *often surpass it*. What could be more important for our knowledge and for the formation of our worldview than those truths that we learn by studying, for example, questions of the history of society? But the truths of historical sciences are justified only by analysing and studying the sources from which they can be drawn, that is, by means of *genetic* evidence.

Substantive evidence

§ 8. The evidence essentially represents, as we already know, an investigation of the content of the grounds and the logical connection between the bases and the thesis. In this evidence, the question is solved: is there a necessary logical connection between the content of these grounds and the content of the thesis which is derived from them.

The evidence is essentially divided into *four* main groups: 1) evidence in which all cases of the thesis being proved are exhausted by complete *induction* ; 2) *separation* evidence, in which all assumptions are excluded, except for one, namely, in addition to the thesis being proved; 3) *refuting* evidence, or rebuttal, in which, from the truth of a known judgment, they conclude that another judgment is incompatible with the first; 4) *conditional* evidence in which, from the presence of all necessary conditions of truth (or falsity), judgments are concluded to its actual truth (or falsity).

§ 9. *Evidence that exhausts all possible cases of the thesis being proved* . In this evidence, the thesis being proved is considered first of all. This consideration is intended to completely exhaust all possible cases of the thesis being proved. It is further proved that the thesis is true for each of these cases separately. Hence, by the method of complete induction, the conclusion is drawn that the thesis is true in general, i.e., irrespective of a particular case.

This kind of *proof* is essentially often used in mathematics, especially in geometry. By means of this form of proof, for example, a theorem is substantiated, according to which no conic section intersects a line at more than two points.

§ 10. *Separation evidence*. In the separation evidence, the truth of the thesis being proved is verified by excluding all hypotheses of the separation conclusion, except for the only one which is the thesis being proved. Since one hypothesis must necessarily be true in its entirety, exhausting possible division, and since all of them, except for the hypothesis that coincides with the thesis being proved, have been refuted, the thesis as the only hypothesis that remains unverified will necessarily be true.

If, for example, it is established that only A, B, C and D could commit a certain crime, and if, in addition, it is established that neither B, nor C, nor D committed it, then it follows that the conclusion recognizing the culprit of crime A, truly.

However, however, as is always the case *justification of the* thesis being proved, the position being proved is limited to one thesis, the characteristic of the thesis itself as true is usually omitted.

The peculiarity of this form of evidence is that the truth of the thesis being proved is not verified directly, but *indirectly*. Indeed, the justification of the thesis being proved is achieved in this case not by direct research or substantiation of this thesis, but only indirectly—by refuting all possible assumptions, except for one that coincides with the thesis.

§ 11. *Refuting evidence*. The proof of this form is not the justification of the thesis, but its *refutation*. A rebuttal is achieved by comparing the thesis with another proposition that stands for the thesis regarding logical incompatibility. The basis for the conclusion that the thesis is false is the confirmation that the proposition incompatible with the thesis is true.

Thus, the falsity of the opinion of old zoologists, who believed that no mammal belongs to the ovipositor, was proved as soon as it was established that some mammals, such as platypuses, belong to the ovipositor. The evidence in this case was rebuttal. It boiled down to comparing the rebuttable thesis with a contradicting one, i.e., with an incompatible judgment.

Refuting evidence is enormous in practical life and in science. Evidence of the innocence of the accused *directly* the commission of the crime attributed to him is achieved by refuting the assumption that the accused could have committed it. Having established, for example, the alibi of the accused, i.e., the absence of the accused at the time the crime was committed, in the place where it was committed, the court thereby verifies the truth of the situation, logically incompatible with the assumption that the accused is guilty of *direct* commission of the crime. This refutes the assumption that the person charged with the crime is indeed the *direct* perpetrator of the crime.

§ 12. *Conditional evidence*. In this evidence, the study begins with the establishment of all necessary conditions for the truth of the thesis. It is further verified that all these conditions are present. From here they conclude to the truth of the thesis.

An example of conditional evidence has already been considered in explaining the difference between evidence and simple inference. Another example of conditional evidence: it is required to prove the thesis that some arthropods are not insects. This very thesis can be inferred from the following conclusion: “All spiders are arthropods, not one spider is an insect, therefore, some arthropods are not insects.” But our task is not only to perceive the necessary logical connection between the found premises and the thesis. Our task is to prove that the thesis “some arthropods are not insects” is true.

To confirm its truth, we develop the following conditional proof: “If the premises” all spiders are arthropods “and” not a single spider is an insect “are true and if the conclusion is correct, then the position” some arthropods are not insects “is true.” But both premises are really true, and the conclusion is also correct. Therefore, the thesis “some arthropods are not insects” is true.

§ 13. As can be seen from both examples, the conditional proof is an inference about inference. In both cases, the first conclusion was found justifying the thesis. Then an inference was found proving that this thesis is true.

The conclusion by which a logical connection is established between the foundations and the thesis is called the *main* inference of conditional evidence. In our last example, the main conclusion is the *first* conclusion: “All spiders are arthropods, not a single spider is an insect, therefore, some arthropods are not insects.”

That inference, by means of which the thesis is verified as follows from the presence of all the conditions of its truth, is called *conditional* inference.

By no means always the proving evidence contains both of these conclusions: *basic* and *conditional*. Usually, conditional inference is not expressed in the text of the proof itself and is only implied. Only the main conclusion is fully formulated. But since in the main conclusion only the necessary logical connection between the grounds and the thesis is revealed, the truth of the thesis is confirmed only by conditional inference, this last is the main component of the conditional evidence.

By the example of the proving evidence, it is better than by the example of any other form of proof, it is clear that there is a difference between the proof in the special sense of the concept and simple inference.

§ 14. Since the proving evidence consists of two conclusions and since one of them, namely the conditional, is usually only implied, it is often difficult to determine in the case of the proving evidence which judgments are the *basis of the* evidence. Indeed, since in the conditional proof usually only the *main* conclusion is fully expressed , the idea easily arises that its premises constitute the basis of the whole proof. But since the characteristic of the thesis as true (or false) is contained only in the *conditional* inference (regardless of whether it is expressed or only implied), then, strictly speaking, the premises of the conditional evidence are the premises of the conditional inference: the premise indicating the necessary conditions for the truth of the thesis, and the premise certifying that in this case all these conditions are present.

§ 15. The most widespread variety of conditional evidence is evidence in which, having ascertained the truth (or falsity) of the premises of the main conclusion and the correctness of the logical connection, they conclude from this the truth (or falsity) of the thesis.

Suppose, for example, you want to prove that not a single fern reproduces by seed. We conclude: “not a single spore is propagated by seeds, all ferns are spore, therefore, not a single fern is propagated by seeds.” Consider the premises and the logical connection between them. Since this consideration reveals that both premises are true and that the logical connection between them is correct, we have the right to deduce that the basic conclusion is true. From the truth of the main conclusion, it follows that the resulting judgment “no fern reproduces by seed” is true. But this proposition is a proven thesis.

§ 16. The second widespread variety of conditional evidence is the proof in which, having ascertained the falsity of a certain judgment, they conclude from this the falsity of the main conclusion from which this proposition follows.

But the falsity of the conclusion can be due to: 1) either the falsity of the premises, 2) or the incorrectness of the logical connection between the premises, 3) or the combination of the falsehood of the premises with the fallacy of the logical connection established between them.

Therefore, having established on the basis of the falsity of the thesis — the falsity of the inference substantiating this thesis, we still do not know which of these three conditions exactly causes inaccuracy of the inference in each given case. To solve this issue, first, all the premises of the main conclusion must be investigated, and secondly, the logical connection between them.

In this study, *two* cases are possible . *The first* of these is when the study establishes that the logical connection between the premises of the main conclusion is correct and that all the premises, with the exception of the only one that is not considered, are true. The result of the study in this case will be a *separation* inference: “The premises themselves, or the logical connection between them, could be erroneous. But since neither the logical connection between the premises, nor the premises—except for one that we have not considered—are erroneous, the only premise that has not been considered is erroneous."

§ 17. An example of this case is evidence called apogological, or "leading to absurdity" (reductio ad absurdum). If, in considering this proposition, we could immediately contrast it with another proposition, logically incompatible with the first and at the same time obviously true, then we would thereby refute this proposition. This would be an ordinary case of the so—called "refuting" (see paragraph 11 above), and not a conditional proof.

But if we cannot immediately find such a judgment, which, being incompatible with the given one, would be obviously true at the same time, then the refutation of the thesis takes the form of the conditional evidence discussed above. Namely: a conclusion is being constructed in which the thesis, i.e., a refuted judgment, is one of the premises. All other premises of the conclusion are selected true, the logical connection between them is established correct. Having received—according to the rules of the conclusion— the conclusion, then they find another judgment with such a calculation that it is logically incompatible with our conclusion and at the same time that it is true. Having found such a judgment, they thereby refute the conclusion. In turn, the refutation of the conclusion reveals the fallacy of the conclusion from which the conclusion was deduced. But what can be the fallacy of the conclusion in this case? Since the logical connection in it is correct and since all the premises, except the one that is the thesis of proof, are true, only the thesis should be false.

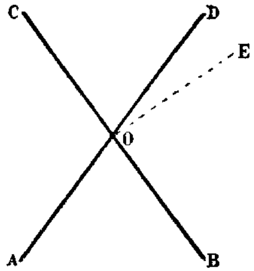


Fig. 67

An example of apagogical evidence. In geometry, a theorem is proved (see Fig. 67), according to which, provided that two equal angles *AOB* and *COD* have a common vertex *O* and two sides of *OB* and *OC* in one straight line, then the other two sides of *OA* and *OD* make up one straight line , and therefore the angles *AOW* and *COD* are vertical. The theorem is proved as follows. *Assume* that *AOD* is not a straight line, but a *broken* line. We assume further that *OE* is a continuation of the side of *AO* . Then the angles *AOW* and *SOE* as angles made up by the intersection of two straight lines, the angles will be vertical and, therefore, equal to each other. But by position $\angle DOC$ is equal to $\angle AOB$. Two quantities equal separately to the third are equal to each other. Therefore,$\angle EOC$ should be equal to $\angle COD$ (since $\angle EOC$ and $\angle COD$ are equal separately for each $\angle AOB$).

But $\angle EOC$, obviously, cannot be equal to *DCOD*, since $\angle COE$ is only a part of $\angle COD$. So, the assumption that AOD is not a straight line, as an assumption leading to an absurd conclusion that a part is equal to its whole, is false. But if it is false that *AOD is* not a straight line, then it must be true that *AOD* is a straight line and that the angles *AOB* and *COD* are vertical.

Looking closely at the course of this argument, we see that it is quite suitable for the scheme of the variety of conditional evidence under consideration. The purpose of the argument was to prove the theorem by refuting the thesis that contradicted it. The refuted thesis was made by one of the premises of the conclusion. All other premises, except for the thesis, turned out to be true. The conclusion itself was also correct. The conclusion drawn from this conclusion (the equality $\angle EOC \quad \angle COD$), compared with the axiom that the whole is larger than its part, turned out to be incompatible with it.

Thus, it was confirmed that the conclusion that *OEOC* equals $\angle COD$ false. But the falsity of the conclusion means the falsity of the conclusion from which this conclusion is derived. In turn, the study of the falsity of inference leads to the following dividing syllogism: “The source of an error in our inference could be either the falsity of the premises or the erroneousness of the logical connection between them. But in this case, the logical connection was correct, all the premises, except for the one which is a rebuttable thesis, are also correct. Consequently, the refuted thesis is false.”

§ 18. The logical scheme of the considered variety of conditional evidence in itself is completely simple and clear. However, in its implementation in practice often have to overcome significant difficulties.

These difficulties usually arise in that part of the proof where the conclusion drawn from the main conclusion must be contrasted with another, which is incompatible with it and at the same time is obviously true.

Indeed, the successful solution of this problem requires that the conclusion drawn from the main conclusion must be *false* , and the judgment opposed to it and incompatible with it must be *true*.

As for the falsity of the conclusion, generally speaking, as the conclusion of the conclusion, which includes a false premise (a rebuttable thesis), this conclusion should be false. However, sometimes with a false larger premise, the conclusion of the syllogism may accidentally turn out to be true. For example, from the premises “all students study French” and “Nikolaev is a student”, the conclusion “Nikolaev is studying French” is obtained. It may happen that, in spite of the falsity of the larger premise, which claims that *all* students are studying the French language, Nikolaev will accidentally belong to that part of the students who, without exhausting all the students, really learn the French language. In this case, the falsity of one of the premises does not preclude the truth of the thesis. This is not explained by the fact that this truth is logically *follows* from the falsity of the premise, but by the fact that it *does not depend on the quantity of the* larger premise: for the student Nikolaev to be a student of French, there is no need for *all* students to learn this language. For this, it is enough that at least some of the students learn this language and that Nikolaev turns out to belong to this particular part.

Knowing that under certain conditions the presence of one false in the number of premises can be combined with the truth of the conclusion, we must reckon with this possibility when developing apagogical evidence. Since in this evidence the conclusion of the main conclusion must be *false*, then the premises of this inference must be selected in such a way that the combination of a false thesis, which is one of the premises of the inference, with its other true premises, will inevitably give a *false* conclusion in conclusion.

On the contrary, a judgment contrasted with imprisonment as incompatible with it must necessarily be *true.*. However, far from always the truth of the judgment, which is opposed to the conclusion and incompatible with it, turns out to be undeniable for those to whom the evidence refers. In many branches of knowledge, the judgment that is true in the eyes of some seems false or at least doubtful to others. But if the judgment, which is opposed to a thesis incompatible with it, seems false, then the thesis itself will no longer be assessed as false, and thus the refutation of the thesis, which forms the centre of the whole proof, will not be reached.

§ 19. We examined the *first* case study of the fallacy of the main conclusion. In this case, the study establishes that the logical connection between the premises and the conclusion is correct and that all premises are true, except for the one that is the refuted thesis.

The second case of the study of the main conclusion takes place when the study establishes that it is not the premises that are erroneous, but the *logical connection* between the premises and the conclusion.

To verify the erroneousness of the logical connection, the studied basic conclusion is compared with another conclusion. This last one is selected so that all the premises in it are true, that the logical connection between the premises and the conclusion is erroneous, and that the conclusion is clearly false.

The fulfillment of all these conditions gives the right to the following conclusion. The conclusion with which we compare the main conclusion of our proof has a false conclusion. Therefore, it is erroneous. Its error can be caused either by the error of the premises, or by the error of the logical connection. Since all the premises in it are undoubtedly true, only a logical connection can be erroneous in it. But our basic conclusion has the same structure as the conclusion with which it is compared. Since the conclusions in these conclusions are false, and all the premises are true, in the main conclusion, only the logical connection is erroneous.

For example, it is required to investigate the error in the conclusion: “All the great artists were impressionable, N is impressive, therefore, N is a great artist.” If the fallacy of the logical connection in this conclusion is not striking and does not lend itself to an exact logical definition, due to a lack of logical knowledge in the researcher, then it can be detected as follows.

The studied conclusion is compared with another, having the same structure, the same false conclusion, but containing only true premises: "All great artists have two arms and two legs, H has two arms and two legs, so H is a great artist ". Since both conclusions have absolutely the same structure, since all the premises in them are true and the conclusions are false, only the logical connection between the premises and the conclusion is erroneous in both.

Genetic Evidence

§ 20. We already know that the second group of evidence after the *evidence is essentially the* so—called *genetic* evidence, or evidence *by source of origin*.

In genetic evidence, the truth (or falsity) of a thesis is proved by examining: 1) the conditions for the *occurrence of the* thesis and 2) the conditions for its *transfer* from one person to another.

The need for genetic evidence arises everywhere where the question of the source of judgment is of particular importance, and where the question of whether the judgment that has come down to us by transmission coincides with the original judgment that served as the source for it.

Thus, the historian is constantly forced to verify the truth of the opinions expressed by historical figures, eyewitnesses, and memoirists. To do this, he turns to the study of the source of these judgments, the awareness, conscientiousness and accuracy of the people who expressed them, etc. The historian of ancient or ancient Russian literature uses genetic evidence in assessing the truth of direct and indirect information about certain authors that has come down to us. about their life and work, about their works, etc. The judicial investigation verifies the truth of the testimony of a particular fact or action, examining possible intentional or involuntary distortions of the truth when transferring judgments from one person to another, etc.

In many cases, when deciding on the truth or falsity of judgment, we have no other way but to prove by the source of origin.

§ 21. Genetic evidence, like any evidence, is either the establishment of the truth of the thesis (its justification), or the discovery of its falsity (its refutation).

The justification of the thesis in genetic evidence has the following structure. In the *first* part of the proof, it is established that the initial judgment, by virtue of the very conditions of its occurrence, could not be wrong. In the *second* part of the evidence, it is established that the thesis being checked coincides with the original judgment, since the judgment could not be distorted when transferring the initial judgment from person to person. This part proves that: 1) the initial judgment could not change due to memory errors; 2) the person who submitted the judgment did not intend to intentionally distort it; 3) this person accurately expressed the meaning of the judgment; 4) a person who has adopted a judgment in someone else's transfer correctly understood the meaning of the transmitted.

If the answer to all these questions is positive, then it follows that the thesis being verified really coincides with the original judgment.

In the *third* part of the proof, as a result of previous studies, a conclusion is drawn on the truth of the thesis.

Through genetic evidence, *only probable* , but not reliable, judgments are justified . The degree of probability of judgments proved in this way, generally speaking, varies widely—from a very small probability to a probability that practically borders on reliability. The more links of the transmission pass the reported initial judgment, the more easily various distortions of its meaning can arise, the less the probability of the thesis being proved becomes.

§ 22. The *refutation of* judgments in genetic evidence, as in all others, is the establishment of the falsity of the thesis being proved. Typically, the establishment of this is achieved as follows. The *first* part of the refutation establishes *falsity* initial judgment. In the *second* part of the refutation, it is established that the judgment in question coincides with the original judgment, since in any of the links of the transfer the initial judgment could not be subjected to any distortion.

Since the initial judgment, according to the first part of the proof, is false and since the considered judgment, according to the second part of the proof, is identical with the original, the considered judgment is also false.

§ 23. One should not think that the falsity of the initial judgment makes it unnecessary to consider the conditions for its transfer from person to person. Although, generally speaking, the falsity of the initial judgment also means the falsity of the judgment that results from the transfer, there may be cases when, as a result of some changes that have occurred in the links of the transfer, the considered consideration is accidentally true.

So, for example, a person expressing a judgment can deliberately tell a lie, that is, give out a deliberately false message as true. But if, wanting a lie to pass off as truth, the person himself will be mistaken and mistakenly consider what is really true to be false, then as a result of the transfer the considered consideration may turn out to be true.

§ 24. A genetic rebuttal is possible not only where the initial judgment is *false* . If the initial proposition is true in itself, but if at the same time it has experienced a change in the links of transmission, and if the change makes the initial proposition incompatible with the underlying judgment, then it is proved that this last proposition is false. For example, an accused of accepting a bribe on behalf of N claims that he did not take a bribe.

If this statement is true, then in this case, of course, he could not be mistaken, that is, the initial judgment relating to this case should have been true. But testimony revealed that in this case the accused is lying. Since the lie is nothing but the replacement of the original judgment with an incompatible judgment, it follows that the statement of the accused is false.

The Role of Practice and Experience in Evidence

§ 25. In all sciences and in all scientific evidence, all the concepts that make up the evidence have their origin in the final analysis from practice, from experience. In this respect, the proofs of the mathematical sciences are no exception. True, the concepts used by the mathematician are abstracted from a number of properties that belong to the objects of these concepts in our experience. The mathematical circle, cube, ball, etc. do not exist in experience in the form in which the mind of the geometry thinks. And yet, even the most abstract concepts of mathematics arose ultimately from experience and from experience. The same is true for mathematical *definitions* and for *axioms*, i.e., directly obvious truths that underlie all mathematical knowledge. No matter how distant from experience, and sometimes even contrary to experience, these definitions and axioms, all of them are ultimately products of distraction from certain aspects of experience and could not form in thought otherwise than on the basis of experience.

§ 26. This is the case with the concepts, definitions and axioms of mathematics. More difficult is the case with evidence. In all sciences, except mathematical, the proof is always *directly* connected with experience. This means that, in addition to the connection with experience, without which there could have been no concept, no axiom, in these sciences the evidence always includes parts and data that *directly* involve a reference to experience: observation, experiment, etc.

On the contrary, in the mathematical sciences, evidence— if we consider one *logical* side of them, rather than the *origin of the* concepts that make up the evidence — are always carried out in such a way that mathematics in the course of the proof does not one has to turn *directly* to experience, in addition to those elements of experience that are already contained in its concepts, definitions, and axioms. In other words, experience is *not directly* included in mathematical evidence as it is included in the evidence of a physicist, chemist, biologist, but only *through concepts* that were once formed on the basis of experience, but in their modern content are abstract in relation to this experience.

§ 27. This distinction between *mathematical* sciences and *empirical* sciences, that is, proving their position on the basis of a direct appeal to experience, gives rise to a difference in the types of evidence.

The proofs of the mathematical sciences, which do not require direct evidence of experience *in the course of the proof* and are based on experience only through the elements of experience that are contained in the basic concepts, definitions and axioms of these sciences, are called *mathematical* proofs.

Evidence of nature sciences that require involvement *direct* evidence of experience *in the course of the proof* and, therefore, not limited to those elements of experience that are contained in their basic concepts, are called *empirical* evidence.

From these definitions and explanations it is clear that the difference between the two types of evidence is not that the proofs of the mathematical sciences are supposedly outside the experience, but the evidence of the empirical sciences is based on experience. *All the* proofs of *all* sciences — mathematical as well as empirical — presuppose experience as the necessary last basis and test instance of all their truths and propositions.

The difference between these two types of evidence is due only to the fact that in one course of the proof a direct access to the data of experience is required, in the other for the implementation of the evidence that connection with the experience, which is already given in the very content of the concepts that make up the evidence, is sufficient.

It can be seen from the foregoing that the difference between mathematical and empirical evidence is not unconditional. A number of nature sciences, proving their truths through a direct appeal to experience, contain in themselves also those parts in which evidence is conducted by the method of proofing of the mathematical sciences. On the other hand, in the mathematical sciences, the mathematical form of proof is often preceded by justification, which involves a *direct* appeal to experience, so that the mathematical form of the proof is developed later, when the thesis being proved, i.e. the result of the proof, has already been known from experience. An example of such a transition from the result found in the experiment to its mathematical and deductive justification in form can serve as the history of the Archimedean definition of the parabola area already mentioned above.

Finally, even in strictly mathematical proof form, the last grounds on which these evidence is based, namely the definitions of the basic concepts of science and axioms, arose ultimately on the basis of experience, although in the content in which they are currently thought by science, they due to their extreme abstraction, they may seem independent of any experience.

The division of evidence into mathematical and empirical depends, as has been shown, on whether the evidence is conducted without a *direct* appeal to the experiment or whether the evidence also includes a *direct* reference to the experimental data in one volume or another.

§ 28. Evidence also differs in the course of thought in *the argument itself*.. A proof in which reasoning proceeds from established or recognized propositions—through a series of corollaries derived from these propositions—to a thesis or a proven proposition is called *progressive* proof. The name of this shows that the thought in the course of reasoning goes forward all the time — from reason — through reasoning—to the thesis being proved.

So, for example (see Fig. 68), from the Pythagorean theorem ($a^2 + b^2 = c^2$) and from the definition of trigonometric functions of sine and cosine ($\sin(\alpha) = a / c$ and $\cos(\alpha) = b / c$) can be obtained through progressive proof one of the basic formulas of trigonometry.

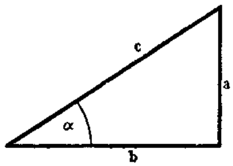


Fig. 68

In fact, by the Pythagorean theorem we have:

$$a^2 + b^2 = c^2 \quad (1)$$

We divide both sides of the equation by c^2 and get:

$$a^2 / c^2 + b^2 / c^2 = 1 \quad (2)$$

On the left side of the equation, each of its terms is a square:

$$(a / c)^2 + (b / c)^2 = 1 \quad (3)$$

But since, according to the definitions, $a / c = \sin(\alpha)$ and $b / c = \cos(\alpha)$, then our equation (3) takes the form: $\sin^2 \alpha + \cos^2 \alpha = 1$.

§ 29. But the course of reasoning in the proof may be the opposite. In some cases, the argument does not proceed from the grounds, but from the consideration of the thesis being proved. Consideration of this shows that from the thesis (if it is accepted), a number of provisions follow, which are already known that they are true, and which were proved in other ways. The proof in which reasoning does not go from the grounds to the thesis, but vice versa — from considering the thesis to clarifying the necessary connection of this thesis with the grounds, is called *regressive*. This title shows that thought in the course of reasoning goes backward: from the thesis to the foundations.

Often the same situation can be proved both in a progressive and regressive way. The same trigonometric formula that we deduced above by means of progressive proof can be deduced by proving regressive.

It is required to prove that $sin^2 \alpha + cos^2 \alpha = 1$. Considering the thesis being proved and recalling that, by definition, $\sin \alpha = a / c$ and $\cos \alpha = b / c$, we can express the thesis in the equation:

$$(a / c)^2 + (b / c)^2 = 1. \quad (2)$$

Having made the a / c and b / c required by formula (2) squared, we obtain:

$$a^2 / c^2 + b^2 / c^2 = 1. \quad (3)$$

Multiplying the parts of equation (3) by c^2, we have: *and* $^2 + b^2 = c^2$ (4), i.e., the formula of the Pythagorean theorem.

In the history of the development of science, quite a few propositions were first found by *regressive* evidence. Often a *hunch* about the truth, *anticipation* truths preceded that form of proof in which the thesis to be proved is obtained as the result of a long series of conclusions going from the grounds to the proved position. In these cases, the evidence takes a regressive form. The researcher, "anticipating" the truth of the thesis, directs his attention to clarifying the necessary connection that exists between the thesis and other truths previously known from other grounds.

§ 30. Mathematical evidence can be distinguished depending on whether the thesis *is* proved *directly* or *by refutation of a judgment contrary to the thesis being proved.* The proof in which the thesis is directly derived from other propositions established or accepted as true is called *direct*.

The proof in which a proposition contrary to the thesis is refuted to substantiate the thesis is called *indirect* . From this definition it is clear that indirect evidence includes the already known apagogical evidence.

Apagogical evidence is also called "reductio ad absurdum" [1] , that is, "leading to absurdity." This title indicates that the conclusions from the assumption adopted at the beginning of the apagogical proof are drawn up until they reach a conclusion that is ridiculous, since it contradicts other — true — premises.

It is easy to see that in the course of this proof the tollens modus is applied, as well as the law of the excluded third. Indeed: the falsity of the admitted position is deduced from the falsity of the consequence to which this assumption leads, i.e., by the tollens modus, and the truth of the thesis being proved is deduced from the falsity of the admitted position, which stands in relation to the contradicting opposite to the thesis and therefore, being false, thereby proving, according to the law of the excluded third, the truth of the thesis.

In mathematics, apagogical evidence is called “evidence from the contrary.” This name, from the point of view of *logical* terminology, is not entirely accurate, since in the proofs of these not *contradictory* to the proved thesis is disproved , but precisely *conflicting* assumption.

Rebuttal

§ 31. The rebuttal, as we already know, is essentially no different from the proof. The refutation consists either in proving that the premises are erroneous or doubtful, or in proving that the conclusion does not necessarily follow from these premises, even though each of them individually was true. At the same time, the refutation does not require the premises to be necessarily *false: it is* enough that they are only *doubtful*—and the conclusion is no longer valid.

The refutation of a well—known *statement*, that is, the proof of its falsity in essence, is at the same time a refutation of any proof of this statement, whatever the forms of proof used in this case.

But rebuttal *of this evidence*, i.e., the discovery of its insolvency, is *not yet a refutation of that thesis, or statement,* *which should have been substantiated by means of this evidence*. It is quite possible that the thesis is essentially true, but the proof by which they try to substantiate it is erroneous. It can be erroneous either because they try to deduce it from false foundations, or because, despite the truth of the foundations, they are not able to show what is the necessary connection leading from these grounds to the thesis.

Therefore, the discovery of inconsistency of *evidence is* not yet the discovery of the falsity of the *proved position*. Since one and the same proposition can be proved, generally speaking, not in one single, but in several ways, there may be a case when, having refuted an inconsistent proof, they then indicate the true, by which the thesis can really be proved. Similar cases are observed in the practice of everyday thinking and in the development of science. It happens that an unsophisticated debater argues for a substantively correct position, but is unable to find proper evidence that would lead to the evidence the thesis he proves. But even in the history of sciences, even as precise as mathematics, it happened more than once that in the evidence that was previously considered perfectly strict, over time—as the concepts were clarified—inaccuracies were discovered, and then these proofs were corrected, i.e. replaced more strict, really revealing the necessary connection between the grounds and theses.

Grounds as Part of Evidence

§ 32. Considering the proofs of any mathematical science, it is easy to notice that all the true propositions of this science form a kind of long chain in which each thesis being proved is based on previously proved bases, and these bases are in turn proved as theses from other bases and etc.

However, this ascent from theses to the foundations and from these foundations, regarded as theses, to other foundations cannot continue indefinitely. Sooner or later, we will come to such provisions that can no longer be proved with the help of other bases and which themselves are the grounds by which they prove — directly or indirectly — all the provisions and theorems of this science without exception.

Direct the participation of these bases in the proofs consists in the fact that these provisions are applied in the proof of some theorems as the *only* bases on which the proof of these theorems is based. So, in geometry, the *first* theorems of this science are proved not on the basis of other theorems, but on the basis of *definitions of the* basic concepts of geometry and on the basis of some *axioms*, or *postulates*, which are not proved anywhere else.

The indirect participation of these bases in the proofs lies in the fact that the theorems proved with the help of these bases alone, in turn, serve as grounds for proving other provisions and theorems of this science.

Since these foundations are foundations for every mathematical science that cannot be deduced from other foundations, and since, having reached them, we can no longer continue to ascend to new foundations, such bases are usually called the *last* or *initial* foundations of this science as a whole, and all the evidence used in it.

But since in the presentation of mathematical sciences in the first place it is precisely the initial foundations of science that are communicated and only then with the help of these foundations first, and then all subsequent theorems of this science are proved, the initial foundations are sometimes also called the *first* foundations.

§ 33. All the underlying grounds are either *definitions* basic concepts of this science, or its *axioms* .

No science—whatever its subject and its field—can prove its position without a precise definition of the concepts included in this science and all its evidence. Geometry, arithmetic, mechanics, physics, chemistry, political economy, etc. begin with a definition of the basic concepts for each of them. Once established in its content, the definition must be thought of in the same content in all the arguments of a given science and in all its evidence. If, undertaking to investigate, for example, the property of flat triangles, in one case we meant one content under the word “flat triangle”, and in another — another that contradicts the first, then we could not prove the properties of these triangles. And in the same way, if, having undertaken to investigate the laws of production and exchange of goods.

§ 34. In addition to *definitions, axioms* also belong to the highest foundations of science . So called bases that are not proved by this science and accepted by it as initial bases. An example of an axiom in arithmetic can be an axiom according to which the sum of these quantities does not change from a rearrangement of the terms of the quantities, etc.

The similarity between the definition and the axiom is that both definitions and axioms are used as the initial basis of evidence, i.e., such grounds that are not derived from other grounds.

The distinction between definition and axiom can be easily clarified. Definition is the establishment of the content of the basic concept for a given science. Determining, for example, a vertical angle implies agreement among all geometers about what content they understand when it comes to vertical angles. The definition of the concept of “product” implies agreement among economists, according to which by “product” they all mean a thing that can satisfy a need and can be exchanged for other things. The establishment of a system of definitions adopted in this science eliminates the inconsistency in concepts that would be inevitable if there were no agreement on the terms meaning these concepts.

The more accurate the definition, the less the danger of logical errors arising from a lack of certainty in thinking. And, on the contrary, in the absence of precise definitions of concepts, a misunderstanding is always possible, consisting in the fact that the interlocutors or debaters only imagine that they are talking about the same subject, in reality, each of them, in the course of reasoning, under the same word, does not mean the very same (and sometimes completely different) content.

§ 35. In contrast to the definition, which only establishes the content of the concept, the *axiom* is a statement that is considered in this science as knowingly true, although it is not proved anywhere.

The definition, taken by itself, does not yet speak of the necessary truth of the determined. True, in the vast majority of cases, definitions express the very content of the subject that actually exists. But an exact definition of such a concept is also possible, which means an object that does not exist and cannot exist in reality. So, the problem of squaring a circle, that is, finding a square whose area would be exactly equal to the area of the circle, is an insoluble problem, but the very concept of squaring a circle can be determined quite accurately.

On the contrary, the axiom is not a condition accepted with respect to the meaning and content of a well—known concept, but a certain statement, which is considered in this science as a known true position.

§ 36. It is sometimes thought that axioms are not proved because the truths expressed in these axioms are so obvious that they do not require any proof. This opinion is not entirely correct. Indeed, the evidence of truth, taken by itself, does not exempt from the need to prove this truth, if only such proof can be found. In geometry, for example, there are many theorems that seem to the non—specialist to be completely obvious in their truth and which nevertheless, the proofs accepted in this science are proved with all severity. Such, for example, is the theorem according to which the diameter of any circle divides this circle into equal parts, etc.

§ 37. But the axioms are not even *absolutely* obvious.

At least some of the axioms of geometry in ancient times seemed far from unconditionally obvious. Such, for example, is the fifth postulate, or the eleventh Euclidean axiom, according to which through point *C* (see Fig. 69), taken outside the given line *AB* , on the plane where *C* and *AB are located* , only one single line can be drawn, for example, the *OS* which, if continued, would not intersect with line *AB*, so that any other line drawn through point *C* and lying in the same plane, if sufficiently extended, intersects line *AB*.

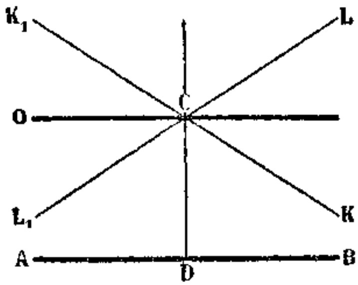


Fig. 69

The independence of a number of propositions proved by geometry from the eleventh axiom, already observed by Euclid himself, the appearance of this axiom in Euclid’s “Beginnings” only after proving 28 theorems of the first book “Beginnings”, inspired geometers to prove this axiom as a theorem. However, the attempt to prove it, made after other geometers by

Lobachevsky and, like theirs, failed, led Lobachevsky to discover that the assumption, which contradicts the parallel axiom, in combination with all other Euclidean axioms, being accepted as one of the initial ones the foundations of geometry, makes it possible to develop a whole system of geometry, which, despite all the contradictions of this foundation, to a direct visual representation of spatial relations.

Having come to this idea, Lobachevsky really developed this system of geometry. In Lobachevsky geometry, instead of the eleventh Euclidean axiom, another axiom is accepted. According to this axiom, through the point *C* lying outside the line *AB* , two parallel lines KSK_1 and LCL_1 [1] pass through it . Each of the equal acute angles *DCK* and DCL_1, which are supposed to be parallel to the perpendicular *CD* from two sides in the Lobachevsky geometry , called Lobachevsky the parallel angles at point *C* relative to the straight line *AB*.

Lobachevsky further showed that at the initial positions adopted by him as the foundations of a new geometry, Euclidean geometry is only a special case of Lobachevsky geometry, namely, the case when the parallel angle has a constant value and is always equal to a right angle.

§ 38. Thus, the axioms are by no means obvious provisions to such an extent that by the obviousness this excludes any possibility of doubt in their truth and any need to demand proof for them. This, incidentally, explains the fact that in the history of mathematics, the largest scientists have repeatedly tried to find evidence for some axioms. Thus, the philosopher Hobbes and the philosopher—mathematician Leibniz tried — though unsuccessfully — to prove the axiom that the whole is larger than its part. Attempts to this kind are prompted not only by the unconditional evidence of axioms, but also by the fact that in the development of mathematical sciences it is always necessary to reduce the circle of unprovable propositions to the smallest possible number. Compared to other propositions, axioms are still the most obvious statements, so it’s easier to see the truth of the axioms, than to see the truth of other provisions that also have evidence. In addition, axioms differ from other obvious provisions in that they represent the smallest number of provisions that, when accepted by a given science without proof as the initial foundations of this science, are combined with the definitions quite sufficient to make them from the definitions all other statements of science could be proved, including some provisions that also have evidence, but are still provable.

§ 39. *Axioms are* sometimes regarded as *postulates* . So—called provisions are not proved, as are axioms, and together with definitions constitute the totality of the initial foundations of science. The difference between the postulate and the axiom is only in that the totality of the postulates considered as the initial foundations of science is established independently of the question of their obviousness and in such a way that the accepted postulates do not contradict each other and thereby make it possible to develop from them also free contradictions the system of truths proved on their basis. The second difference between an axiom and a postulate is that axioms are more general in comparison with postulates.

Along with axioms or postulates, the *lemmas* are included in the system of provisions accepted as true . A lemma is a provision with respect to which it is known that it is recognized as true in the system of some other science and that it is also applied in the system of *this* science.

Moreover, the truth of the lemma can be either directly obvious, or established in this other science by way of proof.

In the system of physics, lemmas are, for example, all the positions of mathematics — regardless of whether they are considered as axioms or proved as theorems.

Sometimes theorems are distinguished and the provisions deduced from them: consequences and additions. From the point of view of logic, these differences are not significant.

§ 40. Not every attempt at proof is successful. In the proofs, as well as in other types of logical activity of thinking, various errors are possible that deprive the proof of its power.

Since any proof consists of: 1) the thesis being proved, 2) the reasons and 3) reasoning, the errors possible in the proofs are: 1) either errors in the thesis, 2) or errors in the grounds, 3) or, finally, errors in reasoning.

Errors Regarding the Thesis Being Proved

§ 41. Errors regarding the thesis being proved arise in cases where, despite the truth and recognition of the grounds, and also despite the correct course of conclusions, that is, despite the presence of the necessary logical connection between the grounds and the conclusion, the conclusion itself does not coincide with that thesis to be proved. In other words, the mistake here is not that they make the wrong conclusion, but that, having correctly drawn the conclusion from the true grounds, they mistakenly believe that this conclusion is the very position that it was taken to prove, while in fact the conclusion this does not coincide with the thesis being proved and is only mistakenly taken for this thesis.

This mistake is called “substitution of the thesis, which must be proved”, or “deviation from the thesis”, “ignoring the thesis, which must be proved”.

Cases of such an error are very common. Especially in disputes one can often see a picture when, in order to refute the enemy, they refute not the position that he actually expressed, but a completely different position, which, however, is mistakenly thought to be the position expressed by the enemy. In such cases, the argument resembles the battle of Don Quixote with the windmills that he took for the giants.

§ 42. No less often in disputes is the fact that, having disproved the evidence by which the adversary tried to substantiate his thesis, it is erroneously assumed that this also refuted the most proved thesis. But, as we already know, a refutation of *evidence* is not a refutation of a proven *position*. It is possible that this provision itself is true and only requires other evidence instead of the erroneous one by which they tried to substantiate it. It is quite obvious that one who accepts a rebuttal of *evidence* as a rebuttal of a *provable provision* makes a mistake in substituting a proven thesis. For example, one of the disputes proves the existence of organic life on Mars on the grounds that the astronomers Schiaparelli and Lovel observed on the surface of Mars a network of regular lines intersecting and converging at known points that were taken by Lovel for "canals", as if constructed by the inhabitants of Mars.

Another participant in the dispute refutes the idea of the existence of organic life on Mars; referring to the considerations developed by the astronomer Antoniadi and others, he argues that there are no correct "channels" on the surface of Mars and that the "channels" of Schiaparelli and Lovel turned out to be incorrect regular thin lines forming a geometric network that could be created only by the labor of intelligent living beings, but by rows of spots of different widths and different lengths, separated from each other by different distances. From here he concludes that organic life does not exist on Mars.

Reasoning is also an example of a substitution of a proven thesis. The mistake here is that a rebuttal of *evidence is* mistaken for a rebuttal *the thesis itself*.

In disputes over the origin of plant and animal species, the opponents of the theory of development in natural science have constantly made the mistake of substituting the proven thesis. The main thesis of Darwinism on this issue is the assertion that all species of plants and animals developed naturally from one or more of the original forms of organisms. This situation was proved by Darwin and his followers, relying on the facts of random changes in organisms, on the survival of the fittest and on the laws of heredity. Opponents of the doctrine of the natural | the origin of the species has repeatedly tried to refute this doctrine, denying Darwin's thought about the development of organisms from random changes fixed by natural selection and transmitted according to the laws of heredity. But they didn't notice that fall into the mistake of substituting a proven thesis. In fact, even if it turned out that the facts indicated by Darwin (random variations, natural selection, heredity), taken separately, are themselves insufficient to explain the development of organisms, proof of their insufficiency for this purpose, of course, there is not yet evidence of the invalidity of the Darwinist thesis, which consists in the correct assertion that all plant and animal species did not exist primordially, but arose and developed naturally from one or several of the original forms. And here the proved thesis is replaced by another, which is presented as the one that must be proved. by themselves, they are still insufficient to explain with their help the development of organisms, proof of their insufficiency for this purpose, of course, there is still no evidence of the bankruptcy of the Darwinist thesis, which consists in the correct assertion that all plant and animal species did not exist primordially, but arose and developed naturally from one or several of the original forms. And here the thesis being proved is replaced by another, which is presented as the one that must be proved. by themselves, they are still insufficient to explain with their help the development of organisms, proof of their insufficiency for this purpose, of course, there is still no evidence of the bankruptcy of the Darwinist thesis, which consists in the correct assertion that all plant and animal species did not exist primordially, but arose and developed naturally from one or several of the original forms. And here the thesis being proved is replaced by another, which is presented as the one that must be proved. but they arose and developed in a natural way from one or several of the original forms. And here the proved thesis is replaced by another, which is presented as the one that must be proved. but they arose and developed in a natural way from one or several of the original forms. And here the thesis being proved is replaced by another, which is presented as the one that must be proved.

§ 43. Sometimes the substitution of a proved position with another, which continues to be accepted or passed off as being proved, goes so far that even the area itself from which the position is drawn, replacing the proved thesis, is completely alien to this thesis. This kind of substitution of the thesis being proved is called "transition to another genus" (literal translation of the Greek term "metabasis neus allo genos"). For example, wanting to prove that an act committed by a given person is ethically impeccable, instead they prove that this act is extremely smart. Here, the substitution of the thesis being proved goes so far that it really results in a "transition to another genus"; proof of the *mind* shown in the act is taken as evidence *moral* dignity of this act.

§ 44. Particularly noteworthy—in its prevalence—is that type of substitution of the thesis being proved, in which the proof of the truth (or falsity) of the situation is replaced by the proof of the merits or demerits (for example, moral) of the person who advanced the thesis. In this case, the thesis being proved is replaced by another one that affects the feelings so much that it incites the insufficiently strictly thinking reader or listener to agree with the thesis, which should have been proved, but turned out to be unproven in practice, since a different situation was proved instead.

For example, instead of proving the truth of a theory, they prove that the author of this theory is a moral good person. This technique is designed for the fact that, citing the moral reputation of the author, they will thereby favour his theory.

Such argumentation, replacing the assessment of a theory (case) with the assessment of the author (person), received the special name "argumentum ad hominem" ("the argument about a person").

The lower the level of logical culture and the logical discipline of a person's thinking, the less able he is to separate the probative power of arguments from the feelings, sympathies and prejudices that they try to inspire him with, the easier this person can succumb to the action of "argumentum ad hominem".

Therefore, in all kinds of disputes, disputes, proofs and refutations, not only for the participants, but also for all those present, it is very important to maintain complete self—control and, tracing the course of evidence, not letting your feelings carry you away so as not to notice the substitution of the thesis being proved to others.

Errors In the Basis of Evidence

§ 45. Errors of the second kind, possible in the proofs, follow *from errors in the grounds* . There are *three* main varieties of these errors.

The first of them is that for proving a well—known thesis, a deliberately false position or one whose falsehood can be proved is used as a basis. For example, based on the premise that all metals are drowning in water and that potassium is a metal, they conclude that potassium is drowning in water. In concluding this, one of the grounds — the assertion that all metals are drowning in water — is a deliberately false proposition, and therefore the conclusion is erroneous. The mistake of using a false base is called a *primary false* or a lie in the starting position of the evidence.

§ 46. Some particular cases of false ground error deserve — in their prevalence — special attention. This is, firstly, a mistake in the fact that a position that is true only *under a certain condition* or in a certain sense is used as a basis, while in proof this foundation is considered as true *in general, certainly, without reference, without any restrictions*.

An example of this mistake can be Lenin ridiculed the reasoning of the economist and philosopher S. N. Bulgakov in his book “Capitalism and Agriculture”. Proving that an increase in the number and size of large farms means *decline* agriculture, Bulgakov as the basis of his evidence referred to the fact that under certain conditions, a decrease in the area of the economy leads to an increase in its productivity. ”You see,” Lenin wrote on this subject, “as our” scientist “remarkably logically argues: *since a* decrease in the area of the economy *sometimes* means , upon intensification, an increase in production, *so* an increase in the number and area of latifundia should *generally* mean a decline!” [1] . The full name of this error is an *erroneous conclusion from what was said under a certain condition to what was said unconditionally* .

§ 47. Another special case of a false ground error is to use a ground by means of which not only the position to be proved can be proved, but another — obviously false position. Being false, this last statement refutes — by means of reductio ad absurdum—the statement of proof as well. For example, they want to prove the law of conservation of energy, relying on the basis according to which no increase or decrease can be obtained with any change. But the foundation is false. Agreeing with him, I would have to accept that the concept of growth and decrease in general contains a logical contradiction, that is, it makes no sense.

A mistake of this kind is called a mistake of “excessive evidence.” They say about a person falling into this error: “whoever proves too much does not prove anything.”

Usually the source of the error of excessive proof is the desire to obtain a conclusion without fail from *general* premises, since this community seems to be the most impressive. But if the premise taken in such a general form is false, then its falsity can easily be detected by reference to conflicting cases.

§ 48. In some cases, for an error of excessive evidence, they accept evidence that in fact does not contain this error. Such are the proofs, as a result of which a justification is obtained not only of the thesis to be proved, but also of some other thesis. For example, the proof of the Pythagorean theorem, developed in the “Beginnings” of Euclid, proves not only that the square constructed on the hypotenuse, in the end, equals the sum of the squares built on the legs. In addition to this provision, the evidence establishes which part of the total result is the square built on each of the legs separately.

This is the *first* a group of varieties of false ground error. All errors of this group are united by one attribute: in all the evidence where these errors are contained, the basis is knowingly false.

§ 49. *The second* group of errors in the grounds consists in using such a reason, which, although it is not knowingly false, cannot be considered indisputable and can only be accepted or presented as indisputable. The Latin name for this error is “petitio rinsirii”, that is, “anticipation of the foundation”. This title shows that for real proof of the thesis put forward, a different basis is required, and not that which was anticipated as the basis, as if justifying the conclusion, but which in fact does not justify it.

So at the beginning of the last century, some scientists tried to prove that substances such as urea, for example, could not be obtained in the laboratory by artificial means. The mistake in the reasoning of these scientists was the mistake of petitio prinsirii: they proceeded, as from an indisputable basis, from an unproven at that time (and subsequently turned out to be false) position that products produced in organisms could not be obtained by laboratory methods.

A special kind of mistake petitio rincipii is formed by the error that the statement regarding a group of objects is true only if this group is considered as a whole, it is presented, without any verification, as a basis true for each of these objects separately. Or, on the contrary, with respect to a group of objects considered as a whole, without any verification, what is true is true only of each of these objects separately, as truth.

For example, it would be a mistake if, from the statement “mushrooms are found in shady places”, we concluded that champignon mushroom also occurs in shady places. As you know, this mushroom often comes across in unshaded wastelands. The mistake here is that a statement that is true only with respect to the group as a whole, we, without proper verification, were also recognized as true with respect to each object of the group.

Reverse example: one thread is easy to break with your hands. But it would be a mistake to conclude from this that a hundred threads woven into fabric can also be easily broken by hands. The mistake here is that the position that is true in relation to a single object of the group is taken as true in relation to the group as a whole.

§ 50. We examined a group of *deliberately false* errors grounds and a group of errors of *doubtful* (unproven) grounds with their main varieties.

The third group of errors in the basis is that as a basis, a position is used which, although it was previously proved, was proved using the same basis. In this case, the position X is proved with the help of Y, which in turn was previously proved with the help of the position X. This error is called the “circle in the proof”, in Latin “circulus in demonstrando”.

The circle in the evidence immediately catches your eye, if the argument is short. But in evidence consisting of long chains of inferences, the “circle” can easily go unnoticed.

Even the most profound philosophers sometimes did not notice the error of the “circle” if the evidence in which this error was present was long enough and if the position justified by this proof was one of those whose proof was considered especially important and desirable.

Errors in Argumentation by Which the Thesis is Proved

§ 51. In addition to errors in relation to the thesis being proved, and in addition to errors in the foundation during the proofs, a *third* type of error is possible : these are *errors in reasoning* or in argumentation, by means of which a transition is made from the grounds to the proved thesis or conclusion.

The error in the argumentation may consist, *firstly* , in the fact that the thesis being proved is simply expressed after the grounds advanced, but in fact does not follow from these bases at all, that is, it does not have any logical connection with them.

This error is often found in the thinking of people who are at a low level of mental development, or in the thinking of people who are careless, unable to concentrate closely on the logical connection of thoughts in reasoning. But even at higher stages of the development of science, this error is possible where the requirement of a strictly necessary logical connection between the foundations and the conclusion has not yet been sufficiently determined. So the history of geometry shows that it was precisely the most elementary theorems of this science that were proved by ancient geometers with the least accuracy, since at that time the requirement of an impeccably strict connection between the foundations and the conclusion was not realised.

§ 52. *Secondly*, the error in the argumentation may consist in the fact that the thesis being proved, although it does not join the grounds without any relation to them, is deduced from the grounds, but is deduced from them by *erroneous* inference.

There is no need to consider in detail all kinds of erroneous conclusions. They have already been examined by us in the chapters on inference and induction. Any violation of the syllogism rules or induction methods already known to us and set forth in these chapters leads to an error in reasoning.

We only note that the errors in the argumentation differ depending on whether we are dealing with *reliable* conclusions (as is the case in syllogisms) or with *probable* conclusions (as is the case in inductive conclusions).

Of the errors in reasoning that are possible in reliable conclusions, the error of “quadrupling terms” (quaternio terminorum) is often encountered. As you know, this mistake lies in the fact that one of the terms of the syllogism (most often—although not necessarily—average) is only *apparently* the same, but in reality it is thought each time with a slightly different, non—identical content.

The mistake of quadrupling terms consists in violating the logical law of identity: appearing again in thought, the term is thought and understood not in the same sense, its identity is violated.

An important way to avoid quadrupling terms is to accurately define all the basic concepts that make up a given argument or proof. The futility of many disputes lies precisely in the fact that those who argue only imagine that they have the same subject in mind. In fact, using the same terms, they put in them a slightly different non—identical content.

§ 53. One of the most important sources of quadrupling terms is the *inaccuracy of the* language. In every highly developed and rich language, there are many homonyms, that is, the *same words* used to express *not quite the same* , but often *completely different thoughts*.

For the proof, those homonyms in which the meanings are clearly different from each other do not pose a danger, refer to completely different, remote from each other areas of phenomena. For example, the term “declination” has several meanings: *grammatical* (declension of names in cases), *physical* (displacement of the magnetic arrow depending on proximity to the magnetic pole), *astronomical* (distance of the star from the celestial equator). Since all these meanings are too different and too obviously relate to different areas of reality and cognition, their displacement or identification in one term, of course, is impossible.

But there are homonyms in which the meanings, although different, however, belong to the same field of phenomena. In these cases, the risk of quadrupling the terms of terms increases significantly, since essentially the same relations are associated with the same content, which are no different in the language. For example, *healthy* , according to the main meaning, is called primarily the body ("healthy heart", "healthy hand"). But healthy is also called in relation to the body and all that *supports the* health of the body ("healthy air", "healthy walking"), and all that this health is *restored* ("healthy medicine"), and even all that having no direct effect on health is a *sign of* health ("healthy complexion").

Such homonyms, called *relationship homonyms*, are often a source of quadruple term errors.

§ 54. In cases where the reason for the quadrupling of terms is homonyms, the elimination of the error is achieved by clarifying the different meanings in which the same term is applied. To do this, it is useful to *contrast the* various uses of the word.

Sometimes the ambiguity of a word clearly appears when trying to translate this word into another language, in which for each of the different meanings there is a special word. Who, for example, translates the Russian word "cage" into English, cannot fail to notice that in some cases the word means what is transmitted through the English word "cage" ("cage for animals"), and in others — through the word "chest" ("chest"), in the third — through the word "cell" ("cell in the biological sense"). In order to understand in what sense this word is used in each individual case, it is necessary to carefully look at the meaning of the statement as a whole or, as they say, the *context*.

§ 55. Another source of quadruple term errors is *synonyms*. So are called different verbal expressions of the same thought.

Since the forms of the language are inseparable from the content that is expressed through these forms, any attempt to convey the same content using different verbal expressions leads in the end to the fact that not exactly the same meaning is transmitted: some part of the transmitted content is lost, and, conversely, some new part is added to the transmitted content, which is absent in the original meaning.

This property of language and verbal expressions is particularly clear when translating from one language to another. So, the word “truth” in Russian expresses the property of a true thought to speak of what is, that is, of what really exists. In Latin, “truth” is conveyed by the word “veritas”, which expresses the property of a true thought to say that is trustworthy. In Greek, the concept of “truth” is conveyed by the word “alethea”, which indicates the property of a true thought to say that it cannot be forgotten, or about an unforgettable, unforgettable, etc. But all three of these meanings express the same concept — the concept of “truth.” It is quite obvious that here is not only *identity* , but also the *difference* in the identity itself.

In this case, the source of all kinds of misunderstandings, ambiguities, quadrupling of terms cannot be the presence in the words of different languages of all these shades of meaning, through which the concept of truth is expressed. These various shades of meaning in the Russian, Latin, Greek words (“truth”, “veritas”, “alethea”) do not in the least prevent us from thinking through these words the very thing that is indicated by these shades — truth. The source of ambiguity and quadrupling of terms can only be such a separation of differences in thought, in which consciousness is lost that the differences denote the same subject of thought.

This is the case with any expression of thought in language. The erroneous evidence into which the error crept in unintentionally, unnoticed by the prover, is called *paralogisms*. Erroneous evidence, which is carried out with a consciousness of their fallibility and in which a violation of the rules of evidence is committed intentionally, since this violation leads to the conclusion that the prover himself is interested in readers or listeners are called *sophisms*.

The difference between paralogisms and sophisms, important from a psychological and moral point of view, does not matter for logic, since the *logical* content of errors in evidence is completely independent of how, intentionally or unintentionally, these errors were made during the proof.

§ 56. If, in the reasoning included in the evidence of the conclusions of *reliability*, the error of quadrupling the terms is often encountered, then in the reasoning included in the proof of the conclusions of *probability* , i.e., inductive conclusions, the error often consists in neglecting cases that *contradict the* generalization.

In many cases, having made a known generalization on the basis of observed facts or cases, the author of the generalization is not inclined to take into account, much less look for cases that *contradict* his generalization, which often seems to be valuable or desirable.

This mistake is extremely widespread in thinking. There is no such superstition, there is no such prejudice, for the proof of which it would be impossible to bring facts favourable to this superstition or prejudice. But these facts, and at the same time the very evidence, are devoid of any evidentiary force, since they also ignore other numerous facts that contradict the conclusion.

There is a story that illustrates this point well. One traveller who visited the coastal city and examined the cathedral there was shown a long list of people who had donated gifts and contributions to the cathedral in thanks to God for their salvation during the shipwreck. The traveller asked: where are the lists of those who also vowed the same donation, but, despite the vow, died. This traveller correctly revealed the main error of the inductive conclusion made in this case: ignoring facts that contradict the generalization.

Tasks

Determine the logical type of evidence below. If there is logical error in this evidence, indicate which ones.

1) *Theorem* . If in $\triangle ABC$ *the* angles *ABC* and *DIA* are equal, then the sides of *AC* and *AB* opposite to these angles are equal.

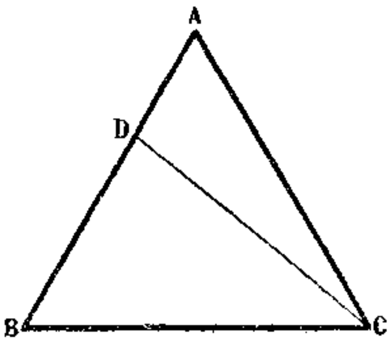


Fig. 70

Evidence. Suppose that the sides of *AC* and *AB* *are* not equal. Then one of them, for example *AB* , will be larger. Postpone on the larger side *AB* , the point *B* , the segment $BD = AC$ and join C with D. In $\triangle ABC$ and $\triangle DCB$ $BD = AC$, BC is the common side and $\angle DBC = \angle ACB$. Consequently, $\triangle DBC$ and $\triangle ABC$, as having an equal angle concluded between equal parties, are equal to each other. But DBC is partDIA. Thus, it turns out that the part is equal to its whole. But this is impossible, since it contradicts the axiom that the whole is larger than its part. It follows from this that *AC* and *AB* cannot be unequal. Therefore *AB* = *AC*.

2) Proof of the existence of emptiness:

"... Everything is not filled with substance and does not keep tightly

Cohesive from different sides: in things there is a void. ... That is why the presence of empty space is undeniable:

Without emptiness, things could not have gone anywhere at all; for what is a sign of the body: To *oppose* and *not to let go* is an eternal obstacle.

There would be things, and then nothing could advance.

For nothing, having retreated, would not give rise to movement.

In fact, in the seas, on the earth and in heavenly heights, many movements take place in front of our eyes; and do not be emptiness, then not only things could never have been in constant motion,

But even nothing could have come into the world,

For the matter would always lie squeezed everywhere.

In addition, and with all its apparent density, things.

Well, as you see now, will always be porous with the body:

So, through the stones of the caves flowing moisture of waters oozes , and they water with abundant drops everywhere;

Throughout the body of living creatures, food disperses;

Yes, and trees grow and bear fruit in due time, since food is spreading from the very roots everywhere,

Passing up the trunk and running along branches everywhere;

Sounds go through the walls of houses and closed doors, Inside flying; frost penetrates cruel to the bones.

If there weren't any voids, no matter what bodies passed, There wouldn't be, you wouldn't see such phenomena in any way.

And finally, why do we see that many things are heavier than others, no less in volume?

After all, since in a ball of wool there is as much body as there is in an ingot of lead, then it should weigh as much,

For pushing everything down is a sign of the body,

On the contrary: the void is by its nature weightless. So, if something is lighter than another of the same size, More obviously it contains the emptiness in itself. On the contrary: if something is heavier, then, therefore, more. There is a body in it, and much less empty.

So, undoubtedly, things are mixed up with what we strive to find with sensitive mind and what we call emptiness " [1]

3) Evidence that the plant nutrient moves not only toward the leaf, but also from the leaf—along the sieve vessels of the secondary cortex.

"What such a movement must exist, obviously a priori, since in the leaf is produced organic matter from which all parts of the plant are built; that it really exists is vividly proved by the following curious experience. Cut the willow branch and put it in the water. After several days or weeks have passed, a growth or sag is formed around the lower section of the branch, and roots begin to break out of this sag. These roots, obviously, should have been formed at the expense of substances obtained from the leaf or already on the road from it in the stem. We will try to determine in what way they went down to the newly formed roots ... Let us make in one branch a circular notch of the bark up to the cambium ... [1] and put our branch in the water for several weeks. Note that this time the roots will appear not at the bottom of the stem, but at the upper edge of the annular notch; obviously, by cutting the bark, we blocked the way for nutrients descending down the stem. This means that the annular cut of the bark, which does not harm the raising of the juice coming from the root, completely prevents the juice going in the opposite direction. So, the juice coming from the root goes through the wood, the juice coming from the leaves goes through the bark. Other experience is convincing in the validity of this conclusion. We select the branch of a plant on which the fruits have just begun to be tied, and cut out the ring of the bark in the place of the branch that separates the fruits from the nearest leaves — the fruits will stop developing. Thus, the annular notch of the cortex, which divides some organ, whether it is a root or a fruit with leaves that nourish it, in advance takes away from this organ the possibility of development. Therefore, there is no doubt that the nutrients used to build organs move along the cortex. But the bark, as we have seen, is a complex structure; we distinguish in it the primary and secondary cortex; Which of the two systems moves nutrient juice? We are doing an experiment with a circular notch again, but this time we carefully cut only the outer part, the primary cortex, trying not to damage the secondary, i.e. the bast part of the vascular bundles. The results are obtained, as in the first experiment, i.e., the roots are formed at the base of the branch. So, the movement takes place along the secondary cortex. Let's try to take one more step — to determine by what elements of the secondary cortex this juice will move. We know that there are mainly two of them: bast fibres and sieve vessels. Already a comparison of the forms of these two kinds of elements makes it probable that this departure belongs to the latter, since the fibres represent very thick walls and an almost complete absence of a cavity, while the sieve vessels represent wide channels communicating through open pores through which not only can pass liquid and semi—liquid substances, but even small grains of starch slip through. This likelihood turns into full certainty thanks to the following experience. We take the oleander branch and do the same with it as we did in the second experiment with the willow branch, i.e. cut the complete ring of the cortex to the cambium itself. It turns out a completely unexpected result: the roots are formed not only on the edge of the notch, but also at the base of the branch, which means nutrients enter it in some other way besides the bark. This apparent contradiction is fully clarified when we learn that the oleander stalk represents an evasion of ... the typical structure of the trunk. In addition to the sieve vessels in the cortex, he also has bundles of these elements in the core, and they, contrary to the annular notch of the cortex, carry juices to the lower part of the stem. Thus, the four simple experiments described with branches of willow and oleander, constantly, systematically limiting the range of possible assumptions, finally, with full certainty point us to the sieve vessels, as to those paths along which the so—called plastic vessels, i.e., serving for building new parts, plant nutrient " that the oleander stalk represents a deviation from ... the typical structure of the trunk. In addition to the sieve vessels in the cortex, he also has bundles of these elements in the core, and they, contrary to the annular notch of the cortex, carry juices to the lower part of the stem. Thus, the four simple experiments described with branches of willow and oleander, constantly, systematically limiting the range of possible assumptions, finally, with full certainty point us to the sieve vessels, as to those paths along which the so—called plastic vessels, i.e., serving for building new parts, plant nutrient " that the oleander stalk represents a deviation from ... the typical structure of the trunk. In addition to the sieve vessels in the cortex, he also has bundles of these elements in the core, and they, contrary to the annular notch of the cortex, carry juices to the lower part of the stem. Thus, the four simple experiments described with branches of willow and oleander, constantly, systematically limiting the range of possible assumptions, finally, with full certainty point us to the sieve vessels, as to those paths along which the so—called plastic vessels, i.e., serving for building new parts, plant nutrient “[2] .

4) The proof of the stillness of the earth, developed by the opponent of Copernicus Simplicio in the Galileo Dialogue on two systems of the world:

Copernicus gives the earth a complex, threefold movement. So that animals and humans can make various movements, they are given joints. But if complex movements without joints are possible, as in the case of the earth, then why nature, which does not do anything superfluous, gave members animals without need. But if the terms for complex movements are necessary, then the earth—a homogeneous, demonic body—cannot have such movements.

5) The proof of the impossibility of a global flood developed in fragments of Leonardo da Vinci:

“In the Bible we read that the flood in question was 40 days and 40 nights of general rain, and that this rain raised the water six cubits above the highest mountain in the world; and if rain really was universal, it would give our earth the appearance of a sphere, and on a spherical surface each part of it is equally distant from the centre of its sphere; therefore, if the sphere of water were in a similar state, it would be impossible for the water to move on it, since the water itself does not move, unless it sinks; therefore, how would the water of such a flood descend if it was proved here that it had no movement? And if she came down, how did she move, if she did not fall? There are no natural reasons here, therefore, to resolve such doubts it is necessary to call for a miracle to help or to say that the water has evaporated from the heat of the sun. “[1].

Page 28, prm. 1

From the Latin word “subject,” meaning “subject.”

Page 28, prm. 2

From the Latin word praedicatum meaning predicate.

Page 42, prm. 1

In order to avoid misunderstandings, it should be noted that the logical concept of “species” should not be confused with a zoological and botanical species. The logical concept of “view” is relative. One and the same concept can be considered both as a “species” and as a “genus,” depending on whether one considers his relation to a subordinate or subordinate concept. So, the concept of “oxygen” is specific in relation to the concept of “gas” and generic in relation to the concept of “ozone”. On the contrary, in natural science the term “species” denotes a very definite degree of kinship between organisms, so that “species” is never called “genus” here, and “genus” is called “species”.